The Astrophysical Journal

THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILLINOIS

THE CAMBRIDGE UNIVERSITY PRESS
LONDON

THE MARUZEN-KABUSHIKI-KAISHA
TOKYO, OSAKA, KYOTO, FUKUOKA, SENDAI

THE MISSION BOOK COMPANY
SHANGHAI

THE

ASTROPHYSICAL JOURNAL

An International Review of Spectroscopy and Astronomical Physics

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VOLUME L

JULY-DECEMBER, 1919

THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILLINOIS

Published
July, September, October, November, December, 1919

Composed and Printed By
The University of Chicago Press
Chicago, Illinois, U.S.A.

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ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

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THE PHYSICAL INTERPRETATION OF ALBEDO. II
SATURN'S RINGS

By LOUIS BELL

Saturn and its rings form easily the most interesting and spectacular feature of the planetary system. The rings from the time they puzzled Galileo to the present have been a favorite subject of study and speculation. Their general character was finally settled by the brilliant dynamical investigation of Maxwell and the no less brilliant spectroscopic study of Keeler, but even these left unsolved the mystery of their intimate structure and the curious phenomena from time to time recorded of them. It is the purpose of this paper to see what added information may be extorted from a study of their reflecting power.

Speaking broadly, Saturn is in itself a rather uninteresting yellowish ball with a bright and broad equatorial band shading off somewhat variously from time to time into rather dull polar caps. Its geometrical albedo is very close to 0.42, and consequently its albedo as a body following Lambert's law is very nearly 0.63. These absolute values play only a subsidiary part in the investigation which follows.

The chief feature here to be considered is that the equatorial belt is notably brighter than the rest of the planet. To account for its proportionate effect on the general brilliancy of the globe

the geometrical albedo must be at least 0.50, indicating a general reflectivity as a diffusing body of about 0.75. In other words, this particular region has approximately the reflectivity of a bright cloud. No proper photometric investigation of the relation of this to the brilliancy of the rings has yet been made. But most observers agree that the outer and brighter part of the *B* ring is both materially brighter and whiter than the bright equatorial belt. Toward the inner edge of *B* this brilliancy is somewhat reduced. The *A* ring is notably less brilliant than the *B* ring, matching the darker portions of the ball, although perceptibly brighter than these toward its inner edge near Cassini's division. The *C* ring is, as every observer knows, ordinarily extremely faint, with a brightness certainly not over a few hundredths of that of the bright part of the *B* ring. Compared with this low figure the *A* ring is immensely bright, although the gross reflectivity of its diffusing material can scarcely exceed 0.3. One has, therefore, in examining the ring system, to account for albedo varying over an enormously great range, from a point well up toward the reflecting limit of known materials down to a faintness comparable with that of black paper.

That the rings are of good diffusing material is clear from their behavior under varying angles of incidence. It is therefore pertinent to examine the limits of reflectivity of diffusing matter, since whatever light the rings return they unquestionably receive either directly from the sun or in meager measure indirectly from the planet. The old suggestion that they may shine in virtue of some occult electrical phenomena may be put aside with the intimation that light from such a source would show a bright-line spectrum if it were really brilliant enough to amount to anything as compared with the reflected light.

It is well known that all substances when finely powdered reflect light diffusely, and sometimes remarkably well. Even some minerals of very dark color show a whitish streak when abrasion powders the surface, and, very generally, colored substances when more and more finely powdered grow lighter in color. This is due chiefly to the fact that the color of any body, be it powdered or solid, is produced by multiple selective reflection from particle to particle in the surface. The finer and more smoothly laid the

particles the less coloration, and also the less modification of hue by shadowing, facts easily seen when one presses flat with a piece of glass a loose surface of dry pigment, like ochre. Hence a smooth flat surface of compressed fine powder shows higher total reflectivity than the same material in any looser aggregation, as Nutting[1] has shown.

Finally, when the dimensions of the particle are small compared with the wave-length of light, the well-known phenomenon of selective scattering appears, as Rayleigh[2] long ago showed, which is as the inverse fourth power of the wave-length, hence highly selective for the blue, violet, and ultra-violet.

When, therefore, a beam of white light strikes a surface of optical dust, meaning thereby matter so finely divided that its reflecting power must be considered with respect to the wave-length of light, whether in free space or confined upon a surface, the amount and character of the light diffusely reflected depend both on the material and on its state of subdivision. A minute amount of the incident energy is spent in producing acceleration of the particles, a portion is selectively absorbed in single or multiple reflections, and if the dust is fine enough a considerable part of the beam is selectively scattered. No substance is wholly non-selective in its reflection, and when finely subdivided every substance is selective in its scattering and always in a direction to lower its luminosity through the λ^{-4} law. Even when the particles are of dimensions to scatter all visible wave-lengths very perceptibly and so produce a whitish ensemble, there is still lack of luminosity as compared with ordinary reflection from fairly non-selective surfaces. More generally the material is selective, and, as soon as it is in a state of aggregation in which ordinary diffuse reflection is predominant over scattering, color appears and the luminosity rapidly decreases. The change due to aggregation can be well seen in smoke or any of the usual experiments in scattering, and under suitable conditions the regions of blue, whitish, and heavily colored material can be separated.

Assuming that a mass of dust is dense enough to transmit no light directly, it will therefore have its highest coefficient of diffuse

[1] *Transactions Illuminating Engineering Society*, **9**, 593.

[2] *Philosophical Magazine*, **41**, 107, 274.

reflection when it is intrinsically fairly non-selective, coarse enough not to produce much scattering effect and presenting as nearly a smooth surface as possible, that there may be the minimum amount of multiple reflection. Hence no dust clouds or loosely aggregated surface can give the same reflective power as a smooth layer of the same material. Table I gives a list of such coefficients of reflection, mainly as determined by Nutting.[1]

TABLE I

COEFFICIENTS OF REFLECTION

Magnesium carbonate	89.4
Magnesium oxide, dry	88.1
Calcium carbonate, smooth	87.9
Calcium carbonate, brushed	86.1
Calcium carbonate, rough	83.4
Aluminum oxide	87.4
Borax	85.2
Snow, fresh, fine	84.7
Sodium chloride	81.9
Snow (Zöllner)	78.3
Cloud (Abbot)	65.0

The method used by Nutting leaves very little to be desired in point of precision, and one must regard the occasional figures given a few per cent higher as due to rough methods of testing. Most known terrestrial substances are noticeably colored and have even under the most favorable conditions much lower coefficients than these.

Therefore, while optical dust in clouds or layers may be more luminous than a rough block of the same material, there is not the slightest reason to suppose that its reflectivity can reach materially above 85 per cent, and, even so, only when the substance is solidly "white" and not in the form of any loose aggregation, cloud or otherwise.

Notably colored substances have reflectivities considerably lower than these mentioned and when massive, rather than in a smooth homogeneous layer, very much lower. Ordinary rocks, even when visibly white, do not come anywhere near the values just given. Table II shows the ordinary range which may be expected of them.

[1] *Transactions Illuminating Engineering Society*, **9**, 593.

TABLE II

Substance	Albedo	Authority
Pumice	0.56	Wilsing and Scheiner
Yellow sandstone	.38	Wilsing and Scheiner
White quartzite	.25	Lowell
Whitish sandstone	.237	Zöllner
Clay shale	.16	Lowell
Marl	.156	Zöllner
Quartz porphyry	.108	Zöllner
Weathered sandstone	.10	J. Herschel
Gray syenite	.078	Zöllner

These, like the others, are specific reflectivities at small angles of incidence and consequently correspond to Lambert's albedo for diffusing material. In brief, the data on reflectivities show that no plausible terrestrial substance when in massive form can give anything like the reflectivity required to account for the observed brilliancy of the bright part of the *B* ring of Saturn. Assuming it to be a first-class diffuser, one therefore has to conclude that, whatever the structure of this annulus may be, the only terrestrial substances which can reasonably account for its albedo are of very light-colored matter finely subdivided so as to give the effect of an exceptionally white cloud. No mere collocation of satellites or similar small bodies, even if closely enough mustered entirely to prevent the transmission of light, can be counted upon to give the albedo approximating 0.80 required in this case. Nor indeed is such a collocation permissible, as we shall presently see, for dynamical reasons. In the case of the *A* ring, with the exception of its brightest portion, the reflectivity falls within possible limits, except for the reasons just stated. With the *C* ring the difficulty, as we shall later see, is not to account for a high albedo, but for one extraordinarily low considering even the well-known transparency of this structure.

The nature of the reflecting matter in the rings is beautifully disclosed by Wood's photographs in monochromatic light.[1] The yellow plate of his series shows the rings much as they appear visually, but in the violet and ultra-violet plates the *A* ring was very nearly as bright as *B*, and both notably brighter than any

[1] *Astrophysical Journal*, **43**, 310, 1916.

part of the ball, while in the ultra-violet plate the relative brightness of *A* is very conspicuous and Cassini's division is nearly obliterated. The *C* ring is almost unchanged, but shows a very slight brightening, which Wood suggested may be due to dust. In fact extremely fine optical dust is the only known material which is strongly selective for the violet and ultra-violet. The few substances which are fairly good reflectors for the ultra-violet are much better for the bright part of the spectrum, while from the λ^{-4} relation extremely fine dust would scatter more than seven times as much light in the 350 $\mu\mu$ to 300 $\mu\mu$ region as in the yellow. Considering the relative weakness of the incident ultra-violet radiation, selectivity of the order of magnitude necessary to account for the differences shown in Wood's photographs is not to be found in any massive substance. One therefore has to conclude that, whatever the general structure of the *A* ring from the visual standpoint, it is charged with the finest optical dust, which is also present about the *B* ring and permeates Cassini's division as well—visible evidence of the grinding process from collisions which makes up the history of the rings. The absence of any large quantity of the same dust in the *C* ring bears witness to its comparatively loose structure.

ISOPHOTAL LINES OF THE RINGS

The distribution of light in the rings is difficult to estimate visually, but photographically it is easy. The denser parts of a negative, however delicately graded, print white for a short exposure. More time drives through the detail preserved in the denser regions, while obliterating half-tones in the weaker regions in a common blackening. Hence if one starts with a well-timed and graded negative, prints of progressively lengthened exposure disclose the sequence of brightness in the various regions of the original very definitely, although it may be recognizable with difficulty in the primary image.

One can thus obtain for any photograph a set of isophotal lines representing the distribution of density in the original, a process which might be made of considerable service in the study of nebulae. Figs. 1, 2, 3, and 4 of Plate I are examples made from an enlargement of Barnard's Mount Wilson plate of November 19, 1911. The

PLATE I

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Albedo of Saturn's Rings

writer could not conveniently lay his hand on a complete positive from this plate, but one from the same negative, containing part of the images, was available. This contained six images with major axis about 13.5 mm. The first thing notable is that in each image the ball is distinctly seen through the *A* ring, a fact pointed out by Hepburn[1] in measuring a positive from another of Barnard's photographs of November 19, 1911. In the plate examined by the writer the difference in contrast was small, only 2 or 3 per cent, but unmistakably showing that the albedo of the planet's ball at the point of crossing was sufficient to add something of brightness even in spite of the shadow of the ring. The same phenomenon is shown in some beautiful negatives of Saturn taken by Lowell in 1911, 1912, and 1916. Visually this small contrast seems to be spoiled by irradiation.

Fig. 1 shows *A* and *B* well, but the original exposure suppressed *C*. *A* is merely slightly narrower than usual. Fig. 2 shows *A* as a mere ghost, only its more brilliant annulus showing at all. Ring *B* in this figure is narrowed about 20 per cent. Fig. 3 shows *A* obliterated and *B* still further narrowed, and Fig. 4 discloses only a shred of *B*, the most brilliant part. In carrying out this printing process for isophotal lines the outer edge of *A* shrinks rapidly, accompanied by a widening of Cassini's division at the expense of both *A* and *B*, the latter rather the less, and a slow expansion of the inner diameter of *B*. Cassini's division in all photographs, in fact, is abnormally wide, about double the visual figure, showing the tenuity of the reflecting matter at its margins. It is as if the sweeping action of the satellites were somewhat slovenly, leaving careless margins and dust scattered in the path. Fig. 5 shows the situation to scale. The faint solid lines upon ring *A* mark the residual bright annulus. The thin solid line just within Cassini's division is the outer edge of the bright annulus of *B*. The broken line within is the inner edge of this residual, and the broken, and thin solid, lines near the inner rim of *B* show a late and early stage of the progressive disappearance of the light. These facts show plainly the cause of the discrepancy as between such measurements as Hepburn's, however carefully made, and the visual figures. The

[1] *Monthly Notices*, **74**, 721, 1914.

structure disclosed, then, is a very tenuous outer edge of *A*, gradually and then rapidly increased optical density up to a point just outside of Cassini's division. Within this is a faint portion, a similar faint portion on the extreme exterior of *B* followed by an annulus of great density fading out at first slowly and then more rapidly toward the inner edge of B. Within this lies the tenuous ring *C* fairly dense at its outer rim and fading into transparency at the inner.

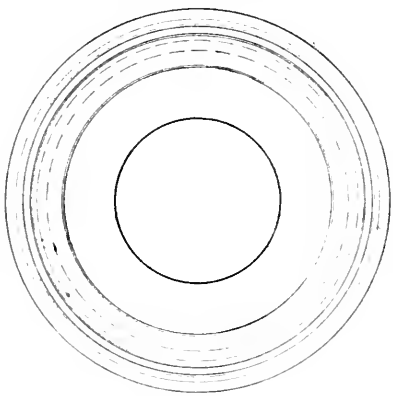


FIG. 5

The observations of Barnard,[1] Lowell,[2] and earlier of Bond[3] show that when the rings are seen nearly edge-on they become invisible save for certain condensations at two points, the outer having its center just inside Cassini's division substantially at the location of the residual annulus of Fig. 5 and extending outward just across Cassini's division. The inner has its maximum of light nearly midway of *C* and fades out in both directions. The several observers do not agree precisely, although substantially, as might be expected in viewing so faint and difficult objects. All agree in their nebulous appearance (Barnard describes it as pale gray) and apparent thickness when the main line of the rings was beyond vision. This thickness cannot be wholly charged to irradiation, which is a function of brightness certainly not conspicuous in mere nebulous ghosts of lines. One is therefore forced to the conclusion that whatever difference of density there may be there is also a heaping up of scattered dust which reflects the light. Such heaping also is indicated in some of Trouvelot's observations and by others, particularly Pratt,[4] in observing the shadow of the ball on the rings. The observations, like numberless others regarding striations on

[1] *Monthly Notices*, 68, 346, 1908.

[2] *Lowell Observatory Bulletin*, No. 2.

[3] *Harvard Annals*, I.

[4] *Monthly Notices*, 44, 411, 1884.

the rings, and particularly concerning Encke's division, make it clear that the ring structure is far from invariability.

THICKNESS OF THE RINGS

Save for the condensations, the rings when fairly edge-on are beyond the reach of all but the very largest telescopes and of doubtful visibility in these. Since the matter of the rings is equally reflective at all angles, an edge-on view with a deeper mass of reflecting matter in the line of sight should be at least as bright as in any other aspect. Hence the disappearance of the rings indicates that the thickness is too small, at their effective albedo, to render them visible as linear bright objects.

The *minimum visible* for a bright line on a dark background is a quantity roughly ascertainable. The experiments of W. H. Pickering at Arequipa with markings on artificial disks[1] show that in general a line of eight or ten times its breadth can be recognized as easily as a round spot of the same total area. These were black markings on a white disk, by daylight, through 18 km of air. The experiments of Aubert[2] half a century ago showed that, as between black on white and white on black, the latter was the more easily visible, so that for a bright line on the background of a dark sky the visibility-ratio should be more favorable.

Next, one can turn to the dimensions of celestial objects actually visible. The smaller satellites furnish examples having albedo of the order to be ascribed to Saturn's ring structure. From photometric data W. H. Pickering[3] shows that Rhea does not exceed $0''.1$ in diameter, i.e., about 600 km, while Hyperion has a diameter of about one-fourth this, i.e., $0''.025$, 150 km. On this basis a line of similar brightness should be visible down to 15 km. Deimos and Phobos from the photometric data have diameters of the order of 10 km, Deimos probably less, approximately $0''.02$, indicating again visibility of a bright line at the distance of Mars not exceeding 1.0 km, and at the distance of Saturn not over 15 km. Other objects, e.g., the smaller visible asteroids, lead to similar results.

[1] *Harvard Annals*, **32**, 144.

[2] *Physiologie der Netzhaut.*

[3] *Harvard Annals*, **61**, 85.

but these examples are taken as not excessively difficult for fairly large telescopes.

Another line of attack is through the visibility of thin lines to the naked eye, having regard to the loss of visibility in telescopic vision. Lowell[1] records experiments on the visibility of a dark wire against the sky and finds that it was plainly perceptible down to an angular diameter of 1″ and with some difficulty down to 0″.7. A rusty iron wire against the sky does not present a very favorable contrast, and visibility increases with contrast very notably.[2] At the distance of Mars, Lowell reckoned that, taking into account the losses in telescopic vision, he could detect lines down to less than 1 km, which at Saturn's distance would be substantially 14 km, and in general this would be a safe estimate for a bright line against dark sky in good seeing. Earlier, Barnard[3] records the observation of a dark wire against the sky, subtending an angle of but 0″.44, which makes the case still stronger. The writer has obtained figures down to 0″.46 for white thread against black paper. From these data it seems highly probable that the substantial layer of Saturn's rings does not exceed 15 km in thickness. This value, 0″.0025, concurs closely with Russell's suggestion of 0″.003, made on a different basis (*Astrophysical Journal*, **27**, 233, 1908).

The condensations appear much thicker but a great deal less brilliant than a genuine edge-reflection at anything like the normal albedo of the rings. Barnard[4] records their apparent thickness as a few tenths of a second, perhaps up to 0″.5. As nearly all observations on them were made when the rings still presented a minute angle of obliquity the real thickness is doubtless materially less, but there is clear evidence of annular heaping up of matter differing from that in the central ring body. Light, from whatever source, cannot be reflected from a void, and since the brightness of the condensations is from Russell's[5] figures less than 1 per cent that of a satellite, the albedo of the condensation as a whole is extremely

[1] *Lowell Observatory Bulletin*, No. 2.

[2] Aubert, *loc. cit.*

[3] *Popular Astronomy*, **5**, 2, 1897.

[4] *Loc. cit.*

[5] *Loc. cit.*

small and the matter so scattered that it becomes visible only at extreme obliquity, like a layer of mist, invisible save when seen edgewise.

The inner condensations indicate such a mass of diffused matter surrounding the central plane of ring *C*, and this would account for the repeated observations, especially from its shadow, that ring *C* is thicker than the general plane, e.g., Wray.[1] These facts lead to a cross-section of the ring system approximating Fig. 6, including all the space occupied by visible matter irrespective of its character, on the scale of $1'' = 10$ mm horizontally and somewhat exaggerated vertically.

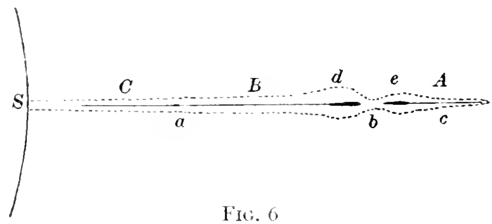


FIG. 6

The situation presented is that of a very thin, nearly plane ring of relatively dense matter with a permanent discontinuity at Cassini's division (*b*) and possibly temporary ones at *a* and *c*, Encke's division, and varying in density rather than thickness, save perhaps at the points *a*, *d*, and *e*, where increased density is likely to lead to more frequent collisions and departures from the average ring plane. Surrounding this much thicker stratum of extremely tenuous matter there are fine dust and detritus from collisions in the ring proper, somewhat more dense and extensive at *d* and *e*, where the grinding chiefly goes on, greatly and more uniformly scattered about ring *C*, where collisions are rare and particles can pass more freely out of the ring plane. Note that the thicker portions reflect visibly only at very oblique incidence.

SOURCE OF LIGHT IN CONDENSATIONS

With rings nearly edge-on the outlying matter of *C* receives sunlight directly in one aspect and by penetration of the tenuous ring in the other. In either case there is back reflection which, as will be presently seen, may be of very perceptible amount. Such is

[1] *Monthly Notices*, 23, 86.

the explanation offered by Barnard[1] and concurred in by W. H. Wright.[2] Bond postulated reflection from the inner edge of *C*, a condition demanding far too great density of matter there. Russell[3] shows in addition that the light reflected to the ring by the ball of Saturn is sufficient to be a perceptible quantity and doubtless this co-operates with the direct sunlight reflected in the ring haze. The sources must evidently coact and their summation is what we see. Barnard did not catch the condensation when the earth was slightly on the sunlit side of the ring, and hence laid aside the idea of a thickened mass. He shows, however, the condensations extending on both sides of the faint edge line of the ring, and they seem to have been so seen by Lowell[4] both as luminous objects and in the shadow on the ball. But neither observer could see any trace of them from the sunlit side.

Bond[5] in 1848 saw the same objects only from the shadowed side of the ring, but changing sides with reference to the central plane of the rings as the earth passed from one aspect to the other, a dissymmetry shown in some of his drawings. Wray,[6] on the other hand, saw a nebulous band of light about the *C* ring on both sides of the line of the edge reflection. His description leaves small chance for any effect of irradiation as the cause, which indeed is altogether unlikely in so faint an object. The outer condensation escaped him, though both sets were contemporaneously seen by Otto Struve.[7]

The weight of the evidence points to outlying regions of very much scattered matter lying chiefly about the *C* ring and on each side of Cassini's division. The latter piles furnish a simple explanation of the outer condensations. Cassini's division is not entirely clear of matter, as many observers have noted, e.g., Secchi; the adjacent piles are anything but dense at the edge, and there is ample opportunity for the sunlight not only to be reflected from the masses of haze but to stream through Cassini's division and light

[1] *Loc. cit.*

[2] *Astrophysical Journal*, **27**, 363, 1908.

[3] *Ibid.*, 230, 1908.

[4] *Loc. cit.*

[5] *Harvard Annals*, **1**.

[6] *Monthly Notices*, **23**, 85.

[7] *Ibid.*, **32**, 86.

them up all down the line. The situation seems to combine the effect of deep reflection of the penetrating light to the shadowed side, with inner edge reflection, not as Bond pictured it, but again as deep reflection from a stratum of mist seen edgewise. The double inner condensations seen by Aitken[1] hint at lessened density at the point *a*, such as was earlier noted by Secchi. That the condensations disappeared when the earth was exactly in the plane of the rings[2] is to be expected in view of the great extent of feebly reflecting optical dust, some 15,000 km wide, drifting about ring *A*, interposed between the observer and any point at which perceptible light could stream through. The faint condensations could be seen only in a fairly unobstructed line of sight. Then, as Barnard[3] notes, the great obliquity gives by perspective enhanced apparent brightness.

This implies deep reflection from a tenuous cloud of highly diffusing matter like optical dust, for neither a compact diffusing surface nor an aggregation of stony masses for which in general Lambert's law does not hold could give the increased brightness observed by Barnard and others. The larger the proportion of dust to massive matter the more striking this increase; hence the relative brilliancy of rings *A* and *C* in very oblique view.

No mere meteoric constitution of the rings can sufficiently account either for the observed albedo of their brighter portions or for the complete invisibility of the condensations from the sunlit side of the structure.

LIGHT-TRANSMISSION OF THE RINGS

Ring *C* was found by Bond to be transparent enough to show the limb of the planet through its whole extent, and, while both *A* and *B* were long considered wholly opaque, *C* is evidently only very slightly absorbing, else with any probable material it would show much higher albedo than it does. The only quantitative data on the transparency of ring *C* rest on Barnard's photometric estimates of the eclipse of Japetus by the rings, made November 1, 1889.[4]

[1] *Lick Observatory Bulletin*, No. 127.

[2] *Monthly Notices*, **69**, 623, 1909.

[3] *Loc. cit.*

[4] *Astronomy and Astrophysics*, **2**, 120, 1893.

Visual estimates were made by comparison with two other satellites in the same field, so that the conditions were fairly good for recording the change in magnitude as Japetus emerged from behind the shadow of the ball, entered the shadows of the crape ring, and then disappeared in the *B* ring. Barnard's estimate of the mean loss of light for the body of the crape ring was $0^m.412$. This means that the net transmission coefficient was 0.68 and the extinction 0.32. But the obliquity of the ring plane in this case is about 11°; hence, considering the component bodies to be distributed uniformly and roughly spherical, the total light transmitted at normal incidence would be approximately 0.94 for ring *C*. The data on the photographic transparency of ring *A* do not permit estimating it even roughly, merely showing that the part of the ball covered by the ring has albedo high enough to return enough light through the ring to give a small contrast with the light directly reflected by the rings.

There is, however, a recent observation by Ainslie[1] which gives a clue to the visual transparency of ring *A*. Ainslie observed on February 9, 1917, an occultation of the star B.D.+21°1714, of about magnitude 7 by Saturn's ring, also noted by Knight (*loc. cit.*). It was seen just grazing ring *B* in Cassini's division, along which it passed very obliquely with small diminution of brightness, and then passed behind ring *A*, through which it remained visible, but dimmed, until its emergence. Ainslie estimated that the star lost 0.75 of its brightness in its occultation, but it twice showed brightening, not to its full magnitude, once in passing the approximate place of Encke's division and again slightly farther out, the changes in brightness being rapid but not instantaneous. Clearly Encke's division, like Cassini's, is not abrupt and contains reflecting material. Taking the transmission of the ring, then, at about 0.25, the transmission at normal incidence should be from the ring angle of about 26° somewhere about 0.6, a figure fairly consistent with the albedo of a mass containing considerable optical dust.

As to ring *B*, there is no direct evidence of transparency, although no occultation of a fairly bright star has yet been observed. In view of the faintness of the inner part of *B* sometimes observed,

[1] *Monthly Notices*, 77, 456, 1917.

e.g., Secchi,[1] it may well be permeable to light. Perhaps at such times as shown in Secchi's curve of estimated brightness a division between B and C appears (Fig. 7), together with enhanced brightness of C.

STRUCTURE OF THE RINGS

The data cited merely show the general shape of the rings, a thin, relatively plane annulus tenuous in the C region and overlaid there with a mist of scattered dust, a denser structure nearly opaque at considerable angles of incidence in the B region and thickened heavily near its periphery by optical dust, a stable, somewhat clouded division beyond, and then the A ring thickened near its inner edge and fading off to nothing outside, overlaid by the optical dust conspicuous in its photographic albedo. Besides the Cassini division there are the somewhat unstable Encke's division in the A ring and now and then impermanent traces of others, especially in A and at the inner limit of B. On this last score Fig. 7 is conclusive. As to the general constitution of the rings, Cassini's guess, Maxwell's analysis, and Keeler's physical proof unite upon a loose structure of discrete particles, whether of satellites or sand. Telescopic evidence merely states that there are no satellites separately visible. The faint sparkling noted by Trouvelot[2] is a familiar phenomenon in physiological optics which on divers occasions deceived Sir W. Herschel in studying nebulae.

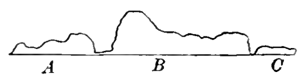


FIG. 7

Nor does Roche's famous paper,[3] more quoted than read, give any substantial help in solving the problem. Roche was dealing with the case of a fluid mass held together by its own gravitational forces, and determined the limits of its stability. In the case of Saturn he showed that the limiting distance from the planet's center was, for equal density of the two bodies, 2.44 times the radius of Saturn—a point slightly outside of the ring system. For unequal densities $r' = 2.44\sqrt[3]{\frac{d}{d'}}$, r' being the new critical radius

[1] *Monthly Notices*, 16, 50, 1856.

[2] *American Journal of Science* (3), 11, 447.

[3] *Memoirs of Montpellier Academy*, 1, 243.

and δ and δ' the densities of planet and satellite. Taking δ' at the average density of such small celestial bodies as we know, 3.6, $r' = 1.42$, reckoning the density of Saturn at 0.7. This point lies just within the inner edge of ring B. Perhaps the only conclusion to be drawn is that inside this limit there can be no large stable aggregation of particles maintained by gravitational attraction alone, but it is interesting that this limit falls just at the point which divides the tenuous from the dense. At least as bodies drew in toward the critical radius they would be subjected to gravitational disruptive forces which would facilitate their breaking up under collisions.

The relations between the light reflected and transmitted by a body of discrete particles furnish, however, some clues to the fineness of the structure concerned.

Conceive the whole mass, assumed uniform, to be divided into laminae containing on the average one layer of reflecting bodies, i.e., of thickness equal to the average separation of the bodies in space. Let T_0 be the coefficient of light-transmission by such a lamina, S_0 the space-factor of the lamina, i.e., the proportion of the area intercepted by the solid matter, and k the apparent albedo. Then for light incident at any angle for which S_0 is found or computed, the intensity I' of the light, reflected backward, neglecting terms in higher powers of k, is:

$$I' = IS_0k(1 + T_0^2 + T_0^4 + T_0^6 \;.\;.\;.\;.\;.\;.\; T_0^{2n-2})$$

while the total transmission coefficient of the structure is $T_n = T_0^n$. The foregoing series must be regularly summed term by term if the terms be few or converge rapidly. For a very large number of slowly converging terms the sum to a first approximation is, since $S_0 = 1 - T_0$, and the series is a familiar one with the odd powers absent.

$$I' = I(1 - T_n^2)\frac{k}{2}\,.$$

Unluckily the difference in I' as computed for few and many laminae is too slight to give a trustworthy value of n from photometric data alone, thus blocking an otherwise promising line of attack on the problem of structure.

Barnard's observations give for ring C, at 11° 10′ inclination, $T_n = 0.68$, and assuming k at 0.2, the average for small celestial bodies as we know them, we find for a loose structure of many laminae $I' = 0.054$ as the apparent albedo of the swarm seen from the given angle. This is little in excess of that from black paper, and is perhaps no higher than is required to make the ring visible against a sky far from devoid of light, and in the form of a narrow strip between two rather brilliant surfaces. Obviously the scattered particles about the ring, which give edgewise the faint shimmer of the inner condensation, cannot, if Russell's estimate of its intensity is near the truth, add perceptibly to the light of ring C save at very large angles of incidence.

Considering now the light transmitted through the mass and reflected downward instead of backward at a similar angle, this component, neglecting terms in k^2 and higher powers, is

$$I'' = S_0 k T_n.$$

For the case of many slowly converging terms this approaches I'. There may therefore be illumination of the shadowed side of ring C, by percolation of light, something approaching the brightness of the sunlit side at the same angle, but percolation cannot account for an increase of physical brightness, save as k increases, as it does for fine particles.

In similar fashion, taking Ainslie's rough estimate[1] of the light absorbed in ring A, we have

$$T_n = 0.25,$$

angle of ring 26°. At $k = 0.2$, as before, the apparent albedo of the swarm would be 0.0937, which is much lower than other observations indicate. To account for an albedo of 0.3, which seems approximately right, the actual reflectivity of the matter should be about

$$0.2 \frac{0.3}{0.0937} = 0.64,$$

a value much too high for any plausible massive solid but a perfectly good value for dust, of the existence of which in ring A the photographic evidence leaves no doubt. In this instance the

[1] *Loc. cit.*

brightness of the occulted star was likely to be underestimated from its bright background, leading to a larger value of T_n, which could hardly exceed 0.5, however, without bringing the derived albedo too high for dust known to scatter light selectively and hence lacking conspicuously in brightness.

Concerning ring B, it is so far as known opaque at the available angles of incidence, and hence of greater volume-density, if of material similar to rings A and C. That such is the case follows from the necessary exchange of matter due to changes of velocity, both + and −, from the inevitable collisions due to the gravitational forces. Maxwell[1] has shown the complicated compressional wave systems that arise in the plane of the rings.

Maxwell[2] also shows that the condition of stability in a ring of discrete bodies is that the volume-density shall not exceed 1/300 the density of the planet. This implies that at the very utmost ring B has a volume-density not exceeding 0.00233.

One cannot suppose that the actual density leaves the ring on the verge of complete instability, and must reckon on a moderate factor of safety. Taking this at 2, assuredly small enough, the volume-density of this B ring cannot exceed 0.00116. That is, for each cubic meter of volume there cannot be more than 1160 cc of matter of unit density, and bulk for bulk the ring space is lighter than air, as Maxwell pointed out. At the mean density of small bodies, 3.6, this volume of matter shrinks to 323 cc, equivalent to a sphere 8.5 cm in diameter for each cubic meter. The projected area of this is 57 sq. cm, and a structure thus proportioned, on whatever scale, would show at normal incidence $T_0 = 0.9943$. Ring B is substantially opaque so that one may reckon for normal incidence $T_n < 0.10$. Since $T_n = T_0^n$, one can make at least a rough estimate of n and hence of the scale of the structure supposed uniform. Thus, n is in the neighborhood of 400, i.e., a distribution of the volume-density indicated should be spread in at least 400 laminae to account for the apparent opacity of the ensemble. At the total thickness of ring already found, 15 km, the unit body would have a diameter of approximately 3 m. This figure would fall for any increase of the factor of safety in density.

[1] *Scientific Papers*, 1, 328 ff. [2] *Ibid.*, p. 338.

The fact that there is much optical dust around the brighter part of ring B makes the light transmission there chargeable to the denser structure very uncertain. A better point of attack is the outer portion of ring C, where Barnard's figures give a fair approximation to T_n, and where the structure, except in density, may be taken as comparable with the nearby portions of ring B, which are relatively free of optical dust and into which, except at occasional periods, ring C merges. Taking for example the point where the apparent transmission was 0.50, the real normal transmission amounts to $T_n = 0.903$. The density here is evidently very much less than in the neighboring opaque edge of the B ring. Taking the density-ratio as 10 in round numbers, the material volume per m^3 is 32.3 cm^3, the corresponding sphere a little below 4 cm, the projected area 12 cm^2, and *n circa* 90. This leads to a unit body slightly less than 8 m in diameter.

Similarly, taking Ainslie's rough estimate of the transmission of ring A at 26° inclination, $T_n = 0.57$; taking the volume-density at one-third that of ring B, the mass diameter per m^3 is 5.9 cm diameter, its projected area 27.3 m^2, *n circa* 200, and the unit mass of the ring about 4.3 m.

The figures depend obviously on the assumed values of the factor of safety of stability, but it seems clear that, taking Maxwell's limit as the basis, and reckoning the unit masses as for a homogeneous medium from the approximate light-transmissions, the diameters are of the same order of magnitude in the several rings and do not as a whole exceed a few meters. Indeed, unless ring B is precariously near to instability, considering its permanence, the unit masses are likely to be considerably smaller.

The point of the matter is that the observed albedos and coefficients of light-transmission are not consistent with bodies even approximating satellite dimensions. They indicate rather bodies of meteorite dimensions ranging from a few meters in diameter down to brickbats, chips, and dust, the last named being the most plentiful in the bright zones of rings A and B, where it appears to have its continuous origin in an increased volume-density of the larger masses. Very fine dust cannot remain stable under the pressure of radiation.

TOTAL MASS OF THE RINGS

Maxwell's limit at once assigns a superior limit to the mass of the rings. Leaving the *C* ring out of the reckoning and figuring the rest as uniform, at 15 km in thickness the total mass would be about 1/400,000. Taking into account the factor of safety in stability, and counting the density of ring *A* as above, with due allowance for *C* and noting that *A* fades away to the edge, it appears that the total mass of the rings can hardly exceed 1/1,000,000, and is likely to be much less.

THE RÔLE OF LIGHT-PRESSURE

If we bear in mind the considerable amount of drifting dust, which increases the albedo of Saturn's rings beyond any reasonable figure for massive matter, it is evident that the pressure of radiation may play a very perceptible part in the distribution of the dust. At the density here taken for the material of Saturn's rings and since solar gravitational force is balanced by its pressure of radiation for diameters not greatly in excess of a wave-length, for the much finer matter which takes part in the scattering of violet and ultraviolet light the repulsive force of light-pressure is relatively very much greater, and becomes a perceptible quantity even with reference to the gravitational attraction of Saturn.

With respect to the gravitational effect of Mimas, the perturbative action of which is chiefly responsible for Cassini's division, light-pressure is a very large quantity. It is clear, therefore, that Cassini's division cannot be swept clear of dust, as against the effect of the pressure of radiation in pushing it back and forth across that division in the nearer and farther parts of the ring system. To go farther, the same force tends to push down into the ring plane dust which through collision is projected out of it. Since the cosmic grinding-mill is near the outer edge of ring *B*, most dust will be generated here, and the effect of light-pressure will be to push it back in the nearer part of the orbit and out into ring *A* in the farther. Matter thus transferred will on the whole tend to be driven down into *A* rather than back again into *B*, owing to the obliquity of the rings, and the final result of this constantly though slowly acting force should be to transfer much fine optical

dust from its zone of origin outward, which furnishes one good reason for the high albedo of ring *A* for ultra-violet light. With respect to ring *C*, a certain small amount of matter must inevitably be transferred to it by light-pressure, but in the main would be driven into *B* again. The slight added brilliancy of *C* found in Wood's photographs bears evidence of the existence of such matter there, and a like origin may reasonably be sought for the considerable volume of extremely tenuous matter which seems to be the source of light in the inner condensations.

The outstanding difficulty regarding the condensations is that they have been visible only on the shadowed side of the rings, never from the sunlit side. The effect of light-pressure, however, is to keep the mobile dust-cloud always toward the shadowed side, and hence hidden by the shadow of the main ring when the inclination of the rings is very small. The same force acts to depress fine dust upon the illuminated ring surface. Obviously the dust is most free to scatter over the surface when the rings are nearly edge-on, which may account for the exterior hazy ring observed by Fournier[1] and Schaer.[2]

The relative masses and distances of Saturn and sun from the region of ring *C* indicate that the acceleration due to radiation-pressure for particles, say from μ down, is of the order of 0.1 mm per second, which amounts to nearly 350 km in a day. Hence as the sun rises above the plane of the rings there is a downward component gradually displacing the fine dust into and below the ring plane. Here it would become visible only as the angle of view permitted enough light to filter through the body of the ring to illuminate the matter below. It is unseen from above simply because it is not there. The dust mass would thus oscillate between two positions of stability determined by the attraction of Saturn, though the finer particles with relatively high acceleration would naturally find their way to the planet. There is thus strong circumstantial evidence, throughout the rings, of radiation-pressure acting freely on a very extended mist of fine particles, in ring *C* several hundred km thick and 16,000 km wide—a demonstration of Maxwell's theorem on a truly colossal scale.

[2] *Scientific Papers*, **176**, 64, 1908. [1] *Astronomische Nachrichten*, **176**, 239, 1908.

The ensemble of the ring system thus presented is a substantially plane, very thin stratum of bodies of meteoric size swept permanently clear at Cassini's division, thinned and grooved elsewhere by the perturbations of the satellites, denser near the grinding zones of action, and thinning out away from them. Permeating and overlaying this is a loose body of widely scattered dust more and more tenuous away from the ring plane, and everywhere away from it so thin a cloud that it reflects no visible light save when seen in great thicknesses edgewise. This floating spin-drift of the ether, thin as autumn haze, is billowed and wind-driven by the gravitational waves in the ring planes, and by the ever-acting pressure of radiation which sweeps the particles back and forth over the rings, and drives them down upon and through the sunlit side to be visible below when and where the light can filter through to illuminate them. It is a scene of perpetual change steadied only by the mass of the whirling ring plane itself.

Boston, Mass.
February 4, 1919

THE STARK EFFECT FOR METALS[1]

By T. TAKAMINE

In 1917 J. A. Anderson[2] developed a method suitable for examining the Stark effect for metals having a relatively high melting-point. The apparatus and methods used in the present investigation are essentially the same as employed by him, excepting a slight modification in the construction of the cathode and the grating spectrograph.

APPARATUS AND METHODS

In his experiments Anderson used a cathode having a diameter of about 13 mm. In order to attain a much stronger electric field, the diameter of the cathode, in the present work, was decreased to 2 to 5 mm.

In Fig. 1, *A*, a cylinder of the metal which is to be studied, is put into the iron frame *C*. *B* is a disk of fused silica having a hole in the center. The upper end of the cathode *A* extends about 0.3 mm above the upper surface of *B*. *D* and *E* are silica tubes ground to fit tightly together. *D* has a vertical slot, about 1 mm in width, which extends down to the level of the upper surface of *B*. The upper end of the slot was covered by a small piece of metal. *F* is the platinum wire leading down to the electrode below.

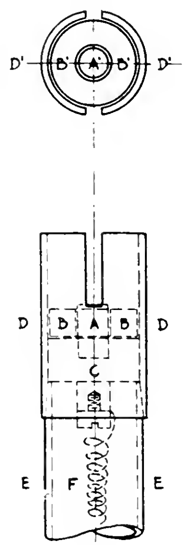


Fig. 1

The grating spectrograph is essentially the same as described by Anderson, excepting that a telescope objective of 5-foot focus, made by Brashear, was used as the collimator lens, and a Bausch & Lomb Tessar of F/4.5 aperture and 40-cm focus as the camera lens. A Voigtländer Heliar lens of 15-cm focus and a double-image prism were placed between the source and the slit of the

[1] *Contributions from the Mount Wilson Observatory*, No. 169.

[2] *Mt. Wilson Contr.* No. 134; *Astrophysical Journal*, **46**, 104, 1917.

spectrograph, so that two images of the uncovered portion of the slot appear one above the other on the slit. The bell jar containing the cathode and the anode is pumped out until the Crookes dark space has a length of 1 or 2 mm. Direct current to produce the discharge is supplied by a set of high-potential generators, each giving about 800 volts, connected in series. The number of dynamos used varied from 16 to 24. A milliammeter and a water-resistance were connected in series with the discharge tube, the current being in general between 20 and 40 milliamperes. The exposures varied considerably according to the cathode metal, ranging from a few minutes up to 2 hours. In order to avoid excessive heating of the cathode, the discharge was passed intermittently, the bell jar being kept cool by means of an electric fan.

The intensity of the electric field was determined by using Stark's data for the "Grobzerlegung" of the Balmer lines $H\beta$, $H\gamma$, and $H\delta$;[1] its maximum value at the cathode ($E_{max.}$) varied from 28,000 volt/cm to 75,000 volt/cm.

With the cathode of smaller diameter used in the present work the distribution of the electric field in the Crookes dark space was found to be parabolic, as noted by Yoshida and the author.[2]

It has already been noticed by Anderson that the spectra of metals used as the cathode are excited to greater brightness when the residual gas is either air or oxygen than when it is hydrogen. In the present work air proved to be preferable to either oxygen or hydrogen. It would seem that the presence of nitrogen acts favorably in producing bright metallic spectra.

RESULTS

The spectra of the following ten metals were examined: Ag, Au, Co, Cu, Fe, Mg, Mn, Mo, Ni, W.

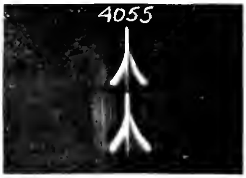
Of these, no affected lines were observed in the spectra of Mn and W. In the course of the experiments a few lines of Na, N, and O were also found to be affected by an electric field. The spectra of Co, Fe, Mn, Mo, and Ni were fairly rich in lines, while the rest of them showed a rather small number of lines. The color of the

[1] *Elektrische Spektralanalyse chemischer Atome*, 1914.

[2] *Memoirs of the College of Science* (Kyoto Imperial University), 2, 137, 1917.

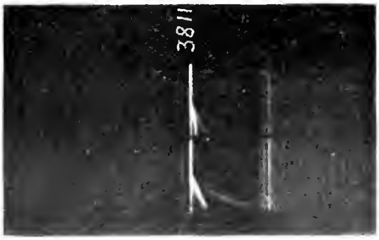
PLATE II

a



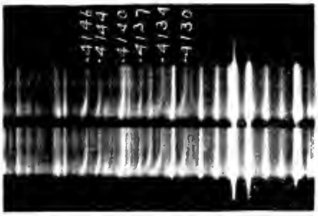
c

b



d

e



f

g

TAKAMINE—THE STARK EFFECT FOR METALS

In each illustration the *n*-components are above, the *p*-components below.

a Silver $E_{max.}$ = 48.000 volt/cm

b Silver $E_{max.}$ = 31.300 "

c Silver $E_{max.}$ = 20.700 "

d Silver $E_{max.}$ = 20.700 "

e Cobalt $E_{max.}$ = 33.300 "

f Copper $E_{max.}$ = 75.200 "

g Copper $E_{max.}$ = 33.000 "

glow in the Crookes dark space was a very bright green in the case of Ag, Cu, and Mg; fairly bright for Fe and Ni; and yellowish for Au, Co, Na, and Mo.

In the tables the first column contains the wave-lengths, the second the series numbers whenever known; the third and fourth give the displacements of the *p*- and *n*-components, respectively, with relative intensities on a scale of 1–10. The displacements were measured at the maximum electric field, $E_{max.}$, which is given in the fourth column. For some lines special remarks are given below the table.

Silver.—With silver as the cathode the brilliancy of the greenish glow in the Cookes dark space was extraordinary, and strong lines came out with a few seconds' exposure. The Stark effect for silver lines is remarkably similar to that for helium lines. Excepting the green lines, the rest of the first subordinate-series lines are spread out very widely on both sides, while those belonging to the second subordinate series are displaced in one direction only. Moreover, the former show a number of detached components just as in the case of helium. This is illustrated for the line λ 3810.85, in Plate II*a*, *b*. The only difference we notice between helium and silver is that in the case of helium all the detached components appear invariably on the violet side, while in silver they usually appear on the red side.

With respect to these detached components, it is important to note that Merton[1] has found that those belonging to helium can be arranged in a combination series. It seems very likely that the detached components of silver may have similar properties.

These detached components are not of sufficient intensity to appear in the negative glow if, indeed, they exist there at all; they appear with sufficient intensity for observation only in the electric field.

It is important to notice that there is another class of lines, first observed by Koch[2] in the helium spectrum, which shows this feature still more markedly. Their approximate wave-lengths are 4519, 4046, and 3930 A, and they begin to appear with an

[1] *Proc. Royal Soc.*, **95A**, 30, 1918.

[2] *Annalen der Physik*, **48**, 98, 1915.

TABLE I

Silver

λ	Series	*p*-Component Δλ	*p*-Component Int.	*n*-Component Δλ	*n*-Component Int.	$E_{max.}$ in Volt/cm
3682.45	IX. II6	$v_3-5.00$ $v_2-1.59$ $v_1-0.69$ 0 $r_1+4.35$ $r_2+13.00$	2 6 1 4 2 1	.... $v_2-1.84$ $v_1-0.84$ 0 $r_1+4.90$	.... 1 5 2 1	48,000
3810.85	IX, I6	$v_3-7.15$ $v_2-1.77$ $v_1-0.41$ 0 $r_1+5.21$ $r_2+11.04$	6 10 2 5 3 3	$v_3-4.78$ $v_2-1.40$ $v_1-0.43$ 0 $r_1+5.36$	3 3 10 5 4	48,000
3840.74	IIX, II5	$+0.15$	2	$+0.08$	2	48,000
3981.87	IIX, I5	$+0.13$	3	$+0.14$	3	48,000
4055.44	....	.... $v_1-5.20$ 0 $r_1+3.33$	...... 8 4 10	$v_2-4.49$ $v_1-3.21$ 0 $r_1+3.29$	2 6 3 10	56,000
4081.7	....	$+0.88$	3	$+0.86$	3	55,000
4206.8	....	$+1.46$	2	$+1.28$	2	55,000
4210.87	IX, II5	.... $v_1-3.80$ 0 $r_2+4.23$	...... 10 6 5	$v_2-3.54$ $v_1-1.91$ 0 $r_1+3.99$	3 10 6 5	56,000
4212.76	IX, I5	$v-3.47$ 0 $r+2.53$	3 5 3	-2.98 0 $+2.70$	3 5 3	56,000
4226.55	....	$+2.33$	3	$+2.14$	3	56,000
4476.31	....	$+0.04$	10	$+0.02$	10	48,000
4668.58	....	$+0.03$	10	$+0.03$	10	48,000

REMARKS

λ 3810.85 The behavior of v_2 in the *p*-component and v_1 in the *n*-component are anomalous. A companion line at +0.86 A has a violet component.

4055.44 In both the *p*- and *n*-components v_1 is detached from the main line by 0.2 A.

4081.7 The line appears only in a strong electric field.

4226.55 The line appears only in a strong electric field.

electric field considerably stronger than is required in the case of the detached components discussed above, and gain in intensity very rapidly as the field-strength is increased. According to Koch these new types of lines, which, for convenience, we may call lines of Koch's type, can be arranged in a combination series. The silver lines at λλ 4081.7 and 4206.8 are evidently of Koch's type.

The line λ 3682.45, which forms a doublet with λ 3810.85, is quite similar to the latter in its mode of decomposition. In both the *p*- and *n*-components of these two lines the violet components v_1 and v_2 show an anomalous behavior. In a weak field they are displaced approximately in proportion to the field-strength; as the field increases this law ceases to hold, and in a strong field the displacement becomes practically constant. As will be seen in Plate II*b*, the phenomenon is especially marked for v_1 of the *n*-component. The intensity of these *p*- and *n*-components is, so to speak, complementary. In fact v_1 is weak and v_2 strong in the *p*-component, while in the *n*-component exactly the opposite holds. As shown in Plate II*a*, *b*, the line λ 3810.85 is accompanied by a weaker line λ 3811.7, which has a violet component only.

The electric effect for the two neighboring lines λλ 4210.87 and 4212.76 is shown in Plate II*d*. These two lines, together with the line λ 4055.44 shown in Plate II*c*, form a doublet of the first subordinate series. The line λ 4212.76 behaves quite like λ 4055.44, while the line λ 4210.87 has its red component detached and at a considerable distance.

Gold.—As shown in Table II, only three affected lines of this element were observed.

TABLE II

GOLD

λ	*p*-Component		*n*-Component		$E_{max.}$ in Volt/cm
	Δλ	Int.	Δλ	Int.	
3796.15	−1.61	6	v_2−1.57 v_1−0.52	3 5	21,000
4084.31	+0.05	7	+0.04	7	30,000
4128.80	+0.32	4	+0.25	4	30,000

TABLE III

COBALT

λ	p-COMPONENT		n-COMPONENT		$E_{max.}$ IN VOLT/CM
	Δλ	Int.	Δλ	Int.	
3664.7	+0.11	1	....	....	33,300
3702.4	+0.04	5	....	....	33,300
3704.17	+0.05	4	....	....	33,300
3732.52	+0.05	6	+0.05	6	33,300
4045.54	+0.11	6	+0.20	5	33,300
4072.43	+0.44	5	+0.35	4	33,300
4096.11	+0.71	5	+0.47	4	33,300
4126.1	−0.84	2	−0.74	1	33,300
4128.3	+1.11	1	....	....	33,300
4129.86	−1.75	3	−1.76	2	33,300
	+1.18	2	+0.98	1	33,300
4130.62	+1.36	4	+1.11	4	33,300
4132.41	+0.20	7	+0.20	7	33,300
4133.94	+1.20	4	+0.96	4	33,300
4135.85	+1.65	5	+1.33	2	33,300
4137.52	+0.52	3	+0.41	2	33,300
4139.60	+0.64	5	+0.49	5	33,300
4140.50	+0.70	4	+0.71	3	33,300
4141.35	+1.08	1	....	....	33,300
4143.98	+0.63	5	+0.52	4	33,300
4144.52	+0.88	5	+0.65	4	33,300
4158.59	+2.06	2	....	....	33,300
4164.7	+0.87	2	....	....	33,300
4177.77	+0.53	3	+0.41	2	33,300
4198.62	−1.71	3	+1.10	2	33,300
4200.0	+1.27	2	+1.00	1	33,300
4201.6	+1.53	4	+1.37	3	33,300
4212.2	+0.78	3	+0.78	2	33,300
4215.70	+0.15	1	+0.08	1	33,300
4220.59	+0.12	2	+0.26	2	33,300
4221.21	−0.06	2	−0.05	2	33,300
4223.93	+1.25	2	+1.25	2	33,300
4307.57	−0.53	3	−0.40	2	33,300
4459.8	−0.40	4	−0.27	3	33,300
4489.1	+0.46	4	+0.60	3	33,300
4512.8	+0.70	2	+0.82	2	33,300
4514.3	−0.06	4	−0.05	4	33,300
4517.24	+0.65	2	+0.10	2	33,300
4550.8	−0.77	2	−1.02	2	33,300
4557.2	+1.23	2	+1.25	2	33,300
4591.8	−0.08	8	−0.10	8	33,300
4660.2	+0.42	2	+0.44	2	33,300
4704.57	−0.24	2	−0.25	2	33,300
5336.36	−0.10	2	−0.15	2	25,200
5418.0	−1.06	1	−0.65	1	25,200
5546.1	−0.45	5	−0.18	5	25,200
5616.5	0	4	0	4	25,200
	+0.37	1	+0.50	1	25,200
5636.15	−0.22	6	−0.17	6	25,200
5637.6	−0.68	6	−0.76	6	25,200

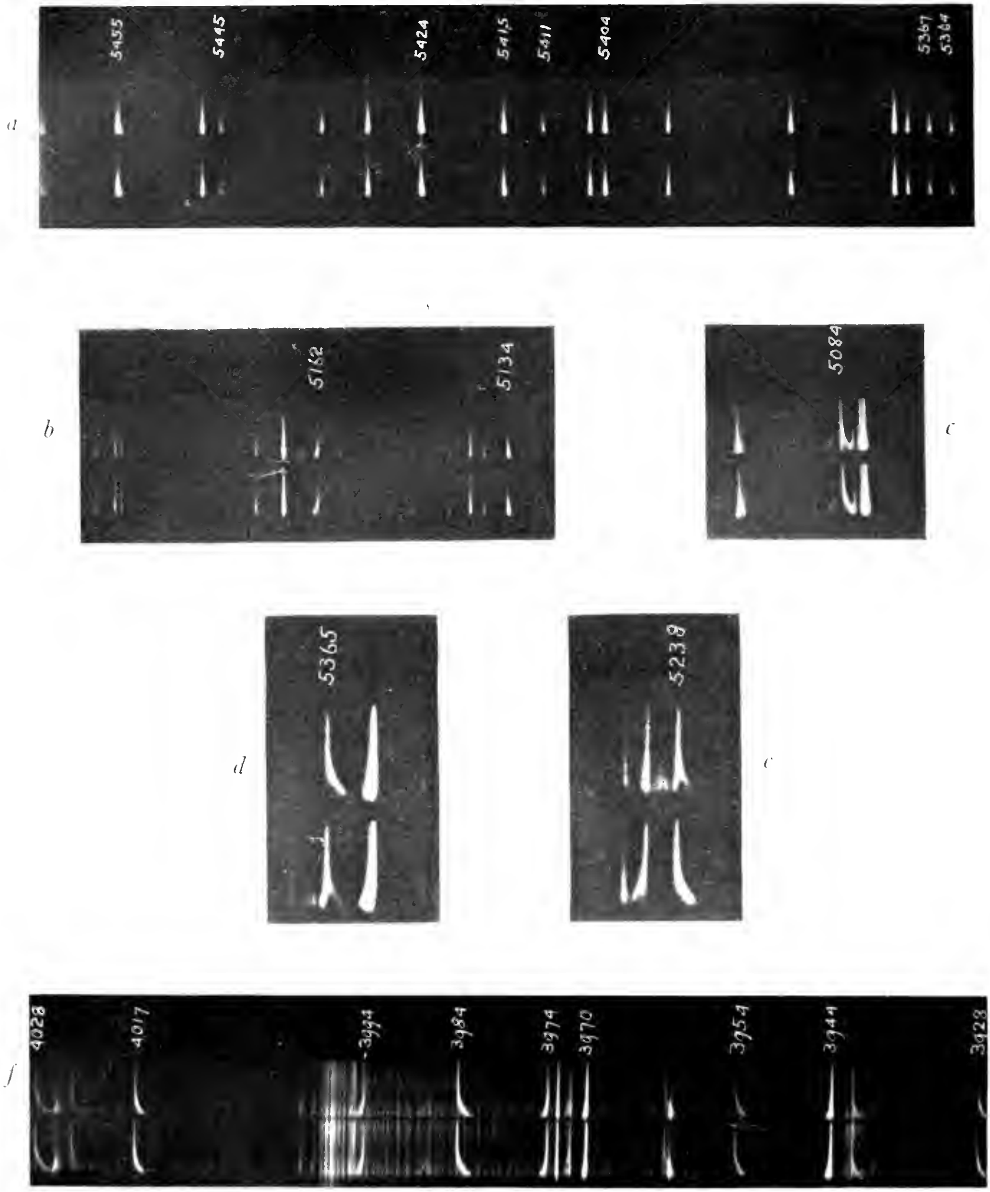
REMARKS

λ 4130.62 Complex?

4133.94 Complex?

4198.62 *p*- and *n*-component displaced in different directions.

PLATE III



TAKAMINE: THE STARK EFFECT FOR METALS

In each illustration the *n* components are above, the *p* components below

a	Iron	$F_{max.}$	39,800 volt/cm
b	Iron	$F_{max.}$	39,800 "
c	Nickel	$F_{max.}$	38,500
d	Molybdenum	$F_{max.}$	38,600 "
e	Molybdenum	$F_{max.}$	38,600 "
f	Nickel	$F_{max.}$	39,000 "

The line λ 3796.15 has a very faint detached component on the violet side, thus showing similarity to the copper lines $\lambda\lambda$ 3654.59 and 3688.60.

The series relation in the spectrum of gold is not known, but judging from their Stark effect the lines $\lambda\lambda$ 3796.15 and 4128.80 behave like diffuse series lines, while the line λ 4084.31 behaves like a sharp series line.

Cobalt.—Like nickel, cobalt shows a considerable number of affected lines. In the case of chromium, Anderson found that many of the affected lines were not given in the ordinary wave-length tables. For several lines in the cobalt spectrum this is also the case, but for reasons similar to those given by Anderson in his paper they are included in Table III. A group of affected lines near λ 4100 is shown in Plate II*e*.

Copper.—The Stark effect for copper lines is in many respects similar to that for silver. Although too faint to be seen in Plate II*f*, the diffuse series lines $\lambda\lambda$ 3654.59 and 3688.60 are accompanied by detached components on the violet side. A series of lines, evidently of Koch's type, was also observed in the copper spectrum. Their approximate wave-lengths are 3652.6, 3686.7, 4015.8, and 4056.8 A, the last two being superposed on the detached components of the diffuse series lines $\lambda\lambda$ 3654.59 and 3688.60.

The lines which form the term $n=4$ of diffuse series, namely $\lambda\lambda$ 4022.83, 4062.14, and 4063.50, are all displaced toward the red, as shown in Plate II*g*. On account of the great intensity, a number of ghosts appear on either side in the reproduction.

Iron.—As will be seen in Table V and Plate III*a*, *b*, the Stark effect for iron lines is exceedingly small compared with other elements, but the fact that it can be observed seems to be of no small importance, especially in connection with the pole effect, which will be discussed later.

In the present investigation, which is of a preliminary character, only the green portion of the iron spectrum was studied.

Magnesium.—In a previous investigation, carried out by N. Kokubu and the author,[1] four magnesium lines, $\lambda\lambda$ 3093.1, 3097.1,

[1] *Memoirs of the College of Science* (Kyoto Imperial University), **3**, 173, 1918.

TABLE IV

COPPER

λ	Series	p-Component Δλ	p-Component Int.	n-Component Δλ	n-Component Int.	$E_{max.}$ in Volt/cm
3654.59	IN, II6	v − 2.51 r + 4.80	2 3	v −2.49 r +3.45	2 3	44,000
3686.67	....	+ 1.26	4	+1.40	4	44,000
3688.60	IN, I6	v_2−10.30 v_1 o r + 6.72	1 3 2 5	.... o r +5.36	.... .. 2 5	44,000
3825.13	IIN, II5	− 0.16	3	−0.10	3	44,000
3830	....	+ 1.79	2	+0.61	2	44,000
3861.88	IIN, I5	− 0.11	3	−0.12	3	44,000
3894.64	....	+ 0.27	1	+0.23	1	44,000
4015.8	....	− 0.47	2	−0.43	2	33,000
4022.83	IN, II5	+ 0.46	7	+0.33	7	33,000
4056.8	P2	− 0.62	3	−0.65	3	33,000
4062.14	IN, I5	+ 0.60	10	+0.44	10	33,000
4063.50	IN, I5	+ 0.37	4	+0.33	4	33,000
4480.59	IIN, II4	+ 0.09	6	+0.11	6	44,000
4531.04	IIN, I4	+ 0.04	6	+0.03	6	44,000
5153.33	IN, II4	− 0.10	6	−0.05	6	25,300
5218.45	IN, I4	− 0.09	10	−0.07	10	25,300
5220.25	IN, I4	....	4	−0.06	4	....

REMARKS

λ 3654.59 The violet component is detached from the main line in both the p- and n-components. Superposed on the end of this violet component is another line which appears only in strong electric field, having the wave-length 3652.0.

3688.60 In the p-components v_1 approaches the main line as the field-strength is increased.

4015.8 The line appears only in strong electric field, and may be regarded as a detached component of 4022.83.

4056.8 The line appears only in strong electric field and may be regarded as a detached component of 4062.14.

4352.2, and 4703.3, were found to be displaced toward the red, and the line λ 4571.3 toward the violet in an electric field. Besides these lines the strong triplet λλ 3829.5, 3832.5, and 3838.4 showed a tendency to shift toward the violet, but as the dispersion was very small (30 A per mm), the fact was stated with reserve at that

TABLE V

IRON

λ	p-COMPONENT		n-COMPONENT		$E_{max.}$ IN VOLT/CM
	Δλ	Int.	Δλ	Int.	
5133.675	−0.14	5	−0.15	5	29,800
5162.312	+0.73	4	+0.28	4	29,800
5324.196	+0.05	2	+0.05	2	29,800
5364.859	−0.06	2	....	....	29,800
5367.455	−0.13	3	−0.05	3	29,800
5404.131	−0.05	5	−0.05	3	29,800
5410.900	−0.02	3	−0.04	3	29,800
5415.189	−0.11	6	−0.06	6	29,800
5424.057	−0.13	10	−0.12	10	29,800
5445.040	−0.06	2	−0.06	2	29,800
5455.435	−0.35	8	−0.38	8	29,800

REMARKS

λ 5455.435 This line was not resolved from the neighboring line 5455.614.

time. In the present work, using a dispersion of about 5.3 A per mm, the results in Table VI were obtained.

TABLE VI

MAGNESIUM

λ	p-COMPONENT		n-COMPONENT		$E_{max.}$ IN VOLT/CM
	Δλ	Int.	Δλ	Int.	
3529.501	−0.08	3	−0.07	3	30,000
3832.450	−0.11	7	−0.06	7	30,000
3838.435	−0.12	10	−0.05	10	30,000

Molybdenum.—The spectrum of molybdenum shows many affected lines, especially in the green and violet regions. In the *p*- and *n*-components of the lines λλ 5238.41 and 5364.50 we notice the same complementary phenomenon in relative intensity which was noted in the case of the silver line λ 3810.85. In addition we have the curious feature that the *p*-component of λ 5238.41 behaves exactly like the *n*-component of λ 5364.50, while the *n*-component of the former behaves just like the *p*-component of the latter. This remarkable feature is illustrated in Plate III*d*, *e*.

TABLE VII

MOLYBDENUM

λ	*p*-Component Δλ	*p*-Component Int.	*n*-Component Δλ	*n*-Component Int.	$E_{max.}$ in Volt/cm
4017.50	−0.44	2	−0.18	2	38,600
4025.64	+0.41	1	+0.39	1	38,600
4028.85	−1.49	4	−1.06	4	38,600
4032.65	−1.51	3	−1.07	3	38,600
4060.4	−1.01	3	−0.62	3	38,600
4071.54	−1.17	1	−1.09	1	38,600
4157.58	0 +0.92 −0.47	2 4 3	0 +0.81	2 2	38,600
4284.79	v_2−1.78 v_1−0.13 0 r_1+1.45	3 2 2 4	v−1.76 o+0.18 r+1.86	3 3 2	38,600
4290.40	+0.47	3	+0.36	3	38,600
4529.53	r_1+0.41 r_2+2.00	2 1	v−0.34 0 r+1.53	1 2 2	38,600
4535.09	+0.53	3	+0.58	3	38,600
4567.87	+0.20	1	+0.22	1	38,600
4582.69	−0.89 0	3 2	−0.49	2	38,600
4052.49	+0.42	4	+0.33	3	38,600
4688.41	−0.20	3	−0.20	3	38,600
4758.71	−0.13	2	.....	.....	38,600
4764.64	+0.25	3	+0.25	3	38,600
5171.33	+0.49	4	+0.94	2	38,600
5173.14	−0.13	5	−0.04	5	38,600
5174.35	−0.33	6	−0.33	6	38,600
5238.41	−1.48	10	−1.41 +0.34	3 4	38,600
5241.09	.....	.....	−0.50 0 +0.47	2 1 1	38,600
5243.01	0 +1.07	2 3	+0.32	3	38,600
5245.71	−0.10	4	.....	.....	38,600
5260.76	+0.81	10	+0.64	10	38,600
5264.50	−1.22 +0.43	2 4	−1.40 +0.43	5 1	38,600
5367.39	−1.22 0 −0.48	2 1 1	+0.48 −0.60 0 +0.57	1 1 1 1	38,600
5406.64	+0.40	1	+0.35	1	38,600
5544.78	+0.34	1	+0.40	1	38,600

REMARKS

λ 5241.09	This line may be regarded as a detached component of 5238.41. The *p*-component is too faint for measurement.
5367.30	This line may be regarded as a detached component of 5364.50. The *n*-component is very weak.

Nickel.—In 1917 Anderson[1] observed a number of affected nickel lines near λ 4000 A. In the present work, using a stronger electric field, the effect was much enhanced, as shown in Plate III*f*. Many affected lines were also found in the green and yellow regions. Plate III*c* shows the effect for λ 5084.20.

The Stark effect for a few lines of nitrogen, sodium, and oxygen was observed rather accidentally in the course of the present work.

Nitrogen.—In our previous paper dealing with the Stark effect of calcium and magnesium[2] it was reported that certain lines of unknown origin came out quite frequently when Al, Ca, Mg, and Ta were used as the cathode, the residual gas being air or hydrogen. Two of these lines, namely λλ 4100.7 and 4110.3, were found to be displaced toward the violet and were marked by the letters *C* and *D* in our paper. Later, Yoshida noticed that these lines had been recorded by Moissan and Deslandres[3] in their investigation of the nitrogen spectrum.

In his study of the Doppler effect in canal rays, Hermann[4] mentions that the two nitrogen lines λλ 4100 and 4110 behave differently from other lines. Since the discharge in pure nitrogen was not tried in the present work, the origin of these two lines may still be open to question. Table IX gives the displacement of these two lines.

Sodium.—The cobalt cubes used contained a small quantity of sodium as impurity, which brought out the sodium lines λλ 5682.90 and 5688.26 fairly strongly. They belong to the first subordinate series of sodium and were found to be displaced toward the red, as shown in Table X.

[1] *Physical Review*, **9**, 575, 1917.

[2] T. Takamine and N. Kokubu, *Memoirs of the College of Science* (Kyoto Imperial University), **3**, 173, 1918.

[3] *Comptes Rendus*, **126**, 1689, 1898.

[4] *Physikalische Zeitschrift*, **7**, 567, 1906.

TABLE VIII

NICKEL

λ	*p*-COMPONENT		*n*-COMPONENT		$E_{max.}$ IN VOLT/CM
	Δλ	Int.	Δλ	Int.	
3826...........	+0.69	3	+0.57	3	....
3831.89........	+0.27	1	+0.17	1	....
3893...........	−0.96	2	−0.83	2	....
3896...........	−0.82	2	−0.82	2	....
3909.10........	+0.17	3	+0.18	3	....
3918.7.........	−0.17	1	−0.23	1	39,000
3921.7.........	−0.93	4	−0.93	4	39,000
3939.3.........	+0.40	3	+0.57	3	39,000
3941...........	−1.01	3	−1.02	3	39,000
3944.3.........	{ 0	2	0	2	39,000
	+0.39	7	+0.40	7	39,000
3953.0.........	+0.65	1	+0.64	1	39,000
3954...........	−1.21	4	−1.21	4	39,000
3955.2.........	+0.57	1	+0.47	1	39,000
3957.8.........	+0.62	1	+0.57	1	39,000
3960.7.........	−1.36	1	....	1	39,000
3962.25........	−1.16	4	−1.22	4	39,000
3964.4.........	+0.54	3	+0.42	3	39,000
3968.3.........	−0.65	1	−0.51	1	39,000
3970.65........	+0.19	8	+0.25	8	39,000
3972.0.........	+0.40	3	+0.40	3	39,000
3974.83........	+0.48	6	+0.44	6	39,000
3984.18........	−1.46	7	−1.31	7	39,000
3987.2.........	+1.12	2	+1.06	2	39,000
3994.1.........	+0.50	5	+0.57	5	39,000
3998.5.........	+0.36	2	+0.36	2	39,000
3999.8.........	+0.18	2	+0.18	2	39,000
4007.8.........	−0.77	1	−0.73	1	39,000
4017.67........	−1.04	8	−0.91	8	39,000
4022.5.........	−1.00	2	−1.00	2	39,000
4024.1.........	{ −0.48	3	0	2	39,000
	0	....	....	....	39,000
4024.9.........	+0.99	2	+0.59	2	39,000
4025.1.........	+0.42	4	+0.41	4	39,000
4025.8.........	−0.70	3	−0.52	3	39,000
4027.8.........	−1.54	5	−1.44	5	39,000
4029.9.........	+0.70	2	....	....	39,000
4034...........	+0.27	2	+0.24	2	39,000
4037.7.........	+0.96	3	+0.70	3	39,000
4410.66........	−0.18	6	−0.08	6	39,000
4937.45........	−1.21	1	−1.34	1	38,500
5018.48........	+0.70	2	....	....	....
5082.55........	+0.44	3	+0.25	3	38,500
5084.20........	{ v −1.29	7	−1.40	4 }	38,500
	0	1	0	......	
	r_1	....	r_1+1.92	......	
	r_2 +4.70	1	r_2+4.62	1	

TABLE VIII—*Continued*

λ	p-Component Δλ	p-Component Int.	n-Component Δλ	n-Component Int.	$E_{max.}$ in Volt/cm
5100.12.........	+0.30	8	+0.15	8	38,500
5142.91.........	−0.09	4	−0.07	4	21,800
5146.61.........	−0.13	3	−0.07	3	21,800
5155.90.........	−0.79	5	−0.79	5	21,800
5176.72.........	−0.15	4	−0.09	4	21,800
5184.78.........	+0.29	3	+0.30	3	21,800
5462.69.........	−0.86	1	−0.56	1	21,800
5588.09.........	−0.15	2	−0.11	2	21,800

REMARKS

λ 3893 Band superposed on this line.
3909.10 Band superposed on this line.
3939.3 Koch's type.
3957.8 Koch's type.
3962.25 Koch's type. Detached component of 3962.25?
3987.2 Koch's type.
4024.9 Detached component of 4024.1?
5084.20 In both *p*- and *n*-components r_1 and r_2 are detached.

TABLE IX

NITROGEN

λ	p-Component Δλ	p-Component Int.	n-Component Δλ	n-Component Int.	$E_{max.}$ in Volt/cm
4100.7..........	−0.15	5	−0.27	5	56,000
4110.3..........	−0.50	5	−0.40	5	56,000

TABLE X

SODIUM

λ	Series	p-Component Δλ	p-Component Int.	n-Component Δλ	n-Component Int.	$E_{max.}$ in Volt/cm
5682.90.......	IN, II4	+1.82	2	+1.72	2	25,200
5688.26.......	IN, I4	+2.21	2	+2.08	2	25,200

Oxygen.—The red oxygen lines λλ 6156.198, 6156.993, and 6158.415, which form a triplet in the first subordinate series, were all found to be displaced toward the red, as shown in Table XI. It may be remarked here that in the discussion given by Schönrock[1] on the width of lines, the data for this triplet show considerable discrepancy, which may perhaps be accounted for by the Stark effect.

In connection with this it may be mentioned that the Stark effect for a number of green oxygen lines was first observed by Yoshida.

TABLE XI

OXYGEN

λ	Series	*p*-Component		*n*-Component		$E_{max.}$ in Volt/cm
		Δλ	Int.	Δλ	Int.	
6156.198......	TNI	+0.37	1	+0.48	1	18,200
6156.993......	TNI	+0.51	2	+0.39	2	18,200
6158.415......	TNI	+0.60	3	+0.39	3	18,200

THE STARK EFFECT AND THE POLE EFFECT

One of the interesting features revealed by the present investigation is that there seems to be an unmistakable connection between the pole effect and the Stark effect. The pole effect for different elements has been studied by various investigators, an elaborate study of the phenomenon having been specially made for iron by St. John and Babcock.[2] Table XII gives the comparison of these two effects for iron lines.

As will be seen from the table, the two effects are in complete agreement as to the direction of displacement. Quantitatively, also, the agreement is fairly good except for the three lines λλ 5162.312, 5424.057, and 5455.435. The last line is a blend, made up of λ 5455.614, which is an "*a*" line in the classification of St. John and Babcock, and λ 5455.435, which is an "*e*" line, and

[1] *Annalen der Physik*, **20**, 995, 1906.

[2] *Mt. Wilson Contr.* Nos. 106, 137; *Astrophysical Journal*, **42**, 231, 1915; **46**, 138, 1917.

TABLE XII

IRON

λ	Pole Effect	Stark Effect ($E_{max.}$: 29,800 Volt/cm)	
		p-Component	n-Component
5133.675	−0.045	−0.14	−0.15
5162.312	+0.030	+0.73	+0.28
5324.196	+0.015	+0.05	+0.05
5364.859	−0.028	−0.06	....
5367.455	−0.025	−0.13	−0.05
5404.131	−0.025	−0.05	−0.05
5410.900	−0.026	−0.02	−0.04
5415.189	−0.030	−0.11	−0.06
5424.057	−0.027	−0.13	−0.12
5445.040	−0.020	−0.06	−0.06
5455.435	−0.016	−0.35	−0.38

hence shifted to the violet in the pole effect. In my spectrograms the dispersion was insufficient to separate the two, and consequently the displacement given is subject to considerable error.

The pole effect for nickel lines was recently examined by Babcock and Merrill (unpublished), and a comparison of pole and Stark effect was also made for this element. In the green part of the spectrum very close agreement was found, as shown in Table XIII; but in the violet part those lines which show Stark effect are so diffuse in the pole effect spectrograms, especially at the negative pole, as to render the measurement almost impossible. An inspection of these diffuse lines, however, indicates that most of them are widened toward the same side as in the case of the Stark effect.

TABLE XIII

NICKEL

λ	Pole Effect	Stark Effect		$E_{max.}$ in Volt/cm
		p-Component	n-Component	
4937.45	−0.006	−1.21	−1.34	38,500
5018.48	+0.006	+0.70	....	38,500
5082.55	+0.009	+0.44	+0.25	38,500
5084.20	−0.009	−1.29	−1.40	38,500
5100.12	+0.015	+0.30	+0.15	38,500
5155.90	−0.007	−0.79	−0.79	21,800
5170.72	−0.008	−0.15	−0.09	21,800
5184.78	+0.011	+0.29	+0.30	21,800

It should be remarked that in discussing the various causes of the broadening of spectrum lines Stark[1] suggested that the pole effect may be due to intermolecular electric fields. On comparing the results for the Stark effect with those obtained by King[2] for furnace spectra, it was found that the lines which are affected in an electric field belong to high-temperature lines in King's classification.

A preliminary search for the possible relations between the Stark effect and the pressure effect on spectrum lines was made also. For this purpose a series of investigations on the pressure effect of the lines of Ag, Au, Cu, Fe, and Ni carried out by Duffield[3] was referred to. Although it is difficult to make any definite statement, still in many cases there seem to be some indications that the Stark effect is involved as one of the causes affecting the pressure shift. For example, in the case of copper, Duffield classifies the lines in his Table VI in two groups, the one being strengthened as the pressure increases, while the other is weakened. On referring to the Stark effect we find that the lines which are affected by an electric field are all included in the latter class. In comparing the pressure effect with the Stark effect, however, we see many lines which are broadened on one side with high pressure, but remain unaffected by an electric field, so that the relation is not so simple as in the case of pole and Stark effect.

The relation between the Stark effect and the broadening of lines in arc and spark spectra was studied by Stark and Kirschbaum,[4] Stark,[5] Wendt,[6] Merton,[7] and others. In the course of the present work the following features were noticed:

1. In the ordinary arc spectrum of copper the lines $\lambda\lambda$ 4022.83 and 4063.50 are usually very diffuse toward the red. In the Stark effect these two lines are also displaced toward the red.

2. Striking similarity was noticed in the appearance of the silver lines $\lambda\lambda$ 4210.87 and 4212.76 as they appear in the arc and in a

[1] *Jahrbuch der Radioaktivität*, **12**, 349, 1915.

[2] *Mt. Wilson Contr.* Nos. 66, 108; *Astrophysical Journal*, **37**, 239, 1913.

[3] *Trans. Royal Soc.*, **208A**, 111, 1908; **209A**, 205, 1908; **211**, 33, 1909; **215**, 205, 1915.

[4] *Annalen der Physik*, **43**, 1017, 1914.

[5] *Jahrbuch der Radioaktivität*, **12**, 349, 1915.

[6] *Annalen der Physik*, **45**, 1257, 1914.

[7] *Loc. cit.*

strong electric field. (Compare Fig. 1. Plate I, in Duffield's paper. *Phil. Trans.*, **211 A,** 33, 1909, with Plate II*c*.)

3. Generally speaking, those diffuse lines which are marked by the letters *u* and *U* in tables of wave-length given in Kayser's *Handbuch der Spectroscopie* were found to be affected by an electric field.

4. In the green part of the spectrum of a condensed spark discharge between iron poles those lines which are affected by an electric field appear invariably very hazy.

STARK EFFECT AND SERIES RELATIONS

Unfortunately the series relations in the spectra of the elements investigated here are not yet known except for silver and copper.

In Table XIV the affected lines of silver and copper are arranged according to their series relation, the + and − signs showing the direction of displacement in an electric field.

TABLE XIV

	Silver		Copper	
	I.N.S.	II.N.S.	I.N.S.	II.N.S.
$n=4$	5471.72?	4668.70−	5220.25−	4531.04+
	65.66?	4476.29+	18.45−	4480.59+
	5209.25?	....	5153.33−	....
$n=5$	4212.76 (complex)	3981.87+	4063.50+	3861.88−
	4210.87 (complex)	3840.74+	62.14+	25.13−
	4055.46 (complex)	....	22.83+	....
$n=6$	3810.85 (complex)	3710.1 +	3688.60 (complex, main line+)	
	3682.45 (complex)	....	54.59 (complex, main line+)	

In general, the diffuse series lines (I.N.S.) show more or less complex decompositions, while the sharp series lines (II.N.S.) are simply displaced in one direction, a feature which is very pronounced in the Stark effect for helium lines.[1]

[1] The fact that some of the sharp series lines of helium are not merely shifted in an electric field, but are resolved into a number of components, is stated in a previous paper (*Proceedings Tokyo Math. Phys. Soc.*, II. **9**, 394, 1918).

With respect to the direction toward which the lines are displaced, silver shows all its sharp series lines displaced toward the red, while in the case of copper the lines of both sharp and diffuse series change sign of displacement as we go from the term number $n=4$ to $n=5$. It is remarkable that exactly the same feature was observed for the diffuse-series lines of magnesium in our previous work,[1] in that the lines $\lambda\lambda$ 3829.5, 3832.5, and 3838.4 corresponding to $n=4$ are displaced toward the violet, while the members of the next term $\lambda\lambda$ 3093.1 and 3097.1 are displaced toward the red.

In general the relation of the Stark effect to the series law seems to be less simple than that of the Zeeman effect. An extension of our study of the Stark effect to the ultra-violet region seems to be highly desirable.

For those elements whose series relations are not yet established it appears that the Stark effect may be utilized as a powerful means of finding the relationships.

It may be worth mentioning that, of the elements studied in the present work, Na, Cu, Ag, and Au are all in the first column of the periodic table, while O and Mo together with Cr, which was studied by Anderson, are in the sixth column. The similarity of the electric decomposition of the lines of the elements in any one column of the periodic table is very striking when we compare, for example, the lines of Ag, Au, and Cu; also those of Mo and Cr as well as those of Fe, Co, and Ni.

In conclusion I wish to express to the members of the Observatory staff my very deep appreciation of the opportunity for my work at the Observatory. I am under particular obligation to Dr. J. A. Anderson for his kind guidance throughout the experiments, and to Miss M. O. Burns of the Computing Division for her careful measurements and reductions.

SUMMARY

1. Adopting the method employed by Anderson, the Stark effect on the spectra of the following metals was investigated: Ag, Au, Co, Cu, Fe, Mg, Mo, Ni. In addition to this a few lines of Na, O, and N were found to be affected by an electric field.

[1] T. Takamine and N. Kokubu, *loc. cit.*

2. Close relations were found between the pole effect and Stark effect in the spectra of iron and nickel.

3. Several instances showing the close relation between the Stark effect and the broadening of lines in the arc and spark spectra were noticed.

4. The connection between the series relationship and the Stark effect seems to be somewhat more complicated than in the case of the Zeeman effect. In some cases the side toward which the line is displaced in an electric field is different for different term numbers.

5. As in the case of helium, detached components and isolated lines, having the peculiar property of showing themselves only in strong electric fields, were observed in the spectra of silver and copper.

6. In the spectrum of molybdenum a curious complementary feature between *p*- and *n*-components of the two lines, λλ 5238.41 and 5364.50, was observed in that the *p*-component of the former was exactly the same as the *n*-component of the latter, and vice versa.

MOUNT WILSON OBSERVATORY
March 1919

STUDIES BASED ON THE COLORS AND MAGNITUDES IN STELLAR CLUSTERS[1]

THIRTEENTH PAPER: THE GALACTIC PLANES IN 41 GLOBULAR CLUSTERS

By HARLOW SHAPLEY AND MARTHA B. SHAPLEY

From evidence presented in earlier papers there can be little doubt that the observed elliptical distribution of stars in the photographic images of globular clusters is really an indication of flattening with respect to more or less symmetrical equatorial planes. The analogy with other flattened sidereal organizations—such as the Galaxy, the spiral nebulae, the solar and planetary systems—leads us to interpret these projected elongations of clusters as indicative not of prolate spheroids but of oblate spheroids or ellipsoids. It is of some interest to know how general this phenomenon is among globular clusters; and since their relation to the galactic system has been demonstrated, it is important to see if their orientation in space betrays any effect of the dominance of the greater system.

A study of star counts by Pease and Shapley[2] has shown the elliptical form for five clusters and has suggested that three others are sensibly circular. The ellipticity of Messier 5 has also been determined from Mount Wilson plates.[3] Further discussion by Shapley[4] has indicated the relation of the elongation in ω Centauri to its variable stars, and in Messier 13 to the stars of bluish color. The present paper continues the work, increasing the number of clusters known to have appreciable elongation from six to about thirty, and showing that several others are sensibly circular, either

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 160.

[2] *Mt. Wilson Contr.* No. 129; *Astrophysical Journal*, **45**, 225, 1917; *Mt. Wilson Communications*, No. 39; *Proceedings of the National Academy of Sciences*, **3**, 96, 1917.

[3] Shapley and Davis, *Publications of the Astronomical Society of the Pacific*, **30**, 164, 1918.

[4] *Mt. Wilson Communications* No. 45; *Proceedings of the National Academy of Sciences*, **3**, 276, 1917.

because of their orientation in space or because of actual sphericity. All globular clusters sufficiently bright (and in other respects satisfactory) to give definite results have been investigated. The accumulated material, though not extensive, is complete enough to permit statistical examination.

TABLE I

COMPARISON OF COUNTS AND ESTIMATES

N.G.C.	MESSIER	POSITION-ANGLE OF MAJOR AXIS		
		Counts	Estimates	Difference
5024	53	160°	170°	−10°
5139	....	105	120	−15
5272	3	Asym.	Asym.	....
5904	5	55	50	+ 5
6121	4	115	Indeterm.	....
6205	13	125	130	− 5
6218	12	Appr. circ.	Appr. circ.	....
6254	10	See note	Appr. circ.	....
6266	62	75	65:	+10:
6273	19	15	15	0
6341	92	Indeterm.	25	....
6402	14	110	70:	+40:
6626	28	50	45	+ 5
6656	22	25	25	0
6779	56	Appr. circ.	150:	....
7078	15	35	20	+15
7089	2	135	150	−15

N.G.C. 5139 (ω Centauri). The direction of the major axis, as estimated from photographs made at Harvard, is 105°.

N.G.C. 5272. Cf. tabulated counts in *Mt. Wilson Contr.* No. 129.

N.G.C. 5904. Cf. discussion of asymmetry in *Publications of the Astronomical Society of the Pacific*, **30**, 164, 1918.

N.G.C. 6205. The image of the Hercules cluster on the Franklin-Adams chart is too distant from the center and too diffuse to permit dependable estimates. The tabulated value is from photographs by Pease.

N.G.C. 6254. Perhaps the counts given in *Mt. Wilson Contr.* No. 129 show a trace of ellipticity, major axis in position-angle 150°.

The Franklin-Adams charts, which were of much value in determining the relative diameters of globular clusters, are also highly useful in the present problem. Although the ellipticity of clusters cannot be accurately determined from ordinary photographs without a special study of the distribution of the individual stars, and generally then only when many faint stars are included in the

counts, it is found that direct estimates based upon photographs of small scale give values of the direction of elongation as definite as those resulting from the extensive counts of stars. The direct estimates do not give the degree of elongation, but that property is also indefinite from counts, for it apparently depends in an uncertain manner upon the brightness of the stars enumerated and upon other factors.

In Table I the direction of elongation, when measurable, is given for 17 clusters, both from estimates and counts, the two determinations being entirely independent of each other. The average difference[1] between the position-angles thus derived is 8°. Better agreement than this is not obtainable from counts of two plates of the same cluster, or from the counts on one plate of stars of different magnitudes, colors, or distances from the center.[2] The consistent agreement therefore justifies the substitution of direct estimates for laborious counts. We are therefore enabled to extend the investigation expeditiously to the bright southern objects and complete the survey of the form of globular clusters.

Ten of the values of the third column of Table I are from published results. Those for N.G.C. 6121, 6266, 6273, 6341, 6402, 6626, and 6656, based on Mount Wilson plates, are from recent counts made by Miss Davis subsequent to the estimations from the charts. New counts have also been made of the stars in N.G.C. 5139 (ω Centauri) on plates kindly lent by the Harvard College Observatory. Some of the more interesting results of the unpublished counts are noted below.

1. The decided asymmetry of N.G.C. 6266 (Messier 62), particularly noted by Sir John Herschel[3] and later by Bailey,[4] is shown by the following values derived from a Mount Wilson plate:

Sector	15°	45°	75°	105°	135°	165°	195°	225°	255°	285°	315°	345°
No. stars	67	56	67	45	35	42	49	56	68	79	72	72

[1] Omitting N.G.C. 6402, which is very open and ragged in outline on the Franklin-Adams chart.

[2] Cf. curves for Messier 13 in *Mt. Wilson Contr.* No. 129.

[3] *Cape Results* (London, 1847), p. 23.

[4] *Harvard Annals*, 76, 74, 1915.

Stars within 30″ of the center could not be counted. Combining opposite sectors, the position-angle of the major axis appears to be about 75°.

2. Probably the most conspicuously elongated globular cluster is N.G.C. 6273 (Messier 19), whose relative stellar density is shown in Fig. 1. The radial co-ordinate is number of stars for each 30° sector of position-angle. Without correction for superposed field stars, there are more than twice as many stars in the direction of the major axis as perpendicular thereto. The plates show no evidence of a double nucleus. We probably see this flattened cluster more nearly edgewise than is usually the case. The major axis is approximately parallel to the Milky Way, the galactic latitude is +9°, and it appears likely, therefore, that the plane of symmetry is nearly parallel to the galactic plane. Apparently the flattening of a cluster, if we judge from this extreme case, is of a much lower order than that of spirals and of the galactic system.

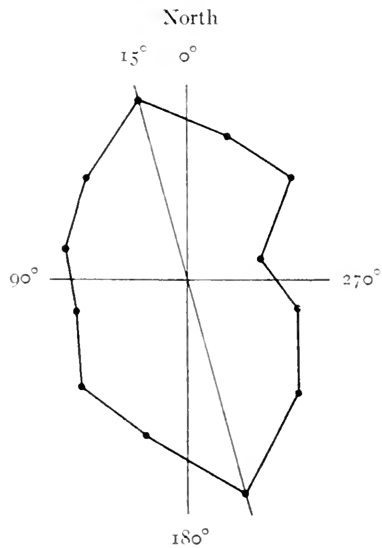


Fig. 1.—Curve of star-density for Messier 19. Radial co-ordinates show the number of stars for each 30° of position-angle.

3. The cluster N.G.C. 6626 (Messier 28) is also conspicuously elongated, with major axis roughly parallel to the Milky Way, but it is hardly as symmetrical as Messier 19.

4. A photograph made with the 60-inch reflector of the bright southern cluster N.G.C. 6656 (Messier 22), which shows more than 70,000 stars, has been counted by Miss Davis.

Nearer the center than 8 mm (218″) the density is too great to give reliable results; under such conditions the large images of the brighter stars occult so many faint images that counts

of the latter may show the ellipticity obliterated or even reversed.

Outside the circle of 14 mm (381″) radius the proportion of field stars becomes so large that the amount of ellipticity is masked, and its observed direction may be affected by local irregularities in the field. Neither of these factors—the interior crowding or the exterior field—is troublesome in estimating the direction of elongation of bright clusters from small-scale plates.

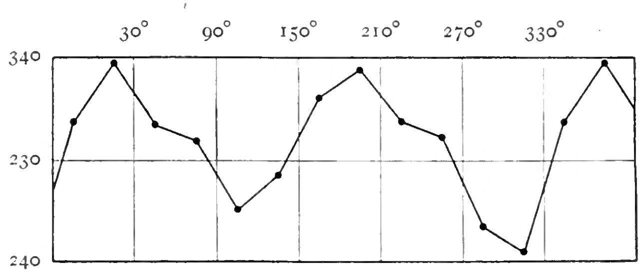


FIG. 2.—Distribution of stars in Messier 22. Ordinates are numbers of stars; abscissae are position-angles.

Counts within the limits 8 and 14 mm give the distribution of stars shown in Fig. 2, the ordinates being numbers of stars in 30° sectors and the abscissae position-angles. A correction for superposed field stars has been made by deducting 30 stars per square minute. Combining opposite sectors we have the following means for the smooth ellipticity-curve shown in Fig. 3:

Sector	15°	45°	75°	105°	135°	165°
No. stars	336	308	300	262	263	314

The number of stars in the direction of the major axis exceeds the number in the direction of the minor axis by nearly 30 per cent of the latter.

Within the errors of measurement the major axis of Messier 22 is exactly parallel with the Galaxy. The high ellipticity suggests that we see the cluster edgewise, and, since it is in low galactic latitude, the result further indicates that the plane of symmetry is

approximately parallel with the galactic plane. This point may be significant in connection with the loose structure of the cluster and its proximity to the central plane of the galactic system.

Table II contains data for 41 globular systems. When the direction of the major axis has been derived by two methods a weighted mean is adopted. The inclination of the observed major axis to the galactic plane, in the sixth column, has been computed from the data of preceding columns and checked graphically. The linear distances from the galactic plane, $R \sin \beta$, expressed in units of 100 parsecs, are taken from Tables V and VIII of *Mt. Wilson Contr.* No. 152.

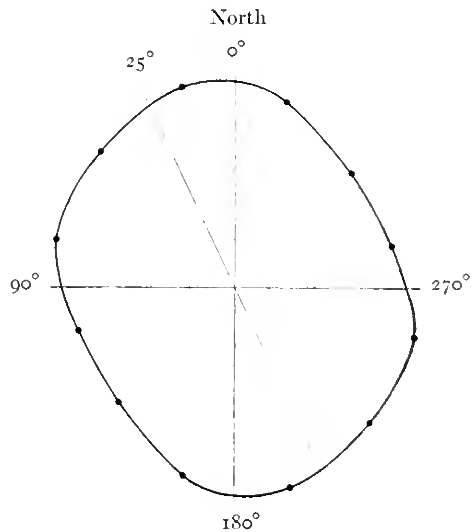


FIG. 3.—Curve of star-density for Messier 22. The radial co-ordinates show the number of stars for each 30° of position-angle.

In order that the equatorial plane of a cluster and the galactic plane may be parallel, the parallelism of the major axis with the galactic circle is a necessary though not sufficient condition. Without knowing another dimension of the inclination we can say little as to the possible influence of the greater system upon the orientation of its secondaries. There are 15 clusters whose major axes are inclined less than 35° to the galactic circle; some of these may be parallel with the Galaxy, but none of the others can be considered as even approximately so.

If the equatorial planes of globular clusters are distributed at random we should find both large and small inclinations of the major axes at all distances. This condition holds for the 25 clusters more distant than 2000 parsecs from the plane, their inclinations

ranging from 0° to 85° with an average of 45°, as shown in Table III. The major axes of all the 8 clusters within 3000 parsecs of the plane,

TABLE II

RESULTS FOR 41 CLUSTERS

N.G.C.	Messier	Galactic Longitude	Galactic Latitude	Adopted Position-Angle of Major Axis	Inclination of Major Axis to Galactic Circle	Distance from Galactic Plane (Unit is 100 Parsecs)
104........	....	272°	−44°	35°	45°	− 47
362........	....	268	−46	110	15	−109
1851........	....	211	−34	Appr. circ.	....	− 96
1904........	79	195	−28	20	30	−120
2808........	....	249	−11	20	65	− 32
4372........	....	269	− 9	30:	65:	− 18
5024........	53	307	+79	165	75	+186
5139........	....	277	+16	110	30	+ 18
5272........	3	8	+77	Asym.	....	+135
5286........	....	279	+11	Appr. circ.	....	+ 37
5904........	5	333	+45	55	15	+ 88
5986........	....	305	+13	Appr. circ.	....	+ 47
6093........	80	320	+18	165	55	+ 62
6121........	4	319	+15	115	70	+ 30
6171........	....	331	+22	65:	30:	+ 60
6205........	13	26	+40	125	65	+ 71
6218........	12	344	+25	Appr. circ.	....	+ 52
6254........	10	343	+22	Appr. circ.	....	+ 45
6266........	62	320	+ 7	70	35	+ 19
6273........	19	324	+ 9	15	20	+ 25
6284........	....	325	+10	Appr. circ.	....	+ 64
6293........	....	325	+ 8	35	0	+ 37
6304........	....	323	+ 5	Appr. circ.	....	+ 28
6341........	92	35	+34	25	15	+ 69
6362........	....	293	−17	110	80	− 38
6397........	....	304	−12	35	5	− 17
6402........	14	349	+14	100	70	+ 56
6541........	....	316	−11	45	20	− 28
6584........	....	309	−16	Appr. circ.	....	− 73
6626........	28	336	− 7	45	20	− 23
6656........	22	338	− 9	25	0	− 13
6681........	70	329	−13	150:	55:	− 41
6715........	54	333	−16	95	70	− 44
6723........	....	327	−18	Appr. circ.	....	− 39
6752........	....	303	−26	125:	65:	− 39
6779........	56	30	+ 7	Appr. circ.	....	+ 30
6809........	55	335	−24	100	80	− 41
7006........	....	32	−20	115	85	−228
7078........	15	33	−29	30	10	− 71
7089........	2	22	−37	140	70	− 94
7099........	30	356	−48	20	5	−128

however, are inclined less than 35°, except for N.G.C. 4372, which is very loose and apparently is partially obstructed by dark nebulosity.[1]

[1] Cf. p. 22, n. 2, of *Mt. Wilson Contr.* No. 152.

There is some evidence, therefore, that the equatorial planes in clusters may be roughly parallel to the galactic plane when near it, but that when distant they are oriented at random in space.

TABLE III

AVERAGE INCLINATION TO GALAXY FOR DIFFERENT DISTANCES FROM GALACTIC PLANE

Interval of $R \sin \beta$	Number of Clusters	Mean $R \sin \beta$	Mean Inclination
0 to ± 20	5	17	23°
± 20 to ± 40	8	31	41
± 40 to ± 60	5	46	64
± 60 to ± 80	5	67	36
$> \pm 80$	7	136	42
$> \pm 20$	25	72	45°

SUMMARY

1. The good agreement of results from counts on large-scale photographs with results from estimates on Franklin-Adams charts justifies the use of the latter for detecting the ellipticity of clusters (Table I).

2. Thirty globular clusters show an apparent elongation due to the discoidal distribution of their stars; a number of others appear circular in outline, due either to actual sphericity or, more probably, to the high inclination of their planes of symmetry to the line of sight (Table II).

3. Probably the most elongated cluster on record is Messier 19, for which the stars are more than twice as numerous in position-angles 15° and 195° as in directions perpendicular to the major axis of the projected ellipse (Fig. 1).

4. The flattening of a globular cluster, while appreciable, is obviously very small in comparison with that of spiral nebulae or with what now appears to be the form of the general galactic system.

5. The equatorial plane of the globular cluster nearest the galactic plane, Messier 22, appears to be parallel to the Galaxy (Figs. 2 and 3).

6. In general, the clusters nearest the dense stellar regions are most nearly parallel to the galactic plane.

MOUNT WILSON SOLAR OBSERVATORY
September 1918

SPECTRAL SERIES AND THE GROUPING OF THE ASTEROIDS

By F. E. KESTER and DINSMORE ALTER

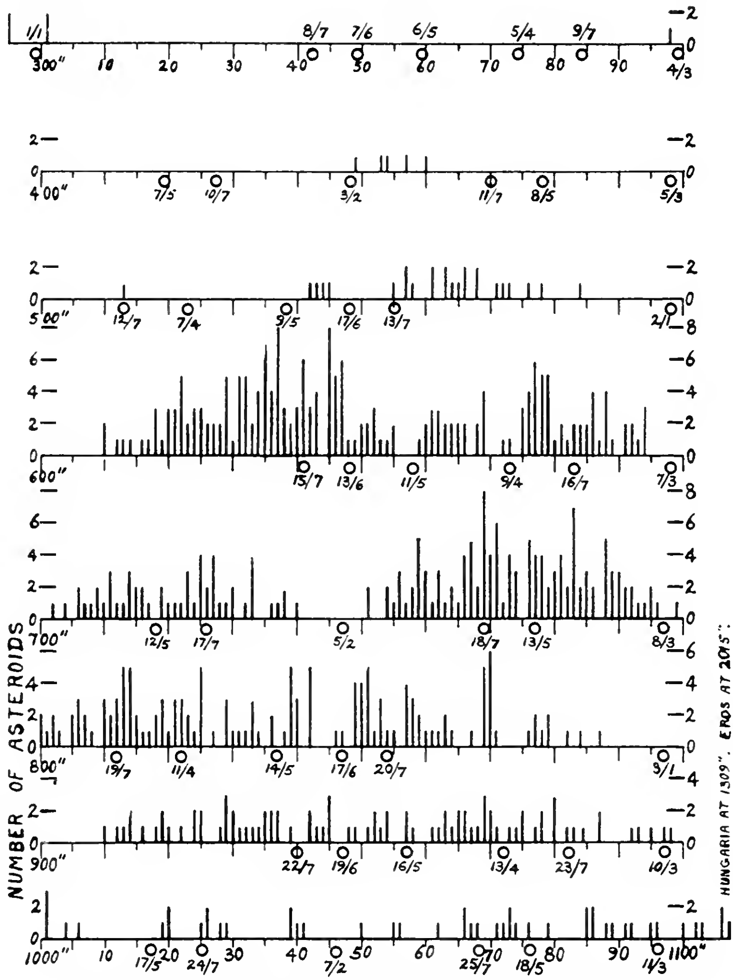
Within the region in the solar system between the orbits of Mars and Jupiter is scattered a swarm of small planetary bodies, known as the asteroids, of which more than eight hundred, ranging in size from a few miles to a few hundred miles in diameter, have been discovered. They are too small and too much scattered throughout this region to influence each other's motion perceptibly, except in possible rare cases. They are near enough Jupiter and that planet is large enough to make the perturbations due to it far larger than those due to any other planet.

It is a fact well known by astronomers that the perturbations of the asteroids produced by Jupiter drive the asteroids from regions of commensurability of frequencies of revolution into regions of incommensurability. This effect is stronger the simpler the ratio of the frequencies of Jupiter and the asteroid.

While it is natural to think in terms of the frequencies of revolution, it will be convenient to work instead in terms of a regularly tabulated orbital element, namely the mean daily motion of the radius vector measured in seconds of arc. In Fig. 1 the horizontal lines are divided to represent such mean daily motions, and an ordinate is erected for each second of this motion to such height as to represent the number of asteroids whose mean daily motions have that particular value. The values are taken, to the nearest second in each case, from the *Berliner Jahrbuch* of 1916 for all asteroids which have been observed at more than one opposition, and for no others.

On this plot there are represented also the ratios of commensurability spoken of above. The asteroids are evidently affected by commensurabilities to the order of six revolutions of Jupiter to an integral number of revolutions of the asteroid, but seem not to respond with equal certainty to more complex ratios.

Much of the recent theoretical work which has been done on atomic structure indicates that there may be some analogy between the solar system and the atom, and suggests that possibly the



MEAN DAILY MOTIONS OF THE ASTEROIDS, IN SECONDS OF ARC.

FIG. 1

grouping of orbits of electrons, as assumed by Bohr[1] to explain spectral series without any mechanical explanation therefor, may be due to perturbations in some manner similar to those that

[1] *Philosophical Magazine* (6), **26**, 1–25, 476–502, 857–875, 1913.

Jupiter produces on the asteroids. An examination of all the asteroids for which the data may be considered sufficiently reliable has been made with this possibility in view.

Difficulties result from two causes as follows: (*a*) Near Jupiter and also for frequencies greater than 1100″ of mean daily motion the number of asteroids is too small to allow one to draw any definite conclusion. In these regions the positions may be represented by several equations of the Rydberg type without any certainty as to which is true. For this reason no representation of these regions has been considered worth publishing. The same thing is true, for a different reason, in the region near the converging frequency of the Rydberg series which we shall use in this paper. Between 850″ and 1100″ the number of asteroids is much smaller than in the preceding three hundred and fifty seconds. Certain maxima seem to appear in this region, but they are not definite enough to be picked out by all observers and therefore are not listed. The reader will notice that in the main such indefinite maxima may be represented by adding Jupiter's mean daily motion to the series tabulated herewith. (*b*) The mean daily motions of all the asteroids are constantly changing, due to perturbations. Elements osculating for the same date for all the asteroids, or preferably mean elements, are needed. The confusion due to this cause amounts probably to three or four seconds, making the observed maxima uncertain by that amount.

TABLE I

I	II	III	IV
			O–C
455″	458″	....	−3″
563″	....	562″	+1″
635″	630″	....	+5″
677″	....	677″	0″
711″	710″	....	+1″
729″	....	734″	−5″
	852″ converging frequency		....

Column I of Table I shows the observed maxima of the asteroid groups. In estimating maxima great care has been taken to be as conservative as possible, and to include no maxima which the

plot of mean daily motions fails to show as certain. To this end the judgments of a number of observers have been taken. The reader must judge for himself how well this has been done.

The first series attempted was that discovered by Balmer for spectrum lines, which for the present purpose may be put into the Rydberg form

$$n=N\left(\frac{1}{p^2}-\frac{1}{q^2}\right),$$

where, for the Balmer series, $p=2$. This proved to be the only one whose presence we could certainly demonstrate. With $p=2$ a fair representation of a number of groups of asteroids was secured. It was then noted that the value used for the constant N was very nearly that of the frequency-constant of the solar system, namely the value of k in the formula

$$n=k/a^{3/2},$$

in which n is the mean daily motion as defined above and a is the semimajor axis of the planet measured in terms of that of the earth. This value was at once substituted and a solution made to obtain a more exact value of p. This turned out to be 2.041. With these values the formula gave a better representation than at first trial.

Columns II and III give maxima computed from the equations

$$n=3548\left[\frac{1}{(2.041)^2}-\frac{1}{m^2}\right] \text{ and } n=3548\left[\frac{1}{(2.041)^2}-\frac{1}{(m+0.5)^2}\right]$$

with $m=3, 4, 5. \; . \; . \; . \; .$ The deviation of any observed value from the corresponding computed value is shown in Column IV. Beyond the maximum at 734″ all further estimates are somewhat open to doubt on account of crowding of maxima.

It will be seen that each group, except that of 663″, is represented up to the region where the difference between successive computed values is of the same order of magnitude as the confusion mentioned above. It should be noted that, between the last computed maximum listed in the table and the converging limit of the series at $n=852''$, there is no pronounced maximum but instead a fairly even distribution of the asteroids, with here and there maxima prominent enough to be counted in any consideration of

weak maxima, as shown by the groups 813″, 825″, and 840″. There is, of course, an infinite number of computed maxima in such a converging region. The number of asteroids in this region is only about a hundred. Therefore there would not be the same possibility of representation as exists in the tabulated region. The maximum at 663″, which is not represented as part of the principal series, may be considered one of a number of minor maxima of whose existence in the main many readers might be skeptical. Most of such minor maxima have been represented by other Rydberg series. More may be said concerning them in a later paper. At first sight the maximum at 763″ may seem as strong as those at 711″ and 734″. If, however, each maximum be divided by the average number of asteroids per second throughout the whole of the immediate group, the two last maxima are seen to be each nearly as important as that at 777″, while that at 663″ is much weaker.

If it be granted that the agreement between the groups of asteroids and the positions calculated from the Rydberg formula be real, we are given thereby a second feature of similarity between atoms and orbital systems. The proof given by Rutherford,[1] by Geiger and Marsden,[2] and by Darwin,[3] from observations on the scattering of alpha particles, that the inverse-square law of attraction holds for interatomic distances, makes it probable that atomic orbits obey the same laws as do those of astronomical systems. The second point of similarity arises from the fact that the Rydberg formula, which was set up empirically to represent spectral series (very probably phenomena of atomic orbits), represents also certain phenomena of astronomical orbits.

While the similarity here suggested between narrow spectral lines and asteroid groups cannot be considered too striking in view of the great breadth of the groups, yet an explanation of the differences may possibly be found in the widely differing frequencies of atomic orbits and planetary orbits. Considering the fact that the frequencies of the vibrations of light are of the order of 10^{15} per

[1] *Philosophical Magazine* (6), **21**, 669–688, 1911.

[2] *Ibid.*, **25**, 604–623, 1913. [3] *Ibid.*, **27**, 504–505, 1914.

second (and orbital frequencies are very probably in simple commensurability therewith), one sees that even in a millionth of a second an atomic orbit is described many more times than an asteroid's orbit during the time since the earliest estimate of its origin. It may be that the sharpness of control of atomic orbits is due to this inconceivably frequent repetition of some controlling influence.

University of Kansas
May 8, 1919

Minor Contributions and Notes

MEASUREMENTS ON THE INDEX OF REFRACTION OF AIR

FOR WAVE-LENGTHS FROM 2218 A TO 9000 A[1]

[Editorial Note.—The valuable researches which have been conducted at the Bureau of Standards on the index of refraction of air appear to be of such importance to active workers in spectroscopy that it has seemed desirable that we should not only call them to the attention of our readers but also extract certain of the data for their convenience. The authors have had the opportunity to smooth out certain very small outstanding differences in the last column of the table (pp. 61–64), so that these values may be regarded as superseding those in the original publication. A few other corrections were made by them in May 1919. See also the article by Professor R. T. Birge in this number.

The practical astronomer will find data regarding astronomical refraction in section 4, which might otherwise escape his attention.

Sections 5 and 6 are of obvious interest to physicists.]

1. CORRECTION OF WAVE-LENGTH MEASUREMENTS MADE IN AIR AT OTHER THAN NORMAL TEMPERATURES AND PRESSURES

The international wave-length standards are represented by specified radiations whose wave-lengths were measured in dry air at 15° C., exerting a barometric pressure equivalent to 760 mm of mercury.[2] These standards serve as a basis for the precise measurement of all other wave-lengths. Variations in the density of the air appreciably affect the absolute values of the wave-lengths, and when wave-length measurements are made in air whose temperature is not 15° C., or whose pressure is not equal to 760 mm, corrections must be applied to the measured values to reduce them to their values in air under normal conditions. If the index of refraction of air were constant throughout the spectrum all the wave-lengths would vary in the same ratio, but the dispersion

[1] This paper by W. F. Meggers and C. G. Peters, Associate Physicists, Bureau of Standards, is an extract from *Scientific Papers of the Bureau of Standards*, S. W. Stratton, Director, No. 327. Issued October 31, 1918. Price, 10 cents; sold only by the Superintendent of Documents, Government Printing Office, Washington, D.C.

[2] *Astrophysical Journal*, **32**, 215, 1910; **33**, 85, 1911; **39**, 93, 1914.

of the air makes this variation a function of the absolute index and of the density of the air at the time the wave-length comparisons were made. For example, let a wave-length λ be measured in terms of a standard wave-length λ' and consider the result if the measurement is made in air whose density is greater than that defined by the normal conditions. The indices of refraction n and n' for these two wave-lengths increase proportionately with the density of the air, and the wave-lengths decrease. But if λ is smaller than λ' the absolute index of refraction n is larger than n', and the increase in index due to the increased density of air will be a larger amount for n than for n'. Consequently λ is proportionately shorter than λ' in denser air and requires a positive correction to make it comparable with λ' in air at 15° C. and 760 mm.

The effect of the temperature and pressure of the air must be taken into account in all accurate comparisons of wave-lengths made either by the coincidence method with diffraction gratings[1] or with interferometers.[2] The necessary corrections are generally small and may be negligible under certain conditions. In order actually to calculate these corrections let us imagine the wave-length comparisons to be made by means of an interferometer in air whose density d is defined by the temperature t and the barometric pressure B. Let λ represent the wave-length of a radiation under these conditions, n the index of refraction of this air for this wave-length, and λ' and n' analogous quantities for the standard wave-length with which λ is to be compared. The same letters with the subscript o may be used to represent the same quantities under standard observing conditions; that is, air at 15° C. and 760 mm. It is desired to obtain λ_o. The orders of interference measured under the actual conditions give $p\lambda = p'\lambda'$ $(=2e)$. The calculation for λ, however, is made from $\dfrac{p'\lambda'_o}{p}$ in which the standard wave-length is used as if the air had no dispersion, and then a correction is applied.

[1] Kayser, *Handbuch der Spectroscopie*, **1**, 719, 1900.

[2] Buisson and Fabry, *Journal de Physique*, **7**, 169, 1908.

This correction, the difference between the exact value and that calculated, is equal to

$$\delta=\lambda_0-\frac{p'\lambda_0'}{p}=\lambda_0\left(1-\frac{p'\lambda_0'}{p\lambda_0}\right)=\lambda_0\left(1-\frac{\lambda\lambda_0'}{\lambda'\lambda_0}\right).$$

But

$$\frac{\lambda}{\lambda_0}=\frac{n_0}{n} \text{ and } \frac{\lambda'}{\lambda_0'}=\frac{n_0'}{n'}.$$

Then

$$\delta=\lambda_0\left(1-\frac{n_0n'}{n_0'n}\right)=\lambda_0\frac{n_0'n-n_0n'}{n_0'n}=\lambda_0(n_0'n-n_0n'),$$

since $n_0'n$ in the denominator may be called unity. Assuming the refractivity to be proportional to the density,

$$\frac{n-1}{d}=\frac{n_0-1}{d_0} \text{ and } \frac{n'-1}{d}=\frac{n_0'-1}{d_0};$$

we have

$$n=\frac{(n_0-1)d}{d_0}+1 \text{ and } n'=\frac{(n_0'-1)d}{d_0}+1,$$

and the correction reduces to

$$\delta=\lambda_0(n_0-n_0')\frac{d-d_0}{d_0}.$$

The factor $\frac{d-d_0}{d_0}$ is easily calculated as a function of temperature and pressure, and is constant for all the waves compared under these conditions.

The quantity $\lambda_0\ (n_0-n_0')$ may be calculated as a function of the wave-length and represented by a curve from which the correction to the wave-length can be obtained by multiplying its ordinate by the appropriate density factor $\frac{d-d_0}{d_0}$.[1]

2. CORRECTIONS FOR WATER-VAPOR IN THE AIR

The refractive index of water-vapor has not been fully investigated, and the corrections necessary to change wave-lengths measured in wet air to their values in dry air are therefore not very

[1] To avoid all these troublesome computations, tables have been prepared and reproduced in the *Bulletin of the Bureau of Standards* to give these corrections for the entire range of wave-lengths and densities of air in which accurate spectroscopic measurements are ordinarily made.

accurately known. Lorenz measured the index of refraction of water-vapor (specific gravity=0.0008061) for sodium light as 1.0002500. From this result he recommends a correction of $+0.000041\frac{m}{760}$ to indices of refraction measured in air containing water-vapor of m millimeters pressure.

No measurements on the dispersion of water-vapor are known to exist, but if the dispersion is assumed to be comparable with that of air the corrections to relative wave-lengths will not be affected, since they depend on the difference of indices n_0-n_0'.

Two of our experiments on the dispersion of air for waves between 5800 A and 7500 A were made with air having a water-vapor pressure of 13 mm. The index of refraction was observed to be diminished by approximately the amount given by the Lorenz expression. Furthermore, this decrease was about the same for different wave-lengths. These results on wet air are not complete enough to add much to the subject of dispersion of light by water-vapor, but they show that the corrections to index of refraction as recommended above for water-vapor in the air are quite proper.

3. CORRECTIONS TO CONVERT TO VACUUM VALUES

Corrections required to convert wave-lengths or oscillation frequencies measured in air at 20° C. and 760 mm to their value in vacuum are found in Watts's *Index of Spectra*, Appendix E (p. 51), and in Kayser's *Handbuch der Spectroscopie*, **2**, 513. These are applicable to Rowland's standard wave-lengths and all wave-lengths determined from them, but must be revised to apply to the international system of wave-lengths, which represents values measured in air at 15° C. and 760 mm.

The dispersion equation representing our observations on the indices of refraction of air at 15° C. and 760 mm was used to construct a new table of corrections. The table (p. 61) gives the values of wave-length in air (λ), refractivity $(n-1)10^7$, vacuum correction to λ $(n\lambda-\lambda)$, oscillation frequency in air $\left(\frac{1}{\lambda}\right)$, and vacuum correction to frequency $\left(\frac{1}{n\lambda}-\frac{1}{\lambda}\right)$ from 2000 A to 10,000 A at intervals

of 50 A. This table shows, for example, that a light-wave of length 5000.000 A in dry air at 15° C. and 760 mm is increased in length by 1.391 A in a vacuum. The frequency (number of waves per centimeter) in air is 20,000.00, but in a vacuum it is diminished by 5.56. The vacuum frequencies corresponding to any wave-lengths in the international scale can be readily determined by the use of the table (p. 61) together with a table of reciprocals with seven-place arguments.

4. ASTRONOMICAL REFRACTION

Newcomb states in his *Compendium of Spherical Astronomy* (p. 223):

> There is, perhaps, no branch of practical astronomy on which so much has been written as on this (astronomical refraction) and which is still in so unsatisfactory a state. The difficulties connected with it are both theoretical and practical. The theoretical difficulties arise from the uncertainty and variability of the law of diminution of the density of the atmosphere with height, and also from the mathematical difficulty of integrating the equations of the refraction for altitudes near the horizon, after the best law of diminution has been adopted.

The practical difficulties involve the relation of refractive index with density of air as depending upon pressure, temperature, and humidity, and on the necessary inattention to the dispersion of light. In practice the astronomer usually disregards all dependence of refractive index on color. This must involve errors in zenith-distance observations on stars whose colors are intrinsically different and especially in observations of stars near the horizon, where the short waves are largely absorbed and scattered by our atmosphere, causing the stars to be represented only by longer waves. Because of these difficulties tables of corrections for astronomical refraction are probably better determined from astronomical observations than from laboratory measures. The refraction tables which have been applied to the major portion of astronomical observations are those of Bessel, founded upon the observations of Bradley and published in *Tabulae Regiomontanae.* In 1870 the Pulkowa tables were published under the title *Tabulae Refractionium in usum speculae Pulcovensis Congestae.* These

TABLE OF CORRECTIONS TO BE APPLIED TO WAVE-LENGTHS IN AIR AT 15° C. AND 760 MM TO REDUCE WAVE-LENGTHS AND FREQUENCIES TO VACUUM VALUES

λ	$(n-1)10^7$	$(n\lambda-\lambda)$ Add	$\frac{1}{\lambda}$	$\left(\frac{1}{n\lambda}-\frac{1}{\lambda}\right)$ Subtract
2000	3255.82	0.6512	50,000	16.274
2050	3220.12	.6601	48,780	15.703
2100	3187.86	.6695	47,619	15.176
2150	3158.63	.6791	46,511	14.687
2200	3132.07	.6891	45,454	14.232
2250	3107.87	.6993	44,444	13.809
2300	3085.75	.7097	43,478	13.412
2350	3065.50	.7204	42,553	13.041
2400	3046.91	.7313	41,666	12.692
2450	3029.81	.7423	40,816	12.363
2500	3014.05	.7535	40,000	12.053
2550	2999.48	.7649	39,215	11.759
2600	2986.00	.7764	38,461	11.481
2650	2973.50	.7880	37,735	11.217
2700	2961.88	.7997	37,037	10.967
2750	2951.08	.8115	36,363	10.728
2800	2941.00	.8235	35,714	10.500
2850	2931.60	.8355	35,087	10.283
2900	2922.80	.8476	34,482	10.076
2950	2914.57	.8598	33,898	9.877
3000	2906.85	.8721	33,333	9.687
3050	2899.60	.8844	32,786	9.504
3100	2892.79	.8968	32,258	9.329
3150	2886.38	.9092	31,746	9.160
3200	2880.33	.9217	31,250	8.998
3250	2874.63	.9343	30,769	8.842
3300	2869.24	.9469	30,303	8.692
3350	2864.15	.9595	29,850	8.547
3400	2859.33	.9722	29,411	8.407
3450	2854.76	.9849	28,985	8.272
3500	2850.43	.9977	28,571	8.142
3550	2846.32	1.0104	28,169	8.016
3600	2842.41	1.0233	27,777	7.893
3650	2838.69	1.0361	27,397	7.774
3700	2835.16	1.0490	27,027	7.660
3750	2831.79	1.0619	26,666	7.550
3800	2828.58	1.0749	26,315	7.442
3850	2825.51	1.0878	25,974	7.337
3900	2822.59	1.1008	25,641	7.235
3950	2819.79	1.1138	25,316	7.137

TABLE OF CORRECTIONS—*Continued*

λ	$(n-1)10^7$	$(n\lambda-\lambda)$ Add	$\frac{1}{\lambda}$	$\left(\frac{1}{n\lambda}-\frac{1}{\lambda}\right)$ Subtract
4000	2817.12	1.1268	25,000	7.041
4050	2814.56	1.1399	24,691	6.948
4100	2812.11	1.1530	24,390	6.857
4150	2809.76	1.1661	24,096	6.769
4200	2807.51	1.1792	23,809	6.683
4250	2805.36	1.1923	23,529	6.599
4300	2803.29	1.2054	23,255	6.517
4350	2801.30	1.2186	22,988	6.437
4400	2799.39	1.2317	22,727	6.360
4450	2797.55	1.2449	22,471	6.285
4500	2795.78	1.2581	22,222	6.211
4550	2794.08	1.2713	21,978	6.139
4600	2792.44	1.2845	21,739	6.069
4650	2790.86	1.2978	21,505	6.000
4700	2789.34	1.3110	21,276	5.933
4750	2787.88	1.3242	21,052	5.868
4800	2786.46	1.3375	20,833	5.804
4850	2785.09	1.3508	20,618	5.741
4900	2783.78	1.3640	20,406	5.680
4950	2782.50	1.3773	20,202	5.620
5000	2781.27	1.3906	20,000	5.561
5050	2780.08	1.4039	19,801	5.503
5100	2778.93	1.4173	19,607	5.447
5150	2777.81	1.4306	19,417	5.392
5200	2776.74	1.4439	19,230	5.338
5250	2775.69	1.4572	19,047	5.286
5300	2774.68	1.4706	18,867	5.234
5350	2773.70	1.4839	18,691	5.183
5400	2772.75	1.4973	18,518	5.133
5450	2771.83	1.5106	18,348	5.085
5500	2770.94	1.5240	18,181	5.037
5550	2770.07	1.5374	18,018	4.990
5600	2769.23	1.5508	17,857	4.944
5650	2768.41	1.5642	17,699	4.899
5700	2767.62	1.5775	17,543	4.854
5750	2766.85	1.5909	17,391	4.811
5800	2766.10	1.6043	17,241	4.768
5850	2765.37	1.6177	17,094	4.726
5900	2764.66	1.6311	16,949	4.685
5950	2763.98	1.6446	16,806	4.644
6000	2763.31	1.6580	16,666	4.604
6050	2762.65	1.6714	16,528	4.565
6100	2762.02	1.6848	16,393	4.527
6150	2761.40	1.6983	16,260	4.489
6200	2760.80	1.7117	16,129	4.452

TABLE OF CORRECTIONS—*Continued*

λ	$(n-1)10^7$	$(n\lambda-\lambda)$ Add	$\frac{1}{\lambda}$	$\left(\frac{1}{n\lambda}-\frac{1}{\lambda}\right)$ Subtract
6250.	2760.22	1.7251	16,000	4.415
6300.	2759.65	1.7386	15,873	4.379
6350.	2759.09	1.7520	15,748	4.344
6400.	2758.55	1.7655	15,625	4.309
6450.	2758.02	1.7789	15,503	4.275
6500.	2757.51	1.7924	15,384	4.241
6550.	2757.00	1.8058	15,267	4.208
6600.	2756.51	1.8193	15,151	4.175
6650.	2756.04	1.8328	15,037	4.143
6700.	2755.57	1.8462	14,925	4.112
6750.	2755.11	1.8597	14,814	4.081
6800.	2754.67	1.8732	14,705	4.050
6850.	2754.23	1.8866	14,598	4.020
6900.	2753.81	1.9001	14.492	3.990
6950.	2753.39	1.9136	14.388	3.961
7000.	2752.99	1.9271	14,285	3.932
7050.	2752.59	1.9406	14,184	3.903
7100.	2752.21	1.9541	14,084	3.875
7150.	2751.83	1.9676	13,986	3.847
7200.	2751.46	1.9811	13,888	3.820
7250.	2751.10	1.9945	13,793	3.793
7300.	2750.74	2.0080	13,698	3.767
7350.	2750.39	2.0215	13,605	3.741
7400.	2750.06	2.0350	13,513	3.715
7450.	2749.72	2.0485	13,422	3.690
7500.	2749.40	2.0620	13,333	3.665
7550.	2749.08	2.0756	13,245	3.640
7600.	2748.77	2.0891	13,157	3.616
7650.	2748.47	2.1026	13,071	3.592
7700.	2748.17	2.1161	12,987	3.568
7750.	2747.88	2.1296	12,903	3.545
7800.	2747.59	2.1431	12,820	3.522
7850.	2747.31	2.1566	12,738	3.499
7900.	2747.03	2.1702	12,658	3.476
7950.	2746.76	2.1837	12,578	3.454
8000.	2746.50	2.1972	12,500	3.432
8050.	2746.24	2.2107	12,422	3.410
8100.	2745.99	2.2243	12,345	3.389
8150.	2745.74	2.2378	12,269	3.368
8200.	2745.49	2.2513	12,195	3.347
8250.	2745.25	2.2648	12,121	3.326
8300.	2745.02	2.2784	12,048	3.306
8350.	2744.79	2.2919	11,976	3.286
8400.	2744.56	2.3054	11,904	3.266
8450.	2744.34	2.3190	11,834	3.246

TABLE OF CORRECTIONS—*Continued*

λ	$(n-1)10^7$	$(n\lambda-\lambda)$ Add	$\frac{1}{\lambda}$	$\left(\frac{1}{n\lambda}-\frac{1}{\lambda}\right)$ Subtract
8500............	2744.12	2.3325	11,764	3.227
8550............	2743.90	2.3460	11,695	3.208
8600............	2743.69	2.3596	11,627	3.189
8650............	2743.49	2.3731	11,560	3.170
8700............	2743.29	2.3867	11,494	3.152
8750............	2743.09	2.4002	11,428	3.134
8800............	2742.89	2.4137	11,363	3.116
8850............	2742.70	2.4273	11,299	3.099
8900............	2742.51	2.4408	11,235	3.081
8950............	2742.32	2.4544	11,173	3.063
9000............	2742.14	2.4679	11,111	3.046
9050............	2741.96	2.4815	11,049	3.029
9100............	2741.79	2.4950	10,989	3.012
9150............	2741.61	2.5086	10,928	2.995
9200............	2741.44	2.5221	10,869	2.979
9250............	2741.28	2.5357	10,810	2.963
9300............	2741.11	2.5492	10,752	2.947
9350............	2740.95	2.5628	10,695	2.931
9400............	2740.79	2.5763	10,638	2.915
9450............	2740.64	2.5899	10,582	2.899
9500............	2740.48	2.6035	10,526	2.884
9550............	2740.33	2.6170	10,471	2.869
9600............	2740.18	2.6306	10,416	2.854
9650............	2740.04	2.6441	10,362	2.839
9700............	2739.89	2.6577	10,309	2.824
9750............	2739.75	2.6713	10,256	2.809
9800............	2739.61	2.6848	10,204	2.795
9850............	2739.47	2.6984	10,152	2.780
9900............	2739.34	2.7119	10,101	2.766
9950............	2739.20	2.7255	10,050	2.752
10,000	2739.07	2.7391	10,000	2.738

are based upon the researches of Gylden[1] and are supplemented by those of Fuss.[2] The Pulkowa tables are founded upon a smaller refraction constant than Bessel's, and Newcomb (*loc. cit.*) says the most recent discussions and comparisons indicate a still greater diminution in the constant. The constants under discussion represent the index of refraction of air at 0° C., 760 mm, for white light as follows.[3]

[1] *Mem. de l'Acad. de St. Petersburg*, **10**, No. 1, 1866, and **12**, No. 4, 1868.

[2] *Ibid.*, **18**, No. 4, 1868.

[3] Winkelmann, *Handbuch der Physik* (2), **6**, 534.

Bessel	1.00029257, from Bradley.
Gylden	29232, from Pulkowa, 1842–1849
Fuss	29161, from Pulkowa, 1867–1869
Newcomb	29195, from Greenwich, 1877–1886
Bauschinger	29152, from Munich, 1891–1893
Courvoisier	29180, from Heidelberg, 1900–1903

Under the same conditions our observations give about 1.0002923, assuming the wave-length 5560 A of maximum visibility for white light.

For photographic work a larger refraction constant must be used than for visual. If the photographic maximum is at 4200 A the refraction constant should be 1.0002968 and the photographic corrections are therefore 1.0154 larger than the visual. The ratio of these corrections has been found experimentally by Wilsing[1] to be 1.01539.

5. OPTICAL TEMPERATURE-COEFFICIENT

If the Dale and Gladstone law, $\frac{n-1}{d}$ = constant, holds for air, the temperature-coefficient of index-variation should be identical with the temperature-coefficient of volume-change at constant pressure and any refractivity of air at temperature $t°$ C. could be expressed in terms of the refractivity of air at 0° C. by the relation

$$(n-1)_t=\frac{(n-1)_0}{1+at},$$

in which $a=0.00367$.

The following values of the optical temperature-coefficient of air show considerable disagreement: Lorenz, 0.00367; von Lang, 0.00310; Mascart, 0.00383; Benoit, 0.00367; and Walker, 0.00360.

Our observations on the refractivity of air at temperatures of 0, 15, and 30° C. show that α is a function of the wave-length and increases rapidly as the absorption band in the ultra-violet is approached. This is shown in the following table.

[1] *Astronomische Nachrichten*, **145**, 293, 1898.

λ	α
8500	0.003672
7500	3674
6500	3678
5500	3685
4500	3700
3500	3738
2500	3872

A decrease in the density of air due to an increase in its temperature either apparently or actually changes the position of the ultra-violet absorption band. As the temperature of the air rises the absorption and dispersion of short waves of light diminish, and this effect is probably explained by the increased kinetic energy of the gas molecules rather than by the decreased density of the gas. It would be interesting to test this by ultra-violet dispersion measurements of air with density reductions at constant temperature.

Within the range of our observations the refractivity of air at any temperature for any particular wave-length λ can be obtained from measurements on air at 0° C. by the relation mentioned above if a correction be applied to α, which is a function of λ. A very simple correction to α is contained in the expression

$$(n-1)_t = \frac{(n-1)_0}{1+t\left(\alpha+\frac{3\times 10^6}{\lambda^3}\right)}$$

in which $\alpha=0.00367$, which allows the calculation of $(n-1)_t$ within the limits of probable error of observation.

6. DISPERSION FORMULAE

A large amount of theoretical work on refraction and dispersion of transparent materials has been done in the past, and various theories have been tested by measurements of indices of refraction of solids and liquids over a wide range of spectrum from the ultra-violet through the visible and far out into the infra-red. It is especially important for this purpose to measure indices of refraction in the neighborhood of absorption bands. Measurements of refractivities of gases have heretofore been confined to a compara-

tively small portion of the spectrum, and although more work has been done with air than with other gases the review of previous work showed how incomplete it was. It was hoped that the present investigation of the dispersion of air for red and infra-red light would give an indication of an absorption band in the infra-red so that the Sellmeier or Ketteler-Helmholtz dispersion formula could be tested in its representation of the observations. The empirical Cauchy equation with three constants was first used to represent the measurements, and although the representation was quite satisfactory there seemed to be a slight deviation in the red which might be explained by an absorption band in the infra-red. An effort was made to locate such a band from our dispersion measurements, but without success.

Water-vapor has a very complicated absorption spectrum in the infra-red, but dry air is known to be quite transparent to long waves. The only constituent of air which seems to absorb much infra-red light is carbon dioxide. Although carbon dioxide constitutes only 0.03 per cent of the volume of ordinary air, the absorption is quite marked for wave-lengths 42,700 A[1] and 147,000 A[2] when these waves are received through several meters of air. Rubens and Wartenburg[3] have found dry air to be quite transparent for wave-lengths 230,000 A, 520,000 A, 1,100,000 A, and 3,140,000 A. The absorption bands due to the carbon dioxide in the air are narrow and weak compared to the absorption band of air for ultra-violet light, and it seems doubtful that they can exert any marked influence on dispersion. The effect of these bands is probably similar to that of the *A* band due to atmospheric oxygen which strongly absorbs light of wave-length 7600 A, but relatively small thicknesses of air freely transmit these waves.

Koch[4] measured the index of refraction of air as 1.00028806 for residual rays from gypsum ($\lambda = 67{,}094$ A) and 1.00028875 for residual rays from calcite ($\lambda = 86{,}784$ A). If no absorption band

[1] Paschen, *Wiedemanns Annalen*, **53**, 334, 1894; Statescu, *Philosophical Magazine*, **30**, 737, 1915.

[2] Rubens, *Wiedemanns Annalen*, **64**, 584, 1898.

[3] *Physikalische Zeitschrift*, **12**, 1080, 1911.

[4] *Nova Acta Soc. Upsala* (4), **2**, 1909.

exists between 9000 A and 67,000 A, and if it is permissible to extrapolate the formula representing our observations, we obtain $n=1.0002876$ for the longest waves.

It is probably necessary, therefore, to conclude that if there is any marked absorption of infra-red waves by air, the band of absorption must lie at extremely long waves.

Moreover, the electromagnetic theory has connected the index of refraction n of a medium to its dielectric constant D by the relation $n^2=D$. This relation was derived for long waves and is not generally satisfied in the visible spectrum, where all substances have more or less dispersion. For certain gases, such as hydrogen, nitrogen, and dry air, which show only small dispersion and no strong absorption in the infra-red, $n^2=D$ approximately, even when n is taken for waves in the visible spectrum.[1] For example,[2] D for dry air is 1.000590 and n^2 for sodium light is 1.000584. The dispersion for such substances is well represented by the simple empirical formula of Cauchy

$$n=a+\frac{b}{\lambda^2}+\frac{c}{\lambda^4}.$$

If n^2 is not equal to D for visible rays, the dispersion cannot be represented by Cauchy's formula, but the Sellmeier or Ketteler-Helmholtz formula, which takes into account the free periods corresponding to absorption bands of wave-length λ_κ, must be used. Neglecting absorption for wave-lengths λ, this formula is generally written as follows:

$$n^2=1+\Sigma\frac{m_\kappa\lambda^2}{\lambda^2-\lambda_\kappa^2}.$$

The effect on n of each species of ions K is represented by a term in the sum Σ in which m_κ is the dielectric constant of the K ions. Then $1+\Sigma m_\kappa$ is the observed dielectric constant D of the substance for infinitely long waves. If there are two families of resonance ions, one producing the ultra-violet absorption band of wave-length

[1] Tangle, *Annalen der Physik* (4), **26**, 59, 1908.

[2] Schmidt, *Annalen der Physik* (4), **11**, 121, 1903.

λ_v and the other responsible for the infra-red absorption band of wave-length λ_r, the dispersion is represented by

$$n^2 = 1 + \frac{m_v\lambda^2}{\lambda^2 - \lambda_v^2} + \frac{m_r\lambda^2}{\lambda^2 - \lambda_r^2}.$$

Miss Howell[1] gave a Sellmeier formula for air having an infra-red absorption band of wave-length 31,800 A, "probably due to traces of water vapor," but this seems very doubtful.

If there is only one family of resonance ions, the formula reduces to

$$n^2 = 1 + \frac{m_v\lambda^2}{\lambda^2 - \lambda_v^2}$$

and this first term of Sellmeier's formula gives a satisfactory representation of the dispersion of all transparent substances in which the infra-red absorption is negligible.

The observational equations previously described were solved by the method of least squares and gave the following expression for the dispersion of air at 0° C. and 760 mm:

$$n^2 = 1 + \frac{0.00057378\lambda^2}{\lambda^2 - 595,260}.$$

This simple Sellmeier dispersion formula represents the observations nearly as well as the Cauchy formula given above.

7. SUMMARY

A survey of previous researches on refraction of air shows that many investigators have worked either with white light or with one monochromatic radiation, and dispersion measurements have been limited to a small interval of the spectrum. No index measurements exist for waves longer than those corresponding to orange light, and in the ultra-violet the dispersion formulae disagree by more than 10 per cent of the refractivity.

Recent work in spectroscopy makes it very desirable to have more accurate and extensive data on the index of refraction and

[1] *Physical Review*, 6, 81, 1915.

dispersion of air. The international system of standard wave-lengths expresses the lengths of waves in air at 15° C. and 760 mm, and all wave-length measurements made under other conditions require small corrections because of the effect of temperature and pressure of the air upon its optical dispersion. Furthermore, it is often desirable to multiply wave-lengths measured in air by the indices of refraction of air for these wave-lengths and thus convert them to their value in a vacuum. An accuracy of one part in several millions is now striven for in the measurement of wave-lengths, and to maintain their relative accuracy in the values reduced to vacuum it is necessary to know the indices of refraction within a few units in the seventh decimal place.

For several years this Bureau has been engaged in the accurate measurement of wave-lengths. Interferometer comparisons of standard wave-lengths have been made throughout a large range of spectrum and the grating spectra of more than 50 of the chemical elements have been photographed and measured in the red and infra-red spectral regions. In connection with these accurate measurements of wave-lengths it was thought advisable to measure the absolute indices of refraction of air for the entire spectral region which is accessible to photography.

Accuracy and efficiency recommended the use of the Fabry and Perot interferometer for this work, since this apparatus can be conveniently inclosed in a chamber in which the temperature and pressure of the air can be regulated as desired, and it also permits simultaneous observations for a large number of different wave-lengths. Sections of the circular fringes, produced by various radiations from a source of light illuminating the parallel plates of the interferometer, were photographed either with a grating or a rock-salt prism spectrograph, first when the space between the plates was evacuated, and then when dry air at measured temperature and pressure was present.

The index of refraction for a particular wave-length was obtained directly from measurements of the photographed interference fringes which allowed the ratio of lengths of this wave in vacuum and in air to be calculated. Observations were made at spectrum

intervals of about 40 A from the extreme ultra-violet at 2200 A, through the visible spectrum, and in the infra-red to 9000 A.

Complete sets of observations were made on dry air at atmospheric pressure and at temperatures of 0, 15, and 30° C. These are quite closely represented by the following dispersion formulae of the Cauchy form:

$$(n-1)_0 \times 10^7 = 2875.66 + \frac{13.412}{\lambda^2 \times 10^{-8}} + \frac{0.3777}{\lambda^4 \times 10^{-16}}$$

$$(n-1)_{15} \times 10^7 = 2726.43 + \frac{12.288}{\lambda^2 \times 10^{-8}} + \frac{0.3555}{\lambda^4 \times 10^{-16}}$$

$$(n-1)_{30} \times 10^7 = 2589.72 + \frac{12.259}{\lambda^2 \times 10^{-8}} + \frac{0.2576}{\lambda^4 \times 10^{-16}}.$$

These observations are used in the construction of a table giving the corrections which must be applied to wave-lengths measured in air whose density is not normal. A table of corrections to convert wave-lengths or frequencies measured in air to their values in a vacuum is also given.

The coefficient of index variation with temperature was found from these measurements to be a function of the wave-length. For long waves this optical temperature-coefficient is identical with the density temperature-coefficient, that is, $\frac{1}{273}$, but as the ultra-violet absorption band is approached it increased rapidly, becoming $\frac{1}{258}$ at 2500 A.

There seems to be no definite evidence of any strong absorption of infra-red light by dry air, and it is therefore possible to represent the optical dispersion of air by the first term of Sellmeier's formula quite satisfactorily.

W. F. MEGGERS
C. G. PETERS

WASHINGTON, D.C.
March 13, 1918

THE DISPERSION OF AIR AND THE REDUCTION OF WAVE-LENGTHS TO VACUUM

As long as the wave-lengths of light were founded on the Rowland system it was quite customary, in reducing to vacuum, to use the table printed in Kayser's *Handbuch* (**2**, 514). Since the new International System (I.A.)[1] has come into general use, there has been no uniformity in this reduction. This has led to different values of the frequency in vacuum, even when founded on the same experimental data. Moreover, in some cases the reduction has been made quite incorrectly. There have been carried out, within recent years, several new investigations on the index of refraction of air, thus giving a new chance for variation in the reduction. Finally, there are a number of apparent inconsistencies in the published work relating to this matter. These have caused the author considerable confusion, and may have affected others similarly.

It thus seems advisable to agree on one method for the reduction to vacuum, and a method is here tentatively suggested, with a table of values based upon it. It is perhaps needless to remark that as far as spectral laws are concerned any consistent method of reduction which is not woefully in error would be satisfactory. It is confusing, however, in trying to correlate various spectral formulae, or in checking the data of others, to have in use so many different systems of reduction to vacuum.

The Rowland[2] system of wave-lengths is founded upon the wave-length of the sodium lines in ordinary air at about 20° C. and 760 mm pressure. The International System resulted from Michelson's[3] determination of the wave-length of the red cadmium line (6438.4722 I.A.) in ordinary air at 15° C. (mercury-in-glass thermometer) and 760 mm pressure. In this work no accurate record of the humidity was kept. The experiment was repeated by Benoit, Fabry, and Perot,[4] and this time the humidity was

[1] See *Transactions of the International Solar Union*, **3**, 135, 1910.

[2] *Astronomy and Astrophysics*, **12**, 323, 1893.

[3] *Travaux et Mémoires du Bureau International des Poids et Mesures*, **11**, 1, 1895.

[4] *Ibid.*, **15**, 3, 1913. See also *Transactions of the International Solar Union*, **2**, 109–137, 1907.

carefully considered. The final result was 6438.4696 I.A., reduced to dry air at 15° C. (hydrogen thermometer) and 760 mm pressure. Michelson's determination, as reduced by the foregoing authors to the hydrogen thermometer and to dry air, is identical with their value, if a humidity of 68 per cent be assumed for Michelson's work. The final value of Benoit, Fabry, and Perot has been adopted as the definition of the I.A., giving

$$\text{I.A.}=\frac{\text{wave-length of red Cd line, in dry air at } 15°,\ 760 \text{ mm, with } g=980.67}{6438.4696}$$

The first really accurate measurements of the dispersion-curve for air were made by Kayser and Runge.[1] They used ordinary air and derived as the least-squares solution of the experimental data, in terms of the usual (empirical) Cauchy formula,

$$\mu-1=0.00028787+\frac{13.16}{\lambda^2 10^{15}}+\frac{3.16}{\lambda^4 10^{24}}, \tag{1}$$

where λ is in cm. and μ applies to 0° C., 760 mm pressure. A more customary manner of writing this is

$$10^7(\mu-1)=2878.7+\frac{13.16}{\lambda^2}+\frac{0.316}{\lambda^4}, \tag{2}$$

where λ is in microns.

Kayser[2] in quoting (2) states that λ is in $\mu\mu$. This error has been repeated in Baly's *Spectroscopy* (p. 583), while on page 146 of the same text the unit is given correctly.

In $\mu\mu$ we would have

$$\mu-1=0.00028787+\frac{1.316}{\lambda^2}+\frac{31600}{\lambda^4}, \tag{3}$$

a form of the equation used by some investigators.

The most preferable form for modern work, however, is that in which λ is expressed in angstroms. This form is

$$10^7(\mu-1)=2878.7+\frac{13.16}{\lambda^2.10^{-8}}+\frac{0.316}{\lambda^4.10^{-16}}. \tag{3'}$$

Now (1) applies to ordinary air at 0° C. and 760 mm pressure. Kayser and Runge (*loc. cit.*) estimated that the vapor-pressure

[1] *Wiedemanns Annalen*, **50**, 293–315. 1893. [2] *Handbuch*, **2**, 513.

under the experimental conditions was 5 to 7 mm, and that this would decrease the index about 3×10^{-7}. Therefore for *dry* air we have

$$10^7(\mu-1)=2881.7+\frac{13.16}{\lambda^2}+\frac{0.316}{\lambda^4}, \tag{4}$$

where λ is in microns.

The original experimental data given by Kayser and Runge (*loc. cit.*) apply to formula (2). The same experimental results, however, corrected by 3×10^{-7}, are given by Runge,[1] together with formula (4) and a brief table of corrections to vacuum. The table of corrections given in Kayser's *Handbuch* (**2**, 514) is based on formula (4), while formula (2) accompanies the table and supposedly applies to it. The values given in the *Handbuch* (**1**, 719) are based on formula (2). Thus the corrections to vacuum that nearly all investigators have used (table, *Handbuch*, **2**, 514) are based on Kayser and Runge's determination of the index of refraction for dry air, while the index for ordinary air should really have been used. The difference ($3\times10^{-7}\lambda$) varies from 0.002 to 0.001 A in the visible spectrum. This difference is immaterial, except for the confusion resulting from the apparent inconsistency in the published data.

Since 1893 a number of investigations of the dispersion of air have been made, the most important being by Rentschler,[2] Cuthbertson,[3] Howell,[4] and Dickey.[5] Some investigators, like Cuthbertson, used dry air and quote Kayser's formula (4) correctly as for dry air. Other observers, from the conditions of the experiment, may have used ordinary air. In comparing the various results, however, no distinction has been made, in some cases, between dry and ordinary air. Thus Miss Howell includes Cuthbertson's formula for dry air, Kayser and Runge's for ordinary air, and Rentschler's (dry air) with her own. The objection to this is that the difference (3×10^{-7}) is of the same order of magnitude as the experimental errors.

[1] *Astronomy and Astrophysics*, **12**, 426, 1893.

[2] *Astrophysical Journal*, **28**, 345, 1908.

[3] *Proc. Roy. Soc.* (A), **83**, 151, 1910.

[4] *Physical Review* (2), **6**, 81, 1915.

[5] *Astrophysical Journal*, **45**, 189, 1917.

Before deciding on the best dispersion-curve to use in the reduction to vacuum it seems pertinent to inquire what values have been used by others. For the Rowland system the actual correction in λ is given by $\frac{273}{293}(\mu-1)_0\times\lambda_{vac}$ or $\frac{273}{293}(\mu-1)_0\times\nu_{vac}$ for the correction in terms of ν. One can of course equally well use λ_{air} and ν_{air} in place of λ_{vac} and ν_{vac}. For the International System the correction should be $\frac{273}{288}(\mu-1)_0\times\lambda_{vac}$, etc.

To make the table of corrections printed in Kayser's *Handbuch* (**2**, 514) available for the I.A. system the printed quantities should be multiplied by $\frac{293}{288}$. This is what Hicks[1] has done, increasing the printed values by $\frac{1}{60}$ of themselves—a close enough approximation. Hicks is here using Wood's[2] experimental data for the principal sodium series. Wood's own reduction to vacuum is incorrect and it is not apparent to the author how it was obtained.

Fowler[3] and Nicholson[4], although the measurements are in I.A., use Kayser's table uncorrected. The error is thus $\frac{1}{60}$ of the reduction-factor and so varies from 0.018 to 0.035 A in the visible spectrum. This, as mentioned, is of no consequence in fitting spectral formulae, but it can only cause confusion and does not seem a wise practice. Curtis[5] also makes this error, and here it *does* make a difference in the results, since it changes the value of the Rydberg Universal Constant N_0. As the author has pointed out in a separate communication in *Science*,[6] the error will not affect the accuracy with which any given formula fits the data, nor will it affect the universal constancy of N_0. The correct value of N_0 from Curtis' data is 109,678.705 instead of 109,679.22. The author[7] has already obtained 109,678.6 by direct conversion from the Rowland system of the best previous measurements. Curtis' work therefore checks with the older measurements much better than he himself

[1] *Astrophysical Journal*, **44**, 230, 1916.

[2] *Ibid.*, **43**, 73, 1916.

[3] *Proc. Roy. Soc.* (A), **91**, 208, 1915.

[4] *Ibid.*, 255, 1915.

[5] *Ibid.*, **90**, 605, 1914.

[6] *Science*, **48**, 47, 1918.

[7] *Astrophysical Journal*, **32**, 114, 1910.

supposed. There are thus being used at least three different methods of reduction to vacuum, only one of which is correct.

The foregoing constitutes the major portion of a paper which was in the printer's hands, and about to be published (July 1918), when the author learned that the Bureau of Standards had, in spite of the war, continued its work[1] on the dispersion of air, and that the results were about to be published. Accordingly the present paper was withheld until now, when it is possible to present it in conjunction with an account of the work of the Bureau.[2] This new work, because of its great accuracy, completely supersedes all former work on this subject. It is therefore unnecessary to give here the table of corrections to vacuum, prepared by the author, nor is it necessary to present the comparative results of previous investigators. The Bureau article gives a complete synopsis of previous work and also an extensive table of corrections to vacuum. This table should be used in all future work when the wave-lengths are measured in I.A.

RAYMOND T. BIRGE

UNIVERSITY OF CALIFORNIA
December 1918

REGARDING THE *BULLETIN ASTRONOMIQUE*

TO ASTRONOMERS AND GEODESISTS:

L'Observatoire de Paris se propose de consacrer désormais son *Bulletin Astronomique* à la publication en langue française de très courts résumés des travaux relatifs à l'Astronomie et à la Géodésie. Nous voudrions que ces résumés fussent faits par les auteurs eux-mêmes; nous les publierons en francais mensuellement, immédiatement après les avoir reçus.

Le titre du recueil sera changé et deviendra:

Revue générale des travaux astronomiques publiée par l'Observatoire de Paris.

[1] See *Scientific Papers of the Bureau of Standards*, No. 312, 380, 1918.

[2] *Ibid.*, full paper, No. 327, October 31, 1918.

Vous faciliteriez grandement notre entreprise en nous envoyant, dès la publication de vos mémoires ou de vos ouvrages, un exemplaire de chacun d'eux avec le court résumé ou l'analyse à insérer dans la *Revue générale des travaux astronomiques*.

B. BAILLAUD, *Le Directeur*

OBSERVATOIRE DE PARIS

NEW GENERAL INDEX OF *ASTROPHYSICAL JOURNAL*

It is the hope of the Editors that a General Index to Vols. XXVI to L of the *Astrophysical Journal* may be issued as promptly as possible after the close of the fiftieth volume with the number for December 1919. It is the present plan to follow the style of the General Index, by authors and by subjects, to Vols. I to XXV, compiled by Professor Storrs B. Barrett and published in 1908 by the University of Chicago Press ($1.50 postpaid). The Editors, however, will be very glad to receive from subscribers any criticisms or suggestions which may tend to make the new index more serviceable, if possible, than its predecessor. Such communications may be addressed to the Managing Editor, EDWIN B. FROST, Williams Bay, Wisconsin.

ERRATA

Vol. **49**, No. 3, April 1919, article on:

"The Magnetic Polarity of Sun-Spots," by George E. Hale, Ferdinand Ellerman, S. B. Nicholson, and A. H. Joy:

Page 158, line 17, *for* "R 28" *read* "V 28."
Page 158, line 20, *for* "V 7" *read* "R_7."
Page 163, line 26 (second line from bottom), *for* "over" *read* "of."
Plate VII, facing page 166, line 5 in legend, *for* "Plate IV *i*" *read* "Plate V *i*."

Reviews

An Introductory Treatise on Dynamical Astronomy. By H. C. Plummer. London: University of Cambridge Press, 1918. American agents, G. P. Putnam's Sons, New York. Pp. 343, figs. 7. $5.50.

Professor Plummer's treatise is intended as an introduction to those portions of astronomy which require dynamical treatment. Hence the main chapters of the book are concerned with the astronomical developments of the Newtonian laws of motion and gravitation. But in a volume of this size it is only possible to treat a limited number of the problems which arise, and the author has confined himself mainly to those which concern the motions of the centers of mass of the bodies about one another under their mutual attractions and of the rigid bodies about their centers of mass. A large portion of the work thus deals with the classic problems of two or three bodies, as represented by the planets and satellites of the solar system, and by the motions of the earth and moon about their respective centers of mass. The more modern portions of the book contain two chapters on the methods by which the orbits of double stars and of spectroscopic binaries are obtained.

Professor Plummer assumes a considerable knowledge of analytical and dynamical processes. The proofs are in general developed quite briefly with the essential steps, but we should imagine that the work will be difficult reading for those who have not had a somewhat extensive training in those parts of mathematics which require symbolic manipulation and the physical interpretation of symbolic results. The reader will also require some considerable familiarity with geometrical astronomy in order to understand in detail the bearing of the various conclusions reached. This is more on account of the brevity, which is perhaps a necessary result of the attempt to compress so much into a single volume, than of omission on the author's part. The book will, we believe, be found more useful to a teacher who wishes to use it as a basis for lectures than to a student who is approaching the subject without assistance.

The various phases of the problem of two bodies naturally occupy a considerable portion of the volume. After giving the elliptic expansions

in several forms as well as the theorems necessary to deal with parabolic and hyperbolic orbits, applications are made to the determination of such orbits from observation. Here, as elsewhere, but little effort is made to deal with the details of the methods which are set forth. For these the reader must go to the special volumes or memoirs dealing with the respective subjects. The dynamical portion of all this work is, of course, the same—namely, motion in a conic section. The chief difficulties are observational and geometrical. For comets and asteroids the given data are positions and the masses of the bodies which mainly determine the motion. For double stars the masses and planes of motion are in general unknown, and we are given only a series of projected distances between the bodies, together with the apparent orientation of these distances. For spectroscopic binaries velocities in the line of sight chiefly constitute the known facts, and so on. The results which can be deduced in this way are exact so far as they go, and they furnish certain relations which must be satisfied if the law of gravitation holds in such systems. This part of the work is, of course, quite independent of the information which may be extracted from the rapidly growing body of knowledge deduced from stellar classification and statistics.

For the planetary theory Professor Plummer adopts the method of the variation of the elements. This method was used by Leverrier in his theories of the planets, and it is certainly the most interesting from the point of view of the mathematician and the most useful for a geometrical description of the motions of the bodies of the solar system. For the major planets, however, the direct method of solution as developed by Newcomb has in the past given an easier mode of approach to numerical results except in the case of Jupiter and Saturn, where G. W. Hill used Hansen's method somewhat modified. But the work in all these cases is simple so long as attention is confined to first-order perturbations. For the general treatment of secular perturbations a chapter contains the method developed by Gauss. The least satisfactory part of Professor Plummer's discussion seems to be that which deals with the connection between the ordinary mode of development in the general theory and the stability of the system. Powers of the time in the coefficients of the periodic terms can be avoided as far as a purely formal expansion of the co-ordinates is concerned, in the same way that they are avoided in the lunar theory. But none of these developments are of much value when we come to a consideration of stability. Practically all of them are divergent series and are only available for a limited time or for a limited

degree of approximation. Thus the old theorems concerning the stability of the major axes, eccentricities, and inclinations, as deduced from the terms of the first and second orders, have now little more than a historic and mathematical interest. Their chief value is a formal one in showing the forms of early terms of the series which are used to represent the positions of the bodies.

The classical methods are well adapted to the calculation of the orbits of the major planets, but very considerable changes are necessary if attempts be made to apply them to many of the asteroids. Professor Plummer only touches slightly on them. An unfortunate slip occurs on page 191, where it is stated that commensurability of the mean motions is inconsistent with stability, evidence for the statement being brought from the fact that the asteroids appear to avoid the regions where their mean motions would be commensurable with that of Jupiter. No proof of this has yet been given. On the contrary, all the investigations so far made indicate that such motions are stable and that the apparent gaps must be explained in some other way. When commensurability is closely approached, the type of motion changes: exact commensurability occurs with an oscillation about that motion, generally called a libration. It is true, as Poincaré has shown, that there are certain conditions under which commensurability cannot occur in the form of a periodic solution; in some of these cases, however, it has been shown that libration still exists but that the oscillation cannot be infinitesimal. In this connection the remark on page 243, that no periodic orbits exist about the triangular equilibrium points when the masses fail to satisfy a certain condition, is open to misconception; it should be added that such orbits exist but that they cannot in general be infinitesimal.

In the later chapters Professor Plummer gives a rapid survey of the Euler-Hill-Adams method for developing the lunar theory, as worked out by the writer, and of the motions of the earth and moon about their respective centers of mass. A useful chapter is the last, on numerical computation, in which the formulae of mathematical quadrature and of harmonic analysis are developed.

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Yale University, New Haven
June 1910

THE

ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME L SEPTEMBER 1919 NUMBER 2

ON THE CAUSE OF CEPHEID VARIATION

SECOND PAPER

By C. D. PERRINE

Several hypotheses have been proposed to account for the Cepheid type of variation, but as yet a decisive test has not been found and the cause remains uncertain. The discovery of the variations of the radial velocities of these stars and the possibility of representing these variations by orbital motion seemed to be a long step toward the solution of their peculiar light-variations.

The closeness with which these variations are represented by orbital motion, usually well within the known uncertainties of the velocities observed, is in itself, in the absence of proof to the contrary, almost conclusive evidence of their binary character.

Some investigators have found difficulties, however, in explaining some of the conditions encountered upon a binary assumption, which have led as far even as suggestions to overthrow entirely the evidence of variable radial velocities and to ascribe the observed shift of the lines to activity in a single body. So far as I am aware, however, it has not been proved conclusively that these stars cannot be binary.

When the early spectroscopic results showed such a startlingly large proportion of binaries, I must confess that I wondered for a time if any pulsation effect were possible in single stars which

could cause such shifts of lines. I finally came to the conclusion that the displacements of the lines must be due to orbital motion, because it was difficult to conceive of internal pulsations being as uniform both in length of period and in amount of variation as is the case in the Cepheid variables, and because the lines, even in the stars showing the greatest ranges of velocity, appear to show few signs of violent disturbance.

The irregularities in the spot period of our own sun and the certainty that the underlying action is a pulsation effect of some sort in a body whose spectral condition is in general closely allied to that of the Cepheid variables, as well as the irregular nature of the variations in brightness of the novae and of many of the variables of long period and late spectral type, lead me to think that pulsation effects in general would probably be much more irregular than is the case in the Cepheid stars. This, it seems to me, is what might be inferred from a consideration of the nature and causes of pulsation effects in general. There can be little doubt that orbital motion would produce much greater uniformity than purely internal action of any kind not due to the presence of a secondary body.

While it is true that there is not entire uniformity in the periods nor in the amount of variation of the Cepheids, the deviations are very small and can hardly affect the foregoing reasoning to any considerable extent.

It is to me inconceivable that pulsations in a single body sufficient to double the light could occur in the short periods of these stars, a few days or hours, with any such regularity as that observed.

I find but little published information as to the character of the lines in the spectra of the Cepheids. In most of the orbital investigations examined no mention whatever is made of the character of the lines. Wright remarks[1] upon the breadth and haziness of lines in the spectrum of η Aquilae, which appear to be more marked in some lines than others, and lays stress on the well-known difficulties due to blends. A careful investigation of the widths and structure of the lines in the spectra of these stars would be a useful piece of work.

[1] *Astrophysical Journal*, **9**, 60, 1899.

If we consider the integrated effect of a general pulsation in a star's atmosphere, we must come to the conclusion, I think, that it would cause a considerable widening of the lines. Suppose, as the simplest case, that the pulsation is uniformly radial and outward. The light of such a star entering the slit of the spectrograph would be made up of a large amount from the central portion of the star's disk, which would give a displacement toward the violet, whereas the light from the edges of the disk would be nearly without Doppler-Fizeau effect and would, therefore, occupy a nearly normal position. As the variations of radial velocity in this class of stars are large, the effect of such an integration should be noticeable on the width of the lines.[1] There are also the pressure effects to be considered.

If the pulsations were uniform it might be conceivable that in some of these stars the lines could be wide enough to mask such pulsations. A consideration of the widenings of lines in the novae, the Wolf-Rayet stars and the bright-line hydrogen stars, stars which are undoubtedly very active, seems to render doubtful an assumption of uniformity of pulsations. Our sun is approximately of the same spectral type as the Cepheid variables, and there are certainly no pulsations in the sun to cause such changes of light. With activity sufficient to cause the general displacement of lines observed in the Cepheid stars, I should expect greater widening of the lines and more distortion generally than appears to have been observed.

However this may be, it is desirable definitely to confirm or negative the binary nature of these stars by independent evidence. To this end I have sought for other facts, and the object of this paper is to consider some such and other points relating to these stars.

IRREGULARITIES IN THE LIGHT-CURVES

Irregularities in the light-curves of such of these stars as have had orbits determined for them have been found, which seem to bear on the matter.

[1] Such an effect would lead to a preponderance of negative velocities from the Cepheids, which is not the case among those for which the velocities of the system are known.

The ratios of the masses of the components of all such stars are small, but there is an appreciable range in the values. If such irregularities in the light-curves agree with the relative masses which are indicated for the secondaries, this would be independent evidence of considerable weight that these bodies are in reality binary systems.

The irregularities which have been observed in the light-curves of some of these stars are in the nature of retardations after maximum or, in the most pronounced cases, an actual recovery of light, forming a "hump" in the curve.

In Table I is given a short summary of these characteristics for the stars in question. The stars are arranged in order of the ratios of masses $\frac{m_1^3 \sin^3 i}{(m+m_1)^2}$.

TABLE I*

Star	Mag. Max.	$\frac{m_1^3 \sin^3 i}{(m+m_1)^2}$	"Humps" in Light-Curves	Light Minima Rounder
α Ursae Minoris	2.1	0.00001	....	....
β Cephei	3.3	.0001	None	Yes
SU Cassiopeiae	5.9	.0003	None	Both rounder
SZ Tauri	7.2	.0004	None	....
RR Lyrae	7.0	.0006	None	Yes, strong
Y Ophiuchi	5.8	.0011	Slight ?	Yes
X Sagittarii	4.0	.0016	None	Yes
RT Aurigae	5.0	.0018	Slight negative curvature	Yes
T Vulpeculae	5.5	.0018	One observer, yes; others, none	Yes
ζ Geminorum	3.7	.0023	None	Both rounder
δ Cephei	3.7	.0028	Yes	Yes
Y Sagittarii	5.8	0040	Strong	Yes, strong
η Aquilae	3.5	.0043	Strong	Both much rounder
S Sagittae	5.6	.0049	Strong	....
W Sagittae	4.8	0050	Strong	Yes
SU Cygni	6.7	0.0058	Strong	....

* The stars α Ursae Minoris and β Cephei appear to satisfy the essential characteristics of Cepheid variation and have been classed with these stars in the present investigations. For the purpose of charting, the eccentricity of β Cephei has been assumed as 0.05.

From these results it is seen that for all stars in which the mass-ratios are 0.0023 or less either there are no "humps" or they are so slight as to be doubtful, whereas those of 0.0028 and higher

all show markedly this characteristic. The evidence from so limited a number of stars could not well be numerically more definite in showing a relation of these humps in the light-curves to the relative masses of the secondaries of binary systems which have been determined from the radial velocities of these same stars on the assumption that the observed variations are due to a true Doppler-Fizeau effect. This is irrespective of whatever may be the way in which the secondary produces the fluctuations of light. As it should be on such an assumption, it is the larger secondaries which produce the humps.

The consistency of the evidence is such as to give confidence in its essential reality, notwithstanding the uncertainties of the orbital inclinations. It would be a strange coincidence that would cause such an agreement, if only fortuitous.

DIFFERENCES BETWEEN LIGHT-CURVES AND VELOCITY-CURVES

The general agreement of the light-curves and velocity-curves of the Cepheid variables has been frequently remarked by investigators. A lack of complete agreement in the times of maxima and minima has been observed also.

In addition to this lack of agreement of phase, I have found differences between the light-curves and velocity-curves of these stars which are probably significant.

These differences for the 13 stars for which I have orbits[1] may best be described as a tendency for the light-curve at minimum to be much broader and rounder than the velocity-curve at the same phase. This characteristic is universal in the 10 stars for which I have velocity-curves. In 8 out of these 10 the light-curves are sharper at maximum than the velocity-curves. These deviations are noted in the fifth column of Table I.

There appear also in several cases to be wider ranges of the individual light determinations at times of minima. A careful study of the accuracy of determinations and perhaps even more accurate observations will be necessary to establish the correctness of this suspicion. Some such condition is not impossible, however.

[1] *Astrophysical Journal*, 41, 308, 1915.

ORBITAL ECCENTRICITIES

An examination of Campbell's Tables I, II, III, and V[1] shows two peculiarities, the first of which is especially striking:

A. The increase in eccentricities of the orbits of the stars in these tables with period, in the spectral classes B, A, and F, is not continuous but is abrupt, changing at periods of about 15 days. In each of Tables I, II, and III a line can be drawn such that all shorter periods have the eccentricities very small and all longer periods have eccentricities much larger. The eccentricities have been separated at the critical point and the results are given in Table II.

B. The amount of eccentricity appears to depend upon the relation of masses to separation, the systems with mass-ratios small in proportion to separations having larger eccentricities than those with more nearly equal masses for the components. This peculiarity needs careful study and is reserved for separate investigation.

After the foregoing discontinuity in the eccentricities was noticed it occurred to me that the eccentricities of the Cepheid variables were considerable on the average, that their periods were short, and that Campbell had omitted them from his Tables I, II, and III because of their peculiarities generally. They were then entered on the chart (Fig. 1) as circles. It is readily seen that they fill the gap satisfactorily. If a horizontal line is drawn at $e=0.16$ as far as periods of 17 days, *all of the Cepheids fall within the region of eccentricity 0.16 or greater and periods of 17 days or less*[2] *and only one star not known to vary in brightness falls well within the same area.* The conclusion is thus suggested that the variations of brightness and the larger eccentricities of the Cepheids separate them from other short-period binary systems which do not show variations of brightness, and that binaries with small eccentricities (except eclipsing stars) for some reason do not vary noticeably in light.

[1] *Lick Observatory Bulletin*, 6, 36, 1910.

[2] The subsequent inclusion of Polaris and β Cephei as Cepheids modifies the literal correctness of this statement but not its essential significance.

TABLE II

Spectroscopic Binaries	Star	Vis. Mag.	Period	Eccentricity Small	Eccentricity Large
Types O and B	β Canis Majoris	2.0	0ᵈ25	0.1±	
	u Herculis	Var.	2.05	.05	
	ψ Orionis	4.7	2.53	.06	
	γ Lacertae	4.7	2.62	.02	
	Algol	Var.	2.87	.04	
	α Virginis	1.2	4.01	.10	
	δ Orionis	2.5	5.73	.10	
	κ Cancri	5.1	6.39	.15	
	α Carinae	3.6	6.74	.18	
	η Orionis	3.4	7.99	.02	
	π_4 Orionis	3.8	9.52	.03	
	α Pavonis	2.1	11.75	.01	
	β Lyrae	Var.	12.91	.07	
	β Orionis	0.3	21.90		0.30
	B.D. −1°1004	5.0	27.16		.76
	ι Orionis	2.9	29.14		.75
	κ Velorum	2.6	116.65		.19
	φ Persei	4.2	126.5		.43
	ζ Tauri	3.0	138.		.18
	π Andromedae	4.4	143.72		.58
	μ Sagittarii	4.0	180.2		.44
	Mean			0.07	0.45
Type A	τ Tauri	4.3	1.50	0.08	
	δ Librae	Var.	2.33	.05	
	α_1 Geminorum	2.8	2.93	.01	
	β Aurigae	2.1	3.96	.00	
	ε Herculis	3.9	4.02	.07	
	α_2 Geminorum	2.0	9.22		0.50
	B.D. +66°664	4.9	11.6		large
	θ Aquilae	3.4	17.11		.69
	α Coronae Borealis	2.3	17.36		.33
	ζ_1 Ursae Majoris	2.4	20.54		.50
	β Ursae Majoris	2.4	27.16		.79
	α Draconis	3.6	51.38		.40
	η Virginis	4.0	71.9		.33
	α Andromedae	2.2	96.7		.51
	β Arietis	2.7	107ᵈ0		.88
	γ Geminorum	1.9	3ʸ5		large
	Sirius	−1.6	49ʸ3		.59
	Mean			0.04	0.55
Type F	13 Ceti	5.2	2ᵈ08	0.06	
	θ Draconis	4.1	3.07	.01	
	ζ_1 Lyrae	4.3	4.30	.00	
	ω Draconis	4.9	5.28	.01	
	ι Pegasi	4.0	10.21	.01	
	ο Leonis	3.8	14.50	.02	
	χ Draconis	3.7	281.8		0.42
	δ Equulei	4.6	5ʸ7		.36
	α Ursae Minoris	2.1	11ʸ9		.35
	ε Hydrae	3.5	15ʸ7		.6
	Mean			0.02	0.43

TABLE II—*Continued*

Spectroscopic Binaries	Star	Vis. Mag.	Period	Eccentricity	
				Small	Large
Types G to M	λ Andromedae......	4.0	20^{d}54	0.11	
	Capella..........	0.2	104.02	.02	
	β Herculis..........	2.8	410.58		0.55
	η Boötis..........	2.8	489^{d}14		.18
	η Pegasi..........	3.1	2^{y}24		.16
	β Capricorni.......	3.2	3^{y}77		.44
	α Scorpii..........	1.2	5^{y}8		.20
	ζ Herculis..........	3.0	34^y		.46
	Mean............			0.06	0.33

It is of interest to examine the exception noted above (α_2 Geminorum), and another (B.D.+66°664), which, because the numerical value of the eccentricity was not given but simply stated to be "large," could not be properly charted. The periods

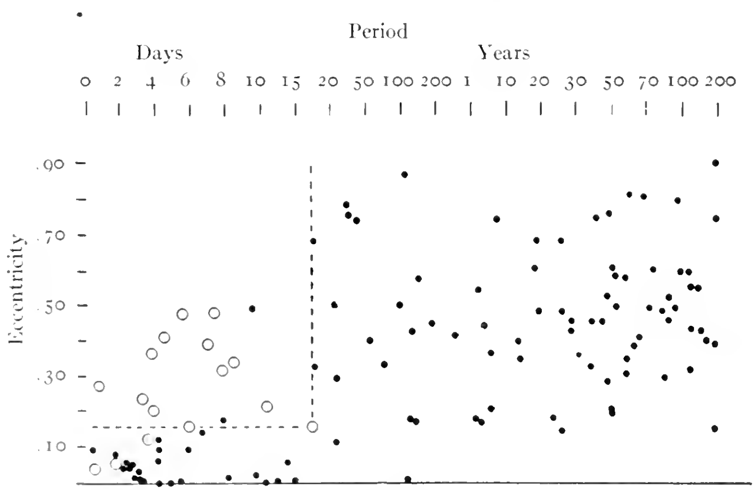

FIG. 1.—Orbital eccentricities of binary stars

○ = Cepheid variables

• = non-variables

of α_2 Geminorum, and B.D.+66°664, are 9.2 and 11.6 days respectively. It is thus seen that for periods of less than 9 days the separation at eccentricity 0.16 is essentially absolute for the data used.

As there are some variables which appear to belong to the Cepheid class with periods longer than 17 days, it does not seem likely that the lines of demarcation noted above are more than general, nor is it to be expected that it would be otherwise. Almost any physical hypothesis, especially if we conceive of a general unity of all these processes of change, would scarcely provide hard-and-fast limits for these phenomena. That Cepheid variation is practically limited to periods shorter than about 17 days is shown by Campbell,[1] who finds that out of 53 known Cepheids only 5 have periods longer than this.

The well-known limitations that the Cepheids, so far as data are available, have very small proper motions and that they are galactic in their preferences, furnish an explanation of these two exceptions. α_2 Geminorum has a galactic latitude no larger than some Cepheids, but its large proper motion, 0."203, differentiates it, probably in the matter of distance. It is also a multiple system. B.D.+66°664 has a small proper motion, 0."025, but its non-galactic tendency exempts it. It is to be noted that the spectral class of these two stars, class A, also tends to place them outside the region of this kind of variation, as practically all true Cepheids are of classes F, G, and early K.

The universally close relation between the velocity-curves and light-curves of all Cepheids spectroscopically observed justifies the conclusion, from their eccentric light-curves, that the orbital eccentricities of these stars are essentially all large.

These characteristics, especially that designated by B, almost undoubtedly have an important bearing on other and general problems related to evolution and to the physical condition at least of binary stars. A consideration of their bearing in these directions may well await advances in other investigations. Attention may be called, however, to the practically entire lack of small eccentricities among the stars of long period in the data above considered. The change in this respect at 17 days is as abrupt as in the complementary case of the non-variable stars of short period and small eccentricity. In fact it is so striking as to suggest the query whether there may not be some relationship—whether by some

[1] *Lick Observatory Bulletin*, **6**, 51, 1910.

process of selection or action the short-period stars have been converted from those of long period and large eccentricity. That the abrupt change of eccentricity with period is not a simple dependence upon length of period is abundantly clear. It seems

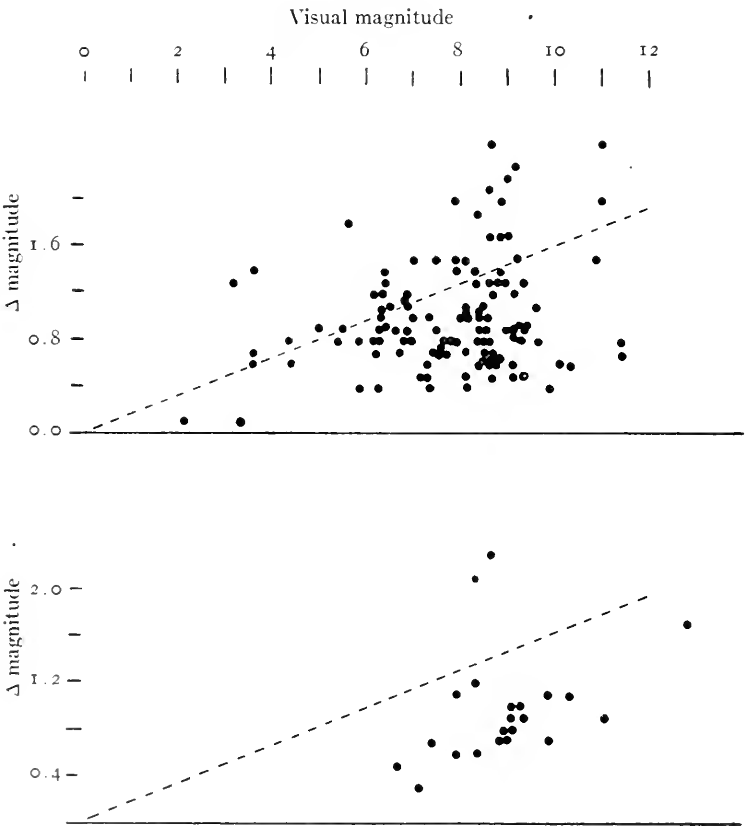

FIG. 2.—Variation of Δ magnitude with visual magnitude
Short-period variables: Galactic (upper figure)
Non-Galactic (lower figure)

to me equally certain that it is not a general physical condition such as might be expected to cause progressive change of spectrum or condition, but that it is probably some condition external to the stars.

The complementary relation of the Cepheids and non-variable stars of short period in the matter of orbital eccentricities is highly significant. As both spectra have been observed in short-period stars of small eccentricity, including Algol and β Lyrae variables, the binary interpretation of their line-shifts is placed beyond question. To assume that the line-shifts of Cepheid stars are not due to orbital motion is to assume that essentially all short-period binaries must have very small eccentricities as well as to reject entirely the interpretation of such consistent line-shifts.

In connection with the objection to duplicity of the Cepheid variables that their orbits appear to be almost impossibly small, the star β Cephei is of especial interest. Frost states[1] that "on some of the plates, certain lines have the appearance of complexity, and indicate the presence of a second-component spectrum." This star is of B type, all of which are under suspicion of being active. The very small size which is indicated for the secondary of this system makes it seem doubtful whether there is sufficient light from the secondary for its spectrum to be detected, unless through emission. This is not impossible, considering the type of spectrum of β Cephei and the fact that emission has been observed in such stars. Should Frost's observation be confirmed with regard to the spectrum of the secondary, we have valuable evidence of duplicity of stars of small orbital dimensions and secondaries, for the value of $a \sin i$ which he finds for β Cephei is only 45,000 km. The corresponding value of $\frac{m_1^3 \sin^3 i}{(m+m_1)^2}$ is 0.0001. These values are smaller than for any other Cepheid for which I find orbits, with the exception of the mass-ratio of Polaris. If this star is double, then small orbital dimensions certainly do not stand in the way of duplicity for the Cepheids. Frost discusses briefly the possible inclination of its orbit to the sight-line. A large inclination of orbit to the sight-line may be correct in the case of this star. Even so, as Frost concludes, its real orbit must be small. It cannot reasonably be urged that all the Cepheid orbits which have been determined have excessive inclinations to the sight-line. To do so would be to admit that these stars all

[1] *Astrophysical Journal*, **24**, 261, 1906.

have most improbable relations, not only to the observer, but to the plane of the Galaxy. Their orbits are undoubtedly small and the secondaries much smaller than the primaries.

β Cephei has been inserted in Fig. 3 as a circle. Its deviation from the other Cepheids in the way of a considerably larger orbital velocity for a small eccentricity, together with its very different spectral type, indicates not only a rather wide spectral range over which Cepheid variation can operate, but very close relations of the underlying cause and the cause of spectral condition generally.

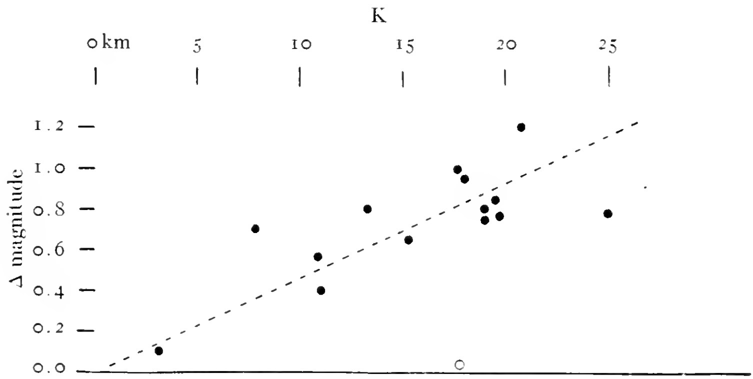

FIG. 3.—Variation of Δ magnitude with orbital velocity

The admission of β Cephei as a true Cepheid implies that this type of variation is largely dependent upon other than a narrow range of spectral condition. In my opinion this is in itself a strong argument against any theory of pulsation alone. A single star cannot be considered of much weight in a matter of the kind, but as far as it goes it is not inconsistent with greater activity and larger masses in the stars of spectrum of type B. It is also noticeable that the orbital velocities of the Cepheids appear not to differ materially from the other binary systems of the same spectral classes (F and G) and that as a rule the B and A stars have higher average orbital velocities than those of later type, indicating also larger masses. Here, with but few exceptions, we are again confronted by the unknown inclinations of the orbits of the binaries. That this uncertainty will to a large extent dis-

appear when we are dealing with considerable numbers of stars seems probable.

R Canis Majoris has a peculiar interest in this connection. It is an eclipsing star both by light-change and velocity-curve, is of Fo type, and the secondary has a mass-ratio of only 0.0027. It is of short period, 1.14 day, and has a small orbit, $a \sin i$ being only 443,000 km.[1] The $a \sin i$ and mass-ratio are about the average of the Cepheids. R Canis Majoris is shown not only by line-displacements but also by eclipsing phenomena to be binary. In other words its duplicity appears to be established in two entirely independent ways. If, then, it is binary there is nothing in small orbits to hinder the Cepheids from being binary also.

Perhaps one of the strongest bits of direct evidence in favor of duplicity of these stars is the fact that the ratios of the masses of the components come out so small; for it is only such stars which, upon the hypothesis of revolution in a resisting medium of some kind, can show a variation in brightness. If the surface areas are nearly equal, little or no variation is possible, for obvious reasons. It would appear to be even more definite in indicating that the brighter star is the larger and that the chief variations of light are in the larger bodies. From the point of view that the known spectroscopic binaries of classes F and G, with the necessary orbital conditions, have without exception been found to vary in brightness, this evidence becomes even stronger for such an explanation of Cepheid variability.

Probability.—On the theory of chance alone some of the Cepheid stars should be binaries. All of these stars investigated have shown variable radial velocities, as usually interpreted, and the opinion is general that all will show these same displacements when observed. Of the sixteen stars of this class whose orbits are available, the probability is strong that, on the theory of chance alone (unless the Cepheids have essentially no relation to other stars), one or more are in fact binary, irrespective of what interpretation may be placed upon the line-displacements. No physical reason has been adduced that I know of to show that none of these stars can be binary. If, therefore, there is no

[1] Jordan, *Publications of Allegheny Observatory*, **3**, 49, 1912.

evidence that these stars cannot be binary, and a strong probability that one or more are in fact binary, there is no reason to assume that the line-displacements of all are not in fact due to orbital motion.

The difficulties of explaining some of the Cepheid peculiarities on a binary hypothesis must be positive and overwhelming before the rejection of all such evidence can be seriously considered.

In my opinion the following four pieces of evidence far outweigh any suspicions to the contrary and leave no room to doubt the binary character of the Cepheid variables:

1. Representation of the observed variable line-displacements satisfactorily by orbital motion.

2. The apparent relation between the relative masses of such secondary bodies and the irregularities in the light-curves.

3. The limitation of non-eclipsing variation in brightness among stars of short period to such as, upon binary assumption, yield large orbital eccentricities, whereas essentially all other short-period spectroscopic binary stars yield nearly circular orbits; and the abruptness of the limits.

4. Other evidence of a more general nature, such as the strong presumption of a similarity in constitution and evolutionary processes among all stars; the repeated observation of both spectra in short-period systems and in which the secondaries usually show much larger masses than in the case of the Cepheids, just as they should do upon such an explanation; spectral relations; orbital velocities, etc.

In the course of this investigation relations have been suspected between the amount of variability and brightness among the short-period variables and between the amount of light-variation and orbital velocities of the Cepheids which have had orbits determined.

Dependence of variability upon magnitude.—The greatest variations of brightness (Δ mag.) among the short-period variables appear to be among the fainter stars. The individual results taken from the list in the *Vierteljahrsschrift der Astronomischen Gesellschaft* for 1913 were plotted for the galactic and non-galactic stars separately, as in Fig. 2. The dependence does not appear

to be very close, as is evidenced by the considerable number of stars from sixth to ninth magnitude, with variations of brightness slightly below the mean. The stars farther from the galactic plane appear to show smaller variations of brightness on the average than do those nearer. Several possible explanations for such a relation suggest themselves. If the primary cause of variability is motion in a resisting medium, such a dependence may be due to a difference of size, the smaller bodies intercepting more matter in proportion to their masses than the larger ones, the activity being consequently greater in the smaller bodies. It might also result from the revolution of the primary of a binary system about the common center of gravity, the larger primary bodies having relatively smaller orbits and velocities than the small primaries. It is conceivable also that it might result from pulsation effects, the pulsation being greater in the smaller stars. The relation to orbital velocity makes it seem more probable, however, that the underlying cause is due chiefly to some external cause rather than wholly to an internal one.

RELATION OF VARIATION OF LIGHT TO ORBITAL VELOCITY

If the variation of light of these stars is largely due to rotation in a resisting medium, a relation between orbital velocity and amount of variation might be expected. The amount of material for such an investigation is limited to 16 stars. An inspection seems to indicate some such effect, as is shown in Table III, and in detail in Fig. 3.

TABLE III

No. of Stars	K	Δ Mag.	$\frac{m_1^3 \sin^3 i}{(m+m_1)^2}$
	km		
8...........	12.0	0.44	0.0010
8...........	20 4	0.80	0.0036

The present data (Fig. 3) indicate a fairly consistent linear relation between orbital velocity and brightness-variation with zero as origin for both factors. This indicates a simple dependence upon the speed with which a body moves through a resisting medium.

It is to be borne in mind that K is not the true orbital velocity except in cases where the orbital plane coincides with the line of sight, but is affected by the factor sin i. Also that there is a relation between velocity in the orbit and eccentricity. In several cases considerable changes (or divergences) have been observed in the light-curves. These do not, however, alter the apparent dependence upon orbital velocity.

Dependence of light-variation upon orbital eccentricity.—There is an apparent dependence of the brightness-variation upon orbital eccentricity, as shown in Fig. 4. If Polaris and β Cephei are

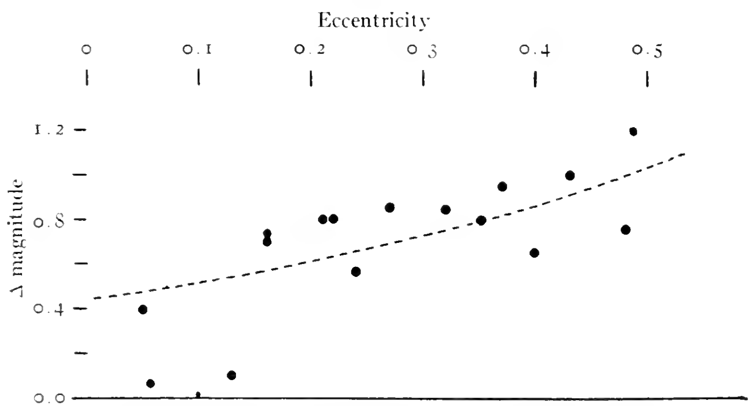

FIG. 4.—Variation of Δ magnitude with orbital eccentricity

omitted the remaining 14 stars show a curve which can be very well explained as a dependence upon eccentricity with the origin, for a circular orbit, at Δ magnitude of about 0.5. The dotted curve represents the variation of brightness with eccentricity on the assumption that it is due simply to the increase of velocity at periastron over that at apastron. The amount of data is too limited to determine the form of the curve, but seems to point fairly definitely to some sort of relation. Such a dependence upon orbital eccentricity is further evidence that the Cepheids are in reality binaries.

Dependence of light-variation upon period.—There appears to be no close relation between the amount of light-variation and length of period. The short-period variables with periods greater

than ten days show, however, a greater light-variation than do those of shorter period, but there does not seem to be any progression; it is apparently abrupt. This conclusion is based upon the 179 short-period variables given in the *Vierteljahrsschrift der Astronomischen Gesellschaft* for 1917. These may not all be true Cepheids, but it does not seem probable that the exceptions will materially affect the foregoing conclusion. The average magnitude-variation for the 179 stars is 1.04; 65 of these have periods of over 10 days, their average brightness-variation being 1.23 magnitude; 10 of these have nearly symmetrical light-curves and their light-variations are small. If these are rejected, the 55 remaining stars give an average brightness-variation of 1.33 magnitude. The 27 stars with periods of over 20 days (average 47 days) give essentially the same light-variation as those between 10 and 20 days. There are no indications of a dependence upon period among the stars with periods shorter than 10 days. Omitting the 65 stars with the longer periods, the remaining 114 stars with periods under 10 days give a mean light-variation of 0.93 magnitude.

The consistency of the larger light-ranges among the longer periods is shown by the fact that 44 of the 65 (or 55) have variations of 1.0 magnitude or greater.

The available data has been examined to determine, if possible, which conditions are the most important in causing this type of variation. As the Cepheids for which orbital data are available all show very small mass-ratios and orbital dimensions, these were taken as the basis, and stars of all spectral classes which have not shown variations in brightness, which come within these limits, were segregated. The upper limit for mass-ratio was set at 0.1, which is nearly twenty times that observed among the Cepheids. This was done with the view of testing the importance of this factor. The results indicate what was to be expected: that all of several conditions are necessary; that when all of these conditions are present the result is Cepheid variability, and, what seems especially significant, that *no star was found, in which all of the conditions are present, whose light still remains constant.* Several B and A stars were found with all the necessary orbital

and other conditions, but without known light-variation, except one of type B with a very small light-variation. On the other hand, not one F or G star with these conditions was found which did not vary in brightness. This seems to establish rather definitely that this limited spectral condition is a prime necessity to the production of full Cepheid variation. This examination further indicates that small proper motion is also an essential. Whether this means that their distances are great or that their motions are small, or both, cannot now be determined owing to the uncertainty which exists as to the relation of distance to proper motion.

The characteristics which stand out so strongly that they appear to be essential to the production of Cepheid variability are the following:

1. Short periods and small orbital dimensions.
2. Considerable orbital eccentricity.
3. Relatively small size of primary.
4. Preference for galactic regions.
5. Small proper motion.
6. Strong preference for F and G types of spectrum.
7. Small light-range.

Of these, after short periods, eccentricity and small mass-ratios seem to be the deciding factors, for we find the other conditions among the stars with periods of less than 10 days without their showing non-eclipsing variability. So far as the data goes the conclusion is very definitely indicated that very small secondaries are essential. Not a single case to the contrary exists among the 16 stars included in this investigation. As it is only the short-period stars with very small mass-ratios which also have large eccentricities, we are led to the conclusion that there is a close connection of some kind. The finding among stars of all periods, separations, and spectral types, of what appears to be a dependence of orbital eccentricity upon mass, mass-ratio, and separation seems to indicate that, in the Cepheids, the large eccentricities may be merely accidental and result from the limitation of this class of phenomenon to small secondaries. It is of much importance to decide this matter if possible, for upon it may depend to a large

extent a proper explanation of Cepheid variability. If large eccentricity is in reality the important factor, then some physical or tidal action either upon the primary or secondary is indicated, some internal cause, whereas if it is the size of secondary and not eccentricity, an external cause seems more probable. The galactic preference of these stars, as is pointed out elsewhere in this paper, favors the hypothesis of an external cause and, upon the foregoing reasoning, dependence upon relative size of secondary.

On account of the uncertainties of inclination in the spectrographic orbits it is difficult to obtain at present any very decisive results as to the law of this apparent relation of eccentricity to masses and separation. A casual examination shows this peculiarity in all spectral classes and among stars of all periods and masses. So far as a hasty examination of the data (the only one possible just now) shows, the phenomenon is entirely general in space distribution as well. If it is a fundamental law of nature, then in all probability the large eccentricities of the Cepheid orbits are entirely incidental and the controlling factor is size of secondary. A more careful study of the phenomenon and probably more data will be necessary for a definite answer. As has been stated already, this peculiarity is under investigation.

The almost complete restriction of the Cepheid type of variation to the intermediate spectral classes, F and G, when so many more stars of short period, at least among the spectroscopic binaries, are found in earlier types, particularly B, is in itself an interesting and important matter. Such a restriction can be accounted for, most readily, it seems to me, upon some theory of activity among the early-type stars due to an external cause. Upon such an assumption the activity is likely to be much more general in the earlier types, particularly B, and to have affected all parts of their surfaces, whereas in the later and less disturbed types the action of a resisting medium might be expected to be in its earlier stages, due perhaps to a lesser density of the medium, and to affect chiefly the advancing face of the star.

In the Cepheids the explanation which seems most plausible on the whole is that proposed by R. H. Curtiss,[1] in which a resisting

[1] *Lick Observatory Bulletin*, 3, 40, 1904.

medium in the system has enhanced the advancing face of the primary. It is necessary to recognize that such a cause will affect the secondary as well and to a considerable extent, because of the rather large eccentricities of the orbits of these stars, that the resulting variation of brightness will be due to at least three factors, viz.: the brightening of the advancing face of the primary, a similar brightening of the secondary, and increased general activity in the secondary due to its higher orbital velocity and near approach to the primary at periastron. This hypothesis may be stated in the alternate form that Cepheid variation is the difference in brightness between the two components of unequal size of a binary system revolving in a resisting medium plus the increased activity of the secondary due to its higher orbital velocity, especially at periastron. If the variation in brightness of these stars were due chiefly to activity in the secondary after periastron passage as a result of the near approach to the primary, more cases of light-variation might be expected in the stars of classes B and A where the periods average even shorter than in the stars of type F and G, and where the orbital velocities are also much higher. This lack of full Cepheid variability among class B stars seems to be almost sufficient of itself to justify the conclusion that it is due to some form of surface activity on the advancing faces of these stars.

In my opinion an almost deciding factor as to the nature of Cepheid variation is that of their strong preference for the Milky Way. The mean galactic latitude of the 168 short-period variables listed by Harvard (in Vol. **56**, p. 191, of their *Annals*) is $16\frac{1}{2}°$. Eighty-eight of them are less than 10° from the plane. This fact is simple and direct and at once indicates that the cause is not wholly due to internal conditions or to the operation of general physical or gravitational laws, but that some external condition is the chief cause. In general the stars of spectral types F and G, to which the Cepheids belong by preference, are distributed nearly uniformly over the sky. That the Cepheids have a strong affinity for the Milky Way indicates that in some way their variability is influenced by conditions peculiar to that region. If it was due

simply to tidal or thermal action as a result of the near approach of the components at periastron, stars with the necessary orbital and other conditions anywhere in the sky should show this type of light-change. The nature of such an external condition is debatable. That it is matter in a form to act as a resisting medium seems most probable. Evidence is accumulating that such invisible matter exists in our stellar system in sufficient quantities to cause some of the phenomena peculiar to the Milky Way. In support of this is, first of all, the large number of meteors which the earth encounters daily. This is fairly positive and direct evidence of the existence of matter in a solid state and invisible. That such matter is confined to our solar system is not probable to say the least. Evidence is not lacking also as to the presence of matter in some sort of gaseous form but invisible. The detection of a considerable number of early-type stars whose spectra contain dark H and K lines of calcium, such lines showing either no radial velocity with respect to our stellar system or generally much smaller velocities than do the stars themselves, is evidence that the absorbing matter is essentially not connected with the stars. The small variations of radial velocity which are generally shown by these calcium lines in such stars and the fact that they usually exhibit the same general phases as for the star itself may conceivably be due to a motion of the calcium matter set up by the star. It is also significant that such calcium absorption has been observed in several of the novae of recent years. That the matter giving rise to this absorption is not connected with the earth's atmosphere or the solar system is obvious from the fact that all stars do not show it. As this characteristic has, so far as I am aware, been observed only among the early-type stars which have a strong affinity for the Milky Way, the inference is suggested that this may be another galactic peculiarity.

There is a growing belief that the dark lanes and "holes" of the Milky Way and the sharply defined dark patches in some of the large irregular nebulae are none other than masses of obscuring cosmical matter. This view is further strengthened by the large number of spiral nebulae which show dark lanes and rings. Just

what the action of a resisting medium would be in causing variations of brightness cannot be easily predicted. R. H. Curtiss[1] suggests simply an enhancing of the brightness of the advancing face of the primary. J. C. Duncan[2] favors a modification of that hypothesis. He considers it unlikely that the light-changes can be due to an actual change in the rate of emission since in that case the character of the spectrum should change radically during the interval between maximum and minimum, and favors a hypothesis based upon a variation of absorption in the star's atmosphere, brought about by a very rare envelope of nebulous matter brushing backward the star's atmosphere from its advancing face, thus reducing the amount of absorption on that face. It is probably too soon to venture an attempt to decide anything more regarding this type of variability than very general questions. There seems to me little doubt that the fundamental cause of these light-changes is to be found in the action of a resisting medium of some kind and that it may be considered to be sufficiently established that these stars are binary. The action assumed by Duncan seems to me less simple and direct than that of a brightening of the advancing face due to the addition of cosmical matter. Nor do I see a sufficient reason to doubt such an effect, especially if the matter encountered is solid and in small particles. If in this form, it seems to me doubtful if there would be sufficient "drag" to carry a gaseous envelope backward to the extent required. It seems probable that a very much smaller quantity of such matter would suffice for the observed increase of light if its entire kinetic energy were converted into radiant energy.

As to the change of spectral type, it is assured that in a number of these stars there is a slight change toward an "earlier" type at maximum. This change is certainly in the right direction. May it not be that it is sufficient also? It seems to me highly probable that this is all the change of spectrum required under an assumption of finely divided solid matter. That such matter is in a gaseous state seems doubtful from the fact that no emission

[1] *Lick Observatory Bulletin*, 3, 40, 1904.

[2] *Ibid.*, 5, 91, 1908.

lines have been detected in this class of star. Loud concludes,[1] I understand, that "only one of the pair is luminous," and "the visible star, then, is the satellite." This I think is contrary to the view now generally held. It would be difficult to explain why a small satellite should be so much brighter as to be the only one visible. This conclusion of Loud's appears to arise from his assumption that "according to the hypothesis to be tested, this component owes its light to the resistance of a diffused medium, to which the other must be assumed to be relatively at rest." It seems to me improbable that the "resisting medium" should be so related to such a system that the primary is relatively at rest with respect to it. If the "resisting medium" were of the nature of the zodiacal light appendage to our sun, then it might be, but there is considerable evidence that such a medium is in that sense disconnected and has an entirely separate existence. I think we may safely conclude that it is the primary which is not only the larger but the brighter, and that any tenable hypothesis must consider both as being luminous, even if one only is dominant. Duncan objects,[2] among other things, to the bombardment hypothesis on the ground that bombardment ought to render the spectrum of the companion visible. If the explanation offered in the earlier part of this paper of the cause of the "humps" in the light-curves of these stars is correct, we have positive evidence on that point. In that case the principal light-variations are due to the brighter body, and the fainter components offer only the small changes represented by the humps. These would undoubtedly be so small relatively as not to affect the spectrum appreciably.

In my opinion some such variations in the amplitude and character of the light-curves as have been observed in some of these stars are to be expected from the hypothesis of rotation in a resisting medium and do not at all militate against their duplicity. It is not to be supposed that almost any such medium of which we can conceive would be perfectly homogeneous. If not homogeneous, variations due to this cause alone must occur in the brightness of a star moving in such a medium.

[1] *Astrophysical Journal*, **26**, 371, 1907. [2] *Lick Observatory Bulletin*, **5**, 91.

It is scarcely necessary to point out the need for more spectrographic orbits of Cepheid stars, even if they were only approximate. This is particularly true of such of these stars as have nearly symmetrical light-curves (small eccentricities) and those with the most unsymmetrical curves (largest eccentricities). Unfortunately most of these stars are faint. There are, however, a number of them which are within reach of the largest telescopes and one-prism dispersion. It would also be of considerable interest to determine orbits of a few stars from velocities derived separately from lines as high in the spectrum as possible and as low as possible, with a view to determining whether there is any difference either in the form of the orbit or the phase, as has been found in the observations of brightness for some of these stars. Refined observations of brightness of any or all of the stars whose spectrographic orbits have been determined would assist greatly in clearing up the questions which have arisen in regard to the cause of Cepheid variability.

CONCLUSIONS

1. Among the Cepheids for which orbits have been determined, those which have the largest relative masses for the secondaries show irregularities in their light-curves.

2. Among 16 Cepheid variables for which orbits are available, only two have eccentricities smaller than 0.13. The mean eccentricity of the 14 stars is 0.30. The eccentric nature of the light-curves of Cepheids confirms considerable eccentricities as a characteristic of these stars as a class.

3. Among 41 stars with periods of less than 17 days whose orbits are available, with but one exception all whose light is sensibly constant have small eccentricities.

4. Among 79 stars with periods from 17 days to 200 years, only three have eccentricities smaller than 0.15, and only one of these is smaller than 0.1. There appears to be among these stars a sharply defined lower limit of eccentricity at about 0.15. This limit is well marked among all periods longer than 15 days.

5. There appears to be among binary stars of all conditions a dependence of the orbital eccentricities upon the masses of the components and their separations.

6. There appears to be in the short-period variables a dependence of the amount of variability upon brightness, the fainter stars showing the greatest variation.

7. Among the Cepheids whose orbits are available, there appears to be a relation between the orbital velocity and the amount of variation, the stars whose orbital velocities are greatest showing the greatest variations.

8. There appears to be a tendency among the short-period variables for those with periods longer than about 10 days to have slightly larger light-variations. The effect appears not to be progressive with length of period, but to be more or less abrupt.

9. There are some indications of a dependence of light-change upon orbital eccentricity among the 16 Cepheids whose spectrographic orbits are available.

10. Evidence which is not entirely dependent upon the construction placed upon the periodic line-displacements in the spectra of the Cepheids confirms the binary nature of these stars.

The spectroscopic orbit of SU Cassiopeiae by Adams and Shapley[1] was received when this investigation was nearly completed. This star also falls into line in respect to relation of humps to size of secondary. Parkhurst's curve[2] shows no irregularities greater than can be ascribed to accidental error. The light-range is small, 0.47 magnitude according to Parkhurst and 0.33 magnitude by Müller and Kempf.[3]

These results are of especial interest on account of the small light-range, the low orbital velocity, and small eccentricity. The small light-range agrees with the other results as to a dependence of light-variation upon orbital velocity.

Small eccentricity in even a single one of these stars is of particular interest as it tends to confirm the explanation of motion

[1] *Astrophysical Journal*, **47**, 46, 1918. [2] *Ibid.*, **28**, 279, 1908.

[3] *Astronomische Nachrichten*, **173**, 307, 1907.

in a resisting medium, for if the variation were entirely dependent upon increased physical action due to a much closer approach at periastron, a nearly circular orbit should yield no sensible variation of light. If, however, the cause is due to orbital motion in a resisting medium, circular orbits might be expected to yield almost as large variations of brightness as highly eccentric orbits.

Observatorio Nacional Argentino, Córdoba
October 25, 1918

STUDIES BASED ON THE COLORS AND MAGNITUDES IN STELLAR CLUSTERS[1]

FOURTEENTH PAPER: FURTHER REMARKS ON THE STRUCTURE OF THE GALACTIC SYSTEM

By HARLOW SHAPLEY and MARTHA B. SHAPLEY

The present paper contains miscellaneous notes relating to or extending the results and arguments of the twelfth paper of this series. To economize space the observations are treated synoptically, and the discussion of interpretations is abbreviated in so far as consistent with a fair presentation. The subjects discussed are the following:

I. The indication of a direct relationship between globular and open clusters afforded by the galactic distribution.

II. The seventeen clusters which recent studies have added to the original lists in *Mount Wilson Contribution* No. 152.

III. Diagrams of the positions in space of all globular clusters.

IV. Evidence that the observed absence of globular clusters from the equatorial segment is not due to obstructing cosmic clouds in the Milky Way.

V. The diameter-parallax relation for globular clusters as derived from other sources than the Franklin-Adams charts.

VI. Holetschek's magnitudes of 40 globular clusters and the possibility of determining relative parallaxes from measures of the integrated light.

VII. Observational data supporting the view that the motions and distribution of spiral nebulae depend upon dynamical causes quite different from those prevailing in clusters of stars.

VIII. The bearing of the cluster studies on the formulation of a tentative hypothesis of the origin and development of the galactic system.

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 161.

IX. Evidence from the *Henry Draper Catalogue* concerning the extent and inclination of the local cluster defined in *Mount Wilson Contributions*, No. 157.

I. THE COMPLEMENTARY DISTRIBUTION IN GALACTIC LATITUDE OF GLOBULAR AND OPEN CLUSTERS

Although we have not as yet determined individual distances for open clusters we may safely accept that, generally speaking, they are much nearer the sun than the globular systems. If we should assume that the two kinds are similarly distributed with respect to the galactic plane,[1] most open groups would appear in relatively high galactic latitude, because of their proximity to the sun. With the exception of such near and luminous groups as the Pleiades and Praesepe, however, they occur only in low galactic latitudes, and they do not show the avoidance of the Milky Way exhibited by globular clusters. In fact, when actual positions in space are considered, they appear to occur in the dense stellar regions where globular clusters are not found; and they do not occur in the extra-galactic realms where the globular clusters are. This apparently complementary distribution of the two types of clusters is of high importance for the hypothesis which derives the galactic system originally from stellar clusters and obtains the open groups of the Milky Way from the extra-galactic globular organizations.

The continuous gradation of the compact groups into the open clusters of the Milky Way might be more clearly shown if we could correlate compactness with $R \sin \beta$;[2] but lacking parallaxes of the open clusters, we must for the present content ourselves with pointing out the complementary character of the frequencies of galactic latitude. In Fig. 1 the full line refers to open clusters—the objects of Classes II and III of Melotte's catalogue.[3] A few clusters have been withdrawn from his classes and two or three added, on the

[1] Cf. *Contributions from the Mount Wilson Solar Observatory*, No. 152, Figs. 4, 5, and 6; No. 157, Fig. 2.

[2] R is the radial distance from the earth, β is the galactic latitude.

[3] *Memoirs of the Royal Astronomical Society*, 60, Part V, 1915. Diagrams by Melotte illustrate the well-known difference for open and globular clusters of the distribution in galactic longitude.

basis of Mount Wilson plates. Table I contains the assembled results.

The dotted line in Fig. 1 refers to globular clusters; the data are from *Mount Wilson Contributions*, No. 152,[1] supplemented by the additional material in Table II of this paper, omitting only the five

TABLE I

FREQUENCY IN GALACTIC LATITUDES OF OPEN AND GLOBULAR CLUSTERS

Galactic Latitude	Number of Open Clusters		Number of Globular Clusters	
	North	South	North	South
0°– 2°	21	25	0	0
2 – 4	19	19	1	1
4 – 6	10	13	4	2
6 – 8	7	4	3	5
8 –10	3	7	4	5
10 –12	5	0	4	3
12 –14	4	2	2	3
14 –16	0	3	4	2
16 –18	3	3	2	3
18 –20	0	0	1	1
20 –22	2	0	0	3
22 –24	0	1	2	0
24 –26	0	0	1	1
26 –28	1	0	0	1
28 –30	0	0	0	3
30 –32	0	0	1	0
32 –34	2	0	0	0
34 –36	0	0	1	2
36 –38	0	0	1	1
38 –40	0	0	1	0
40 –42	0	0	1	0
42 –44	0	0	0	0
44 –46	0	0	0	1
46 –48	0	0	1	1
48 –50	0	0	1	1

unproved clusters. Fig. 2 gives the frequency when the results are treated irrespective of the sign of β, and shows more clearly the apparent relation of open and globular clusters. A few globular systems in high galactic latitude are not represented in the diagrams, and for $\beta \geqq 10°$ the numbers of Table I for both globular and open clusters have been combined into small groups in drawing the curves.

[1] A few of the latitudes of Table V (*op. cit.*) have been slightly revised, as noted in the third section of the present contribution.

If for β, the angular distance from the plane, we were able to substitute the linear distance, $R \sin \beta$, the mid-galactic maximum for the open clusters would be closely restricted around the vertical axis; and the two maxima of the dotted curve would be relatively much steeper on the side nearer the vertical axis, showing that the galactic zone is entirely clear of globular clusters.[1]

Clusters of Melotte's Class II are more compact and populous than those of Class III, but the distribution in galactic latitude is

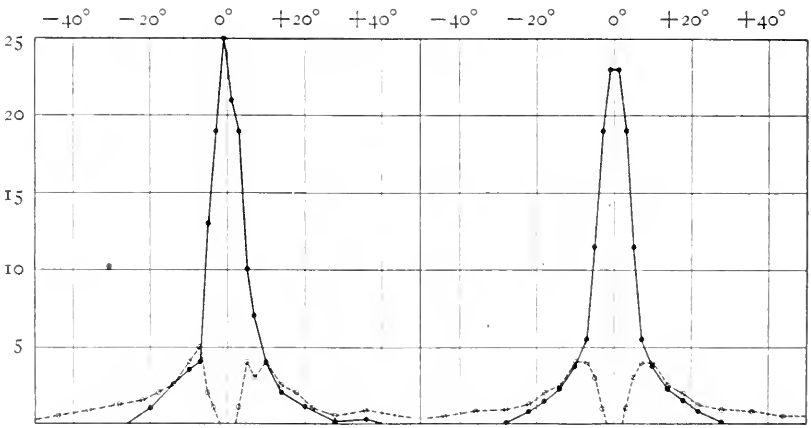

FIG. 1.—Frequency in galactic latitude of open clusters (full line) and globular clusters (broken line). Ordinates are numbers of clusters; abscissae are angular distances from the galactic plane.

FIG. 2.—Reflected frequency-curve of galactic latitudes for open clusters (full line) and globular clusters (broken line).

essentially the same for both types. Choosing from all the data the twenty richest and most regular open clusters, we find the following distribution in galactic latitude:

β	$<-4°$	$-4°, -3°$	$-2°, -1°$	$-0°, +0°$	$+1°, +2°$	$+3°, +4°$	$>+4°$
Number of Clusters	2	4	4	3	5	2	0

This certainly does not show a concentration to the medial plane; but perhaps it does not indicate as marked an avoidance of that plane as might be expected if we assume that globular systems are more closely allied to rich clusters than to the average open group.

[1] Cf. Fig. 6 of the seventh paper of this series.

II. THE DISTANCES OF 17 ADDITIONAL GLOBULAR CLUSTERS

A year ago 69 clusters were accepted as definitely globular.[1] Since that time special efforts have been made to examine with the 60-inch reflector the suspected and doubtful cases north of declination $-30°$ that had not been admitted to the earlier lists. The experience gained as a consequence of photographing faint and abnormal systems has also afforded a basis for better judgment in accepting or rejecting the clusters too far south for observation at Mount Wilson. Some unpublished observations of clusters with the Crossley reflector at the Lick Observatory were generously placed at our disposal by Professor Curtis. While at Mount Wilson during the past summer Professor Duncan, of Wellesley College, kindly made some long exposures on clusters with the 60-inch reflector.

Results of the survey of doubtful cases are summarized in Table II. In addition a number of clusters concerning which some doubt existed,[2] such as N.G.C. 4147, 6144, 6656, 6712, and 7006, have been proved during the year to be entitled to places in the lists; none of the 69 original clusters should be withdrawn, as far as the present observations go.

As in *Mount Wilson Contributions*, No. 152, the treatment of individual cases must be omitted. The distances, which in general cannot claim as high accuracy as those previously studied, depend for the most part upon diameters, since the objects in many cases are so faint that accurate analysis of the magnitudes would have been too laborious. Exposures of several hours were occasionally necessary to test for globularity. Frequently systems that shorter exposures had shown with some definiteness to be open[3] proved upon more persistent observation to be globular. At present there is hardly another object, north of $-30°$ at least, that is suspected with good reason of being a globular cluster.

Three points relative to the large distances are worthy of comment. (1) Practically all of these additional systems are more distant than 30,000 parsecs (100,000 light-years), thereby confirming an

[1] Tables V and VIII of *Mt. Wilson Contr.* No. 152, 1917.

[2] *Op cit.*, p. 9, n. 2.

[3] *Op. cit.*, p. 14, n. 2.

earlier surmise as to the completeness of the survey of globular clusters within that distance of the sun. (2) Every recognized globular cluster except one bears a number from the *New General Catalogue*, thus testifying that, in spite of distance and faintness, all of these remote objects were known prior to 1888. In fact, all but this one exception were known before 1864, the date of the *General Catalogue*, and all but three or four had already been catalogued by the

TABLE II

PARALLAXES OF 17 CLUSTERS

N.G.C.	R. A. 1900	Decl. 1900	Galactic		Distance (Unit is 100 Parsecs)		
			Long.	Lat.	Radial	Projected on Galactic Plane	From Galactic Plane
*5466	14h 1m.0	+29° 0′	10°	+72°	195:	60	+180
†I.C. 4499	14 45.0	−81 49	274	−20	250	235	− 85
*5927	15 20.8	−50 19	294	+ 5	185	185	+ 16
‡5946	15 28.2	−50 19	295	+ 4.0	415	415	+ 29
‡6355	17 17.8	−26 15	327	+ 4.4	500	500	+ 38
*6366	17 22.4	− 4 59	346	+16	320	310	+ 89
*6426	17 39.9	+ 3 13	356	+15	570	550	+150
*6440	17 42.9	−20 19	336	+ 2.5	525	525	+ 23
‡6496	17 51.8	−44 14	315	+11	305:	300	+ 58
*6517	17 56.4	− 8 57	347	+ 5	625	625	+ 55
‡6535	17 58.7	− 0 18	354	+10	370:	365	+ 64
†6539	17 59.4	− 7 35	348	+ 6	400	400	+ 42
†6553	18 3.2	−25 56	332	− 3.4	320	320	− 19
‡6558	18 3.8	−31 47	327	− 6	455	450	− 48
*6569	18 7.2	−31 51	328	− 7	400	395	− 49
†6760	19 6.1	+ 0 52	3	− 4.6	525	525	− 42
*7492	23 3.1	−16 10	21	−64	285	125	−255

* Certainly a globular cluster.
† Almost certainly a globular cluster.
‡ Probably a globular cluster but as yet not proved.

Herschels or earlier observers more than eighty years ago. This is further evidence of the completeness of our lists of globular clusters. (3) N.G.C. 7006, with adopted distance of 67,000 parsecs, still holds its place as the most remote sidereal object of definitely estimated distance.

Further observations are likely to prove that N.G.C. 6355 and 6535 are globular, but present photographs of them are hardly conclusive. The other three unproved systems are too far south for the 60-inch reflector.

Perhaps the most striking result of this special survey is the evidence that every faint, little-condensed cluster in galactic latitude higher than 15° or 20° is really globular, although short exposures and visual observations had in several cases heretofore recorded few stars. On the other hand, the similar faint clusters along the galactic equator, without exception, are open groups with no condensed background of faint stars appearing on long exposures. N.G.C. 7492, for instance, was formerly considered an exception[1]—an open cluster outside the galactic segment—but it is actually globular, containing thousands of stars. The evidence grows continually stronger that open and globular clusters occupy regions of space that are mutually exclusive.

There is also some evidence that an abnormal type of globular cluster exists, one in which the brighter stars are fainter and more scattered than is usually the case. In their luminosity-curves a distinct break appears to occur between the brighter and fainter stars, and for such systems the parallax-diameter relation may not be strictly applicable. The known examples of this type are N.G.C. 5466, 6366, and 7492; it might be well also to place N.G.C. 4372, 5897, and 6144 in this group, although for two of them an alternative interpretation of the apparent discrepancy between diameter and magnitude may be available.[2] The abnormal form possibly represents an early or late stage, or a disturbed condition of a typical globular cluster. In N.G.C. 7492, for which some preliminary colors are available, the brightest stars are yellow.

III. ON THE DISTRIBUTION IN SPACE OF 86 GLOBULAR CLUSTERS

For the reasons apparent from the following remarks it seems worth while to revise the plots and extend the discussion of the distribution in space of globular clusters. Fig. 3 shows the projection on a plane perpendicular to the Galaxy and oriented to include galactic longitude 325°. It gives the appearance of the system of clusters as seen from longitude 235° and is thus merely a revision

[1] *Publications of the Astronomical Society of the Pacific*, **30**, 50, 1918.

[2] *Mt. Wilson Contr.* No. 152, p. 14, n. 2, 1917.

of Fig. 4 of the seventh paper of this series.[1] The values of the abscissae, $R \cos\beta \cos(\lambda-325°)$, were determined graphically for

TABLE III

SPACE CO-ORDINATES OF GLOBULAR CLUSTERS

N.G.C.	$R \sin\beta$	$R \cos\beta$-$\cos(\lambda-325°)$	$R \cos\beta$-$\sin(\lambda-325°)$	N.G.C.	$R \sin\beta$	$R \cos\beta$-$\cos(\lambda-325°)$	$R \cos\beta$-$\sin(\lambda-325°)$
104....	− 47	+ 29	− 39	6341....	+ 69	+ 33	+ 96
288....	−189	− 3	− 7	6352....	− 28	+216	− 66
362....	−109	+ 58	− 89	6355*...	+ 38	+500	+ 18
1261....	−199	+ 8	−161	6356....	+ 67	+375	+ 59
1851....	− 96	− 58	−131	6362....	− 38	+105	− 66
1904....	−120	−145	−173	6366....	+ 89	+290	+111
2298....	− 63	− 88	−219	6388....	− 34	+269	− 62
2808....	− 32	+ 40	−162	6397....	− 17	+ 76	− 29
3201....	+ 26	+ 23	−143	6402....	+ 56	+207	+ 92
4147....	+514	− 23	−107	6426....	+150	+471	+283
4372....	− 18	+ 63	− 94	6440....	+ 23	+516	+100
4590....	+ 97	+ 70	−108	6441....	− 40	+452	− 32
4833....	− 23	+ 96	−132	6496*...	+ 58	+296	− 52
5024....	+186	+ 34	− 11	6517....	+ 55	+579	+234
5139....	+ 18	+ 41	− 46	6535*...	+ 64	+319	+177
5272....	+136	+ 20	+ 21	6539....	+ 42	+368	+156
5286....	+ 37	+134	−139	6541....	− 28	+142	− 22
5466....	+180	+ 42	+ 42	6553....	− 19	+318	+ 39
5634....	+229	+191	− 55	6558*...	− 48	+450	+ 16
I.C. 4499	− 85	+148	−183	6569....	− 49	+395	+ 21
5897....	+ 75	+125	− 31	6584....	− 73	+243	− 70
5904....	+ 90	+ 86	+ 11	6624....	− 40	+282	+ 25
5927....	+ 16	+159	− 95	6626....	− 19	+181	+ 32
5946*...	+ 29	+359	−208	6637....	− 41	+209	+ 15
5986....	+ 47	+191	− 69	6638....	− 48	+337	+ 60
6093....	+ 65	+188	− 16	6652....	− 65	+305	+ 16
6101....	− 55	+156	−135	6656....	− 12	+ 80	+ 18
6121....	+ 31	+109	− 13	6681....	− 41	+177	+ 12
6144....	+ 63	+235	− 25	6712....	− 32	+274	+145
6171....	+ 60	+148	+ 16	6715....	− 44	+153	+ 22
6205....	+ 71	+ 40	+ 75	6723....	− 39	+121	+ 4
6218....	+ 52	+106	+ 34	6752....	− 39	+ 73	− 30
6229....	+274	+ 82	+328	6760....	− 42	+414	+323
6235....	+112	+487	+ 8	6779....	+ 30	+105	+225
6254....	+ 45	+106	+ 32	6809....	− 41	+ 90	+ 16
6266....	+ 19	+150	− 13	6864....	−206	+376	+152
6273....	+ 25	+157	− 3	6934....	−114	+180	+256
6284....	+ 64	+364	0	6981....	−164	+192	+150
6287....	+ 83	+428	+ 15	7006....	−228	+245	+577
6293....	+ 37	+261	0	7078....	− 69	+ 49	+121
6304....	+ 28	+320	− 11	7089....	− 94	+ 70	+104
6316....	+ 46	+524	0	7099....	−128	+101	+ 56
6333....	+ 43	+244	+ 34	7492....	−255	+ 70	+104

* Probably a globular cluster but not definitely proved.

[1] *Mt. Wilson Contr.* No. 152, 1917. The point for M 75 (N.G.C. 6864) is erroneously plotted in the earlier figure.

the earlier plot; they are now computed and entered in the third column of Table III. All globular clusters are represented in the new diagram, including the five unproved objects marked with the double dagger in Table II. The open circles designate clusters for which the provisional distances are marked in Table II by a colon.

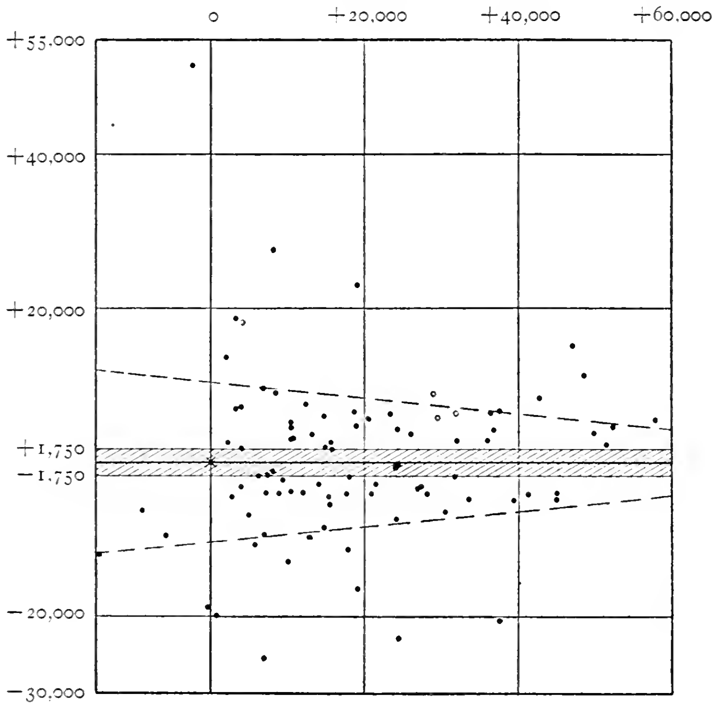

FIG. 3.—Projection of the complete system of globular clusters on a plane perpendicular to the Galaxy and oriented to include the line from the sun to the center of the system. Ordinates are $R \sin \beta$; abscissae are $R \cos \beta \cos (\lambda - 325°)$; unit of distance is one parsec. The sun at the origin of co-ordinates is marked by a cross. See Fig. 4 of *Mt. Wilson Contr.* No. 152.

The parallax of Messier 22 (N.G.C. 6656) has been increased slightly over the value previously adopted. The cluster is fairly open and in a rich field, and it is now found that the stars selected for the study of magnitudes were so near the center that an error of 0.15 mag., due to the Eberhard effect, crept into the earlier results. The values of the galactic co-ordinates for some of the

clusters in Table V of the seventh paper were taken from the catalogues of Bailey, who used a position of the North Galactic Pole slightly different from that adopted for this work. The computations in Table III are all based on the co-ordinates in Melotte's catalogue, with the result that the new values of $R \sin \beta$ differ in a few cases from those previously computed. The revised galactic positions, however, never differ by more than a degree in either co-ordinate, except for the longitudes in high galactic latitude.

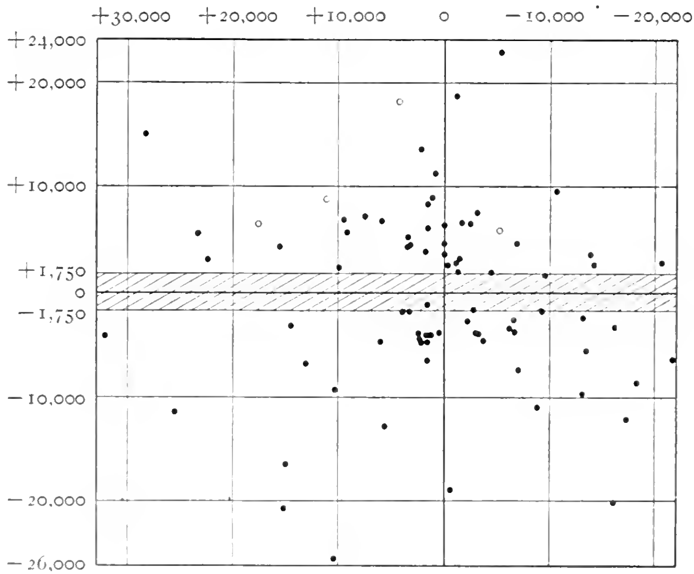

FIG. 4.—Projection of the system of globular clusters on a plane perpendicular to the Galaxy and to the plane of the preceding diagram. Ordinates are $R \sin \beta$; abscissae are $R \cos \beta \sin (\lambda - 325°)$.

Three points formerly not definite are now emphasized by Fig. 3: (1) There is no sensible increase in the minimum distance from the galactic plane with increasing distance from the sun. (2) The additional results have filled in the gaps of the earlier work and show that the provisional estimate of a distance of 20,000 parsecs to the center of the system is not too great. (3) There is evidence that the globular clusters occupy a space shaped somewhat like a split wedge, the base of which passes nearly through the

sun and contains the great circle defined by galactic longitudes 55° and 235°. Only a part of the tapering of the wedge beyond the center of the system is to be attributed to a lack of observations. The inclined broken lines in Fig. 3 suggest the degree of tapering away from the extremely broad and sharply defined base.

Fig. 4 shows the projection of globular clusters on a plane perpendicular to the Galaxy and to the plane of Fig. 3. It represents the appearance of the system of clusters as seen from a great distance in longitude 145°, indicating approximately the apparent distribution as seen from the earth. The co-ordinates for this plot are in the second and fourth columns of Table III. The close approach to symmetry when viewed from this angle is interesting; the numbers of clusters in the four quadrants are 20, 23, 23, and 20. N.G.C. 4147, 6229, and 7006 are outside the limits of the figure.

It is a striking fact that the system of 86 globular clusters listed in Table III is divided into exactly equal numbers by the plane of the Milky Way.

IV. NOTE ON THE ABSENCE OF GLOBULAR CLUSTERS FROM MID-GALACTIC REGIONS

In the seventh and twelfth papers of this series[1] comments have been made upon the absence of globular clusters from the equatorial region of the galactic system. The importance of the phenomenon necessitates a full consideration of its reality and meaning. That the condition is real is attested by such evidence as: (1) the consistent agreement of the results for clusters at various intervals of distance along the galactic plane; (2) the presence of blue stars and open clusters on the galactic equator at distances equal to those of the nearer globular clusters; (3) the absence of appreciable light-scattering in space; and, finally, (4) the apparent dynamical relation of globular to open clusters and their complementary distribution.

Probably the best evidence that dark nebulosity does not obscure globular clusters in the equatorial segment (the nebulous clouds assumed, for instance, to be analogous to the peripheral

[1] *Mt. Wilson Contr.* No. 152, p. 22, 1917; No. 157, pp. 6, 10, 1918.

ring of absorbing matter observed in some spiral nebulae) is afforded by the diagram of Fig. 5. Let us consider first the 31 globular clusters for which the distance projected on the plane of the Milky Way, $R \cos \beta$, does not exceed 15,000 parsecs. Fifteen are north of the galactic plane and sixteen are

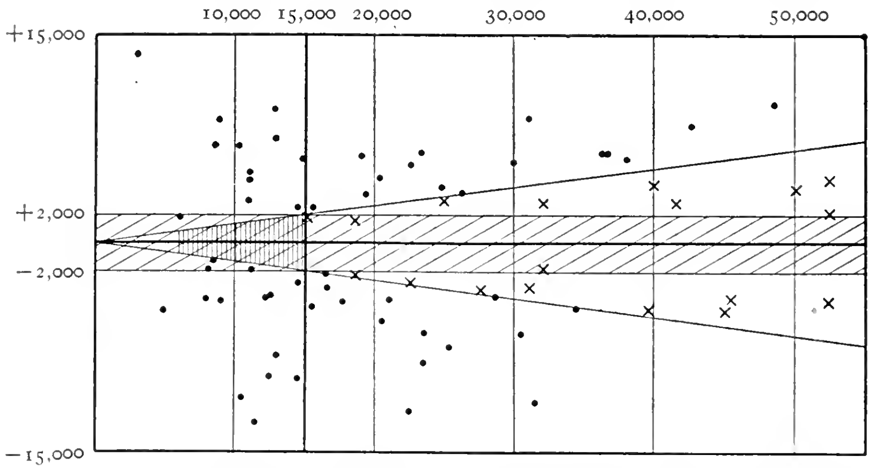

FIG. 5.—Diagram to illustrate that the equatorial segment may not be devoid of globular clusters because of clouds of obstructing matter in the Milky Way. Ordinates are $R \sin \beta$; abscissae are $R \cos \beta$; the unit of distance is one parsec. Twelve clusters fall outside the limits of this diagram.

south. Their frequency in distance from the plane, $R \sin \beta$, is as follows, the unit of distance being 100 parsecs:

$R \sin \beta$	$> \pm 200$	± 200 to ± 150	± 150 to ± 100	± 100 to ± 75	± 75 to ± 50	± 50 to ± 25	$< \pm 25$	-12 to $+18$
Number	3	2	3	5	5	9	4	0

This tabulation, as well as the part of the diagram to the left of the heavy vertical line, shows definitely the absence from the Milky Way of the nearer clusters.

If we attribute the absence of globular clusters to the obscuration by dark matter in low latitudes, we may indicate this hypothetical obscuring material by the vertical shading across the equatorial segment. If the nearby clusters are missing because of these clouds of obstructing matter, then certainly the faint and more distant clusters behind the clouds should be wanting. That

is, the light from every visible cluster with galactic latitude less than $\pm 8^\circ$ would pass through this region, which, by supposition, is capable of concealing twenty or thirty systems that are relatively near. To maintain the assumption, no clusters should be found within the diverging lines. Observation shows nearly a score. The supposition, therefore, that the mid-galactic regions are not transparent appears untenable.

During the six months that have elapsed since the preceding paragraphs were written some doubt has been thrown on the foregoing conclusion, notwithstanding the amount and character of the evidence in its favor. A discussion of the new arguments and the observational data will be given elsewhere in connection with a treatment of the parallaxes of open clusters. For the present the reality of the avoidance of the galaxy by globular clusters must be considered an open question.

We cannot suppose that the 19 clusters[1] within the diverging lines have been estimated too distant because of partial obstruction of their light, and that they are actually the clusters missing from the nearer equatorial segment. The distances of these clusters are based upon angular diameters, some upon magnitudes as well, and we have found that the parallax-diameter relation holds whether the clusters are near or far from the galactic plane.[2]

V. ON THE PARALLAX-DIAMETER RELATION FOR GLOBULAR CLUSTERS

The relation between the distances, as determined both from variable stars and from mean magnitudes, and the apparent diameters, as measured on the Franklin-Adams charts, is unexpectedly definite, if we consider the various constitutional differences among globular clusters.[3] The relatively small deviations from the parallax-diameter curve[4] appear to mean that abnormal clusters are rare. That the correlation of distance and apparent diameter is

[1] One cluster with $R \sin \beta = +5500$ parsecs falls outside the diagram to the right.

[2] In the sixth section of this paper the additional parallax-diameter curves are definite, notwithstanding the large differences in exposure-time and in the brightness of the stars appearing in the photographs.

[3] For instance, the occasional contrast in concentration of the brighter stars, noted in the second section of this paper.

[4] Fig. 1 of *Mt. Wilson Contr.* No. 152.

actually fundamental, however, and is therefore reliably applicable to the estimation of parallaxes not otherwise obtainable, is further attested by the supplementary data represented in the curves of Fig. 6, which we shall presently describe.

The significance of these fairly uniform results must be that, for the large majority of globular clusters, the linear dispersion of the central nucleus[1] is nearly constant, although we cannot as yet decide definitely whether that condition connotes approximately simultaneous origin of all known extra-galactic cluster systems, or rather an essentially permanent dynamical condition in spheroidal stellar groups. The second alternative is supported by the evidence of numerous open galactic clusters. Such groups appear to maintain dimensions of the same order as those of globular systems, but they show, in the scarcity of their red giants and the preponderance of highly luminous blue stars, indications either of much greater age or of more expeditious development.

Without doubt the most homogeneous photographic survey now available for the whole sky is that initiated by Mr. Franklin-Adams;[2] the uniform length of exposure, the quality of stellar images, and the scale of the photographic charts are all particularly suitable for the study of the composite images of clusters, reported in *Mount Wilson Contributions*, No. 152.

We have a second comprehensive photographic survey in the Harvard Map of the Sky, but the copies (positive prints on glass) of the originals show considerable lack of homogeneity in quality and limiting magnitude;[3] in particular, the scale of the plates is so small (aperture 1 inch, focal length 13 inches, exposures 39 to 75 minutes) that high accuracy cannot be expected in determining the apparent diameters of the nearly starlike images of globular clusters.[4] Notwithstanding the difficulties due to faintness, small

[1] The estimates of diameter on the Franklin-Adams charts "refer actually to what appears to be a central core of each system. The scale of the photographs does not permit close differentiation of the outlying members of a cluster from the stars of its surrounding field," p. 25, n. 1, *Mt. Wilson Contr.* No. 152.

[2] *Memoirs of the Royal Astronomical Society*, 60, Part 3, 1915.

[3] Cf. Nort, *Recherches Astronomiques de l'Observatoire d'Utrecht*, VII, 1917.

[4] The Harvard Map is described in *Harvard Circular*, No. 71, 1903.

size, and the resulting confusion with the surrounding fields, the mean results show, in the upper curve of Fig. 6, a definite progression of apparent diameter with adopted parallax. Each point represents the mean of five values. Many of the most distant clusters could not be certainly identified on this scale; and ω Centauri, with diameter 22'.6, parallax 0".00015, is also not plotted. The deviations for the individual clusters average nearly 25 per cent of

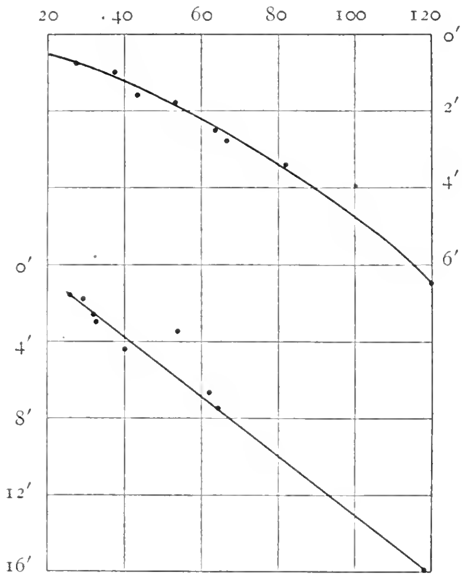

FIG. 6.—Parallax-diameter curves for globular clusters. Above: diameters from plates of Harvard Map of the Sky; below: diameters from Palisa-Wolf charts; ordinates are diameters; abscissae are parallaxes in units of 0".000001.

the adopted parallaxes, a decidedly lower accuracy than shown by results from the Franklin-Adams charts[1] but still of considerable value as a further justification of the method and as a check of the separate values.

The photographs of the Milky Way by Bailey[2] are also serviceable for this work, since many of the globular clusters lie within 10° of the galactic circle. The scale of his photographs is about the

[1] Cf. Table VII of *Mt. Wilson Contr.* No. 152.

[2] *Harvard Annals*, 72, No. 3, 1913; 84, No. 4, 1916.

same as for the Harvard Map, and a casual examination gives comparable results.

The scale of the photographic charts published by Palisa and Wolf is more than double that of the Franklin-Adams charts, but the series is incomplete and the exposure times for the few plates that contain clusters vary from 1^h40^m to 3^h30^m. The results for the nine clusters available are in the lower part of Fig. 6; they suggest the high value of a complete series of long exposures on the larger scale, and they again emphasize the validity of this manner of estimating distance. The discordant point refers to N.G.C. 6626, a compact cluster in declination $-25°$.

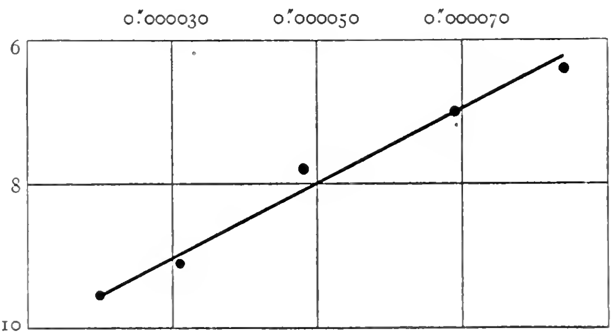

Fig. 7.—Parallax and integrated visual magnitude. Ordinates are Holetschek's magnitudes; abscissae are adopted parallaxes.

Nearly all of the values of diameter used in this section are based upon two independent series of measures by Miss Davis.

VI. TOTAL LIGHT AS A MEASURE OF THE DISTANCES OF CLUSTERS

The mean apparent magnitude of the 25 brightest stars in a globular cluster has been used with satisfactory results as a criterion of distance.[1] The integrated apparent brightness of the same stars, or even of the hundred or so brightest stars in each system, would probably be an equally good measure of the distance. The integrated light of all the thousands of stars in a globular cluster, however, does not greatly exceed that due to its brightest stars alone;

[1] See the fifth section of the sixth paper of this series, *Mt. Wilson Contr.* No. 151, 1917.

and if it were possible, after overcoming some of the obvious minor difficulties, to measure accurately the total apparent brightness of such systems, possibly a valuable method would result for deriving the relative parallaxes of all typical globular groups.

A study has been made by Holetschek[1] of the integrated visual magnitudes of the brighter clusters and nebulae visible at Vienna. His adopted magnitudes for globular clusters, given in Table IV, are of very unequal merit owing to disparity of observation, varying diffuseness of images, and availability of comparison stars. Magnitudes of the comparison stars were taken in general from the B.D.

TABLE IV

HOLETSCHEK'S INTEGRATED MAGNITUDES OF GLOBULAR CLUSTERS

N.G.C.	Mag.	Parallax	N.G.C.	Mag.	Parallax	N.G.C.	Mag.	Parallax
1904......	8.0	39	6229....	8.6	23	6637....	9.0	47
4147......	9.4	19	6235....	9.7	20	6656....	6.2	118
4590......	8.2	62	6254....	6.9	83	6681....	9.5	55
5024......	7.8	53	6266....	7.0	66	6712....	8.9	32
5272......	6.6	72	6273....	6.8	63	6760....	10.5	19
5466......	8.5	51	6284....	9.5	27	6779....	8.3	40
5634......	9.7	33	6287....	9.2	23	6864....	8.0	22
5897......	10.2	67	6293....	8.5	38	6934....	9.0	30
5904......	6.7	80	6333....	7.3	40	6981....	9.5	34
6093......	7.8	50	6341....	6.2	81	7006....	9.7	15
6121......	6.8	88	6356....	8.5	26	7078....	6.2	68
6171......	9.0	62	6402....	7.8	43	7089....	6.7	64
6205......	5.8	90	6626....	7.9	54	7099....	8.5	58
6218......	6.8	81						

catalogues, and high accuracy is not claimed for the results. The parallaxes in Table IV are the adopted values from *Mount Wilson Contributions*, No. 152; the unit is 0."000001.

Omitting the magnitudes for clusters south of declination $-20°$ because of uncertainties connected with low altitude and short observing season, we have combined the remaining twenty-three values into normal groups in order of parallax. The plot of the normal points in Fig. 7 shows the expected uniform decrease of brightness with increasing distance. Obviously a systematic comparison with recognized magnitude standards, using a telescope of

[1] *Annalen der k.k. Universitäts-Sternwarte in Wien*, **20**, 114, 1907.

extremely short focus, or a method that is independent of the angular diameters of the images compared, would contribute important material to the problem of the relative parallaxes of star clusters.

VII. THE CONTRASTED MOTIONS AND DISTRIBUTION OF GLOBULAR CLUSTERS AND SPIRAL NEBULAE

The known radial velocities of globular clusters are predominantly negative.[1] These enormous stellar systems, moving under gravitational attraction with an average speed of more than 100 km/sec., are apparently as a class approaching the sun; and, remembering their extra-galactic positions, we also infer that they are approaching and falling into the dense stellar strata of the general galactic system. To be sure, the spectroscopic study of clusters has not gone far as yet, but the foregoing inference does not rest upon observed velocities alone. The composition of the galactic system and the distribution of clusters in space, especially the relation of globular systems to the open groups of mid-galactic regions,[2] yield more important evidence than is afforded by radial velocities that the galactic system absorbs globular clusters.

On the other hand, the brighter spiral nebulae as a class, apparently regardless of the gravitational attraction of the galactic system, are receding from the sun and from the galactic plane—a remarkable condition that has been little emphasized heretofore. From the published spectroscopic work of Adams, Campbell, Moore, Paddock, Pease, Sanford, Wolf, Wright, and particularly of V. M. Slipher at Flagstaff, we obtain the radial velocities in the fifth column of Table V. The Andromeda nebula (N.G.C. 224) and its companion are treated as a single object. The radial velocity of the nucleus of Messier 33, as determined by Pease from absorption lines, is given in preference to the earlier values derived by Pease and Slipher from bright lines.[3]

Limited as this material is, it yields some important results, which emphasize the contrast between clusters and spiral nebulae

[1] See *Mt. Wilson Contr.* No. 157, 1918, Table I and Section 6.

[2] Sections I and III of this paper.

[3] *Publications of the Astronomical Society of the Pacific*, **28**, 33, 1916.

and bear directly upon the structure and present status of the galactic system. These preliminary results, appearing in the seven numbered divisions below, are the more worthy of mention at the present time because the increase of observational data for spirals will necessarily be slow from now on, due to the extreme faintness of the nebulae not already observed for radial velocity.

TABLE V

POSITIONS AND RADIAL VELOCITIES OF 25 SPIRAL NEBULAE

N.G.C.	MESSIER	GALACTIC		RADIAL VELOCITY	APICAL DISTANCES, θ				VIS. MAG. (HOLETSCHEK)
		Long.	Lat.		$\omega=90°$	$\omega=105°$	$\omega=120°$	$\omega=150°$	
				km					
224......	31	89°	−20°	− 316	20°	25°	36°	63°	5.0
598......	33	102	−30	− 70	32	30	35	55	7?
1023........	....	112	−19	+ 300	29	20	21	42	9.7
1068......	77	141	−52	+1120	67	60	55	52	8.7
2683........	....	158	+40	+ 400	73	63	53	41	9.2
3031......	81	109	+42	− 30	45	42	43	56	8.0
3115........	....	216	+38	+ 600	118	106	95	71	9.0
3379........	....	218	+59	+ 810	108	102	94	79	9.1
3521........	....	225	+54	+ 730	115	107	99	81	9.3
3623......	65	209	+64	+ 800	102	96	90	77	8.9
3627......	66	211	+64	+ 650	103	97	90	78	8.6
4151........	....	118	+76	+ 940	78	76	76	78	10.7
4258........	....	103	+68	+ 500	69	68	69	75	8.7
4526........	....	262	+71	+ 580	109	107	105	97	10.0
4565........	....	215	+88	+1100	91	91	90	89	9.4
4594........	....	267	+52	+1180	128	126	121	106	8.7
4649......	60	265	+75	+1090	105	104	102	96	8.6
4736......	94	76	+86	+ 290	86	86	87	89	7.7
4826......	64	295	+84	+ 150	95	96	96	95	8.6
5005........	....	64	+78	+ 900	79	81	83	89	9.1
5055......	63	69	+74	+ 450	75	77	80	88	9.2
5194......	51	68	+71	+ 270	72	75	78	87	8.4
5236......	83	283	+31	+ 500	147	148	145	126	9.5
5866........	....	59	+52	+ 650	58	65	73	91	10.3
7331........	....	62	−22	+ 500	35	47	61	88	9.3

1. Since the greater number of bright spirals are north of the galactic plane and their positions are more favorable for northern observers, only five negative galactic latitudes appear in Table V. To these should be added that of N.G.C. 1700, for which Sanford estimates a large positive velocity, though, in the absence of a definite numerical value, it is not included in the table. Only one of the twenty velocities for spirals north of the plane is negative,

while two of the spirals on the south side are approaching. But the three negative values depend, as we shall see, on low galactic latitude and high apparent brightness rather than on the sign of the latitude, and hence we conclude that essentially without exception, on both sides of the Galaxy, spiral nebulae recede.

2. The speed of spiral nebulae is dependent to some extent upon apparent brightness, indicating a relation of speed to distance or, possibly, to mass. The six spirals with smallest radial velocity, including all of those with negative values, are not exceeded in brightness by any spirals in Holetschek's list of visual magnitudes:[1]

N.G.C.	224	598	4736	3031	5194	4826
Integrated magnitude	5.0	7?	7.7	8.0	8.4	8.6
Radial velocity	−316	−70	+290	−30	+270	+150 km

The arithmetical mean of these six velocities is ±188 km/sec.; their algebraic mean is +49 km/sec., while for the other 19 spirals of Table V it is fifteen times as large—that is, +726 km/sec. Only three spirals besides these six bright ones are now known to have radial velocities of less than +500 km/sec.

3. Forming means of five in order of galactic latitude, we derive from Table V the first two lines of the following tabulation, which indicate that speed may be related to angular distance from the galactic equator:

Mean galactic latitude, β	24°	45°	59°	72°	82°
Mean radial velocity, V_r	+183	+548	+834	+578	+676 km
$V_p = V_r \operatorname{cosec} \beta$	+450	+775	+975	+610	+680 km

If we should assume that the motion is wholly perpendicular to the galactic plane, the mean velocities would be as given in the last tabulated line above.

4. The correlation of velocity and the latitude co-ordinate, although not very definite, may be of some significance for theories of the spiral nebulae; but, guided by the provisional hypothesis described in Section VIII of this paper, we find some evidence of a more striking relation between the velocities of spirals and a new position co-ordinate. Let λ and β denote the galactic

[1] *Annalen der k.k. Universitäts-Sternwarte in Wien*, **20**, 114, 1917. N.G.C. 4826 is equaled in brightness by two other spirals, according to Holetschek's estimates.

co-ordinates of any spiral, and ω the longitude of an origin on the galactic circle. Then the angular distance, θ, of the spiral from this origin is given by

$$\cos\theta = \cos\beta\cos(\lambda-\omega) \tag{1}$$

In the last four columns of Table V we give for each spiral the values of θ for $\omega=90°$, 105°, 120°, 150°. The angle θ may lie between 0° and 180°, and at the galactic poles is 90°; in the table, however, no value of θ is less than 20° because of the avoidance of the Milky Way by spiral nebulae, and few values exceed 120° because the most southern nebulae have not been observed for velocity.

TABLE VI

THE PROGRESSION OF THE MEAN VELOCITY OF SPIRAL NEBULAE WITH DISTANCE FROM THE GALACTIC APEX

Interval of θ	$\omega=90°$			$\omega=105°$			$\omega=120°$		
	Mean θ	Mean V_r	Number	Mean θ	Mean V_r	Number	Mean θ	Mean V_r	Number
$\leqq 50°$..	32°	+ 77	5	33°	+ 77	5	34°	− 29	4
51°– 75 ..	69	+565	6	66	+588	5	62	+634	5
76 –100 ..	86	+676	5	88	+660	8	88	+641	12
101 –125 ..	109	+751	7	105	+762	5	109	+950	3
>125 ..	138	+840	2	137	+840	2	145	+500	1

For the first three values of ω the progression of the observed radial velocity of spirals, V_r, with increasing θ is shown in Table VI for equal intervals of θ. The interval for θ less than 50° contains in all cases the three bright nebulae with negative radial velocities, and the mean V_r is correspondingly affected. The correlation, if real, is about equally definite for $\omega=90°$ and $\omega=105°$. Its meaning would be that, regardless of galactic latitude, the average radial velocity of spiral nebulae increases with the angular distance from a point in the northern Milky Way—a point which, in anticipation of an explanation proposed in Section VIII, we may call the galactic apex.

In these progressions we have a suggestion that average radial velocity may be roughly predicted from position; but before this relation can be definitely established from spectroscopic results alone, we must have more data, for the wide range of peculiar

velocity in Table V is inadequately reflected by the means of Table VI. Other groupings could alter or even conceal the uniform progression of mean velocities.

If we omit the two brightest nebulae for the sake of greater homogeneity (see p. 126) and combine the others into four groups in order of increasing θ, we obtain for $\omega=105^\circ$:

Mean θ	46°	74°	95°	116°	(2)
Mean V_r	$+458$	$+618$	$+633$	$+780$ km	
Number of spirals	5	6	6	6	

5. On the basis of this more homogeneous material we may make the following computation, which is perhaps to be considered more as an interesting illustration than as an approach to definite cosmic fact. Let V_s be the average systematic recessional velocity of spiral nebulae in the line of sight, and let V_g be a quantity which we may call the velocity of the galactic system toward $\lambda=105^\circ$, $\beta=0^\circ$. Then

$$V_s - V_g \cos\theta = V_r \tag{3}$$

and from the values of θ and V_r in (2) we obtain, provisionally,

$$\begin{aligned} V_s &= +650 \text{ km/sec.} \\ V_g &= +300 \text{ km/sec.}^{1} \end{aligned} \tag{4}$$

Employing these provisional values of V_s and V_g, and setting up conditional equations of the form (3) for each of the 23 spirals in the tabulation (2), we derive the following normal equations:

$$\begin{aligned} 23\Delta V_s - 1.96\Delta V_g - 120 &= 0 \\ -1.96\Delta V_s + 4.34\Delta V_g + 80 &= 0 \end{aligned}$$

The solution gives for corrections to (4), $\Delta V_s = +8$ km/sec., $\Delta V_g = +17$ km/sec. The adopted values with their probable errors are

$$\begin{aligned} V_s &= +660 \pm 45 \text{ km/sec.} \\ V_g &= +320 \pm 100 \text{ km/sec.} \end{aligned}$$

The average difference between an observed velocity and that computed by putting the foregoing values in formula (3) is ± 240

[1] By assuming $V_g = +300$ km/sec., the mean value of V_s derived from *all* the material of Table VI is $+635$ km/sec. for $\omega=105^\circ$, in close agreement with the result above.

km/sec., a quantity that is to be taken as representing, not observational errors, but rather the peculiar velocities of spirals.

From the foregoing result we infer that one way of accounting for the observed increase of average V_r with distance from the so-called galactic apex is by assuming that the galactic system moves toward longitude 105°, approximately, with a velocity of some three hundred kilometers a second, while the spirals of the brightness here involved recede with an average systematic radial velocity of six or seven hundred kilometers a second.[1]

6. If we should assume that the average systematic motion is perpendicular to the galactic plane rather than radial from the center of the galactic system, or radial from the sun as is observed and assumed above, then we would have for this perpendicular velocity

$$V_p = (V_r - V_g \cos\theta) \operatorname{cosec}\beta$$

and a solution of the four mean observational equations derivable from (2) gives, provisionally,

$$V_p = +775 \text{ km/sec.}, \quad V_g = +140 \text{ km/sec.}$$

The least-squares solution of the 23 conditional equations then leads to the following results:

$$V_p = +765 \pm 55 \text{ km/sec.}$$
$$V_g = +110 \pm 100 \text{ km/sec.}$$

and the average value of O − C is ± 250 km/sec. The probable errors give little choice between assumptions involving radial and perpendicular systematic motion, but the conception of radial motion is distinctly preferable from the standpoint of physical probability.

When sufficient data become available, not only should V_g and V_s (or V_p) enter the computations, but also the co-ordinates of the origin of θ.

[1] Following the procedure commonly used to determine the apex of the solar motion, Truman, Young and Harper, and Slipher have computed from the radial velocities of spirals a motion of the galactic system toward a rather vaguely defined southern apex; but they have assumed, explicitly, a random peculiar motion for spiral nebulae and, implicitly, the "island universe" hypothesis. Their result is an obvious consequence of the preferential recession and of the absence of spirals of known velocity from the southern sky.

7. From the present evidence we see that there is a magnitude effect for the very brightest objects; in addition the observed velocities of spirals may be divided into three parts, the first two of which appear to be very definite: (*a*) a systematic radial recession of more than 600 km/sec., which is increased by one-sixth if the systematic motion is assumed to be perpendicular to the galactic plane; (*b*) the peculiar velocities, which average about ± 250 km and are represented by the deviations from the means and formulae; (*c*) a component of the radial velocity whose effect decreases on the average with distance from the "galactic apex," and which may be interpreted as a drift of the whole galactic system with respect to the brighter spiral nebulae. This last component is the least definite, quantitatively, but on the other hand we shall see that its existence seems to be affirmed by the distribution of spirals and by other considerations discussed in the last part of Section VIII.

The apparent distribution of spiral nebulae appears to be sufficiently known for a general statement, though much remains to be done in extending the recent work of Hardcastle, Fath, Sanford, and Curtis. That the spirals approach nearest to the Milky Way in two hours of right ascension ($\omega = 100°$) has been noted by Hinks, Stratonoff, and others. Globular clusters are wholly absent from that region, and it is very significant that in the opposite region of the sky, where globular clusters and clouds of stars most abound, the avoidance of the Milky Way by spirals reaches its maximum.[1]

Thus the region avoided, at least by the brighter spirals, is roughly defined by a wedge, symmetrical in relation to the galactic plane, with its base in the general direction now adopted as the center of the galactic system. A wedge-shaped arrangement (analogous to the wedge-shaped avoidance by spirals) has been pointed out for globular clusters in Section III of this paper, but the base of the wedge lies in a direction approximately opposite to the direction of the center.

[1] See the diagrams by Hardcastle and Hinks (*Monthly Notices*, **74**, 699, 1914) and the bibliography and discussion by Sanford (*Lick Observatory Bulletins*, **9**, 82, 1917).

There is, accordingly, in the contrasted distribution of spirals and globular clusters, as well as in the opposing directions of their motions, an indication that the compelling forces act in opposite directions.

VIII. DATA AND INFERENCES FOR A PROVISIONAL COSMOGONY

The observational results discussed in the foregoing sections of the present contribution and in the thirteen preceding papers of this series permit the statement of five of the conditions that must be considered in attempting to account for the origin of the galactic system and its present relation to clusters and spiral nebulae: (1) the similarity of globular clusters in dimensions and content; (2) the complementary distribution of open and globular clusters; (3) the existence of thousands of suborganizations in the galactic system; (4) the contrasted distribution of spiral nebulae and globular clusters; and (5) the opposed directions of preferential radial motion for spirals and clusters.

In the following paragraphs we offer a brief summary of the observational data bearing on each of these five conditions and a suggestion as to the general interpretation of the evidence.

1. The parallax-diameter curves in *Mount Wilson Contributions*, No. 152 and in Section V of this paper indicate that with few exceptions globular clusters have approximately the same linear diameters. Observations of (*a*) the integrated light of clusters, (*b*) their general luminosity-curves, and (*c*) the phenomena of color and absolute magnitude of their giant stars show also that the stellar content is much the same from system to system.

2. For globular clusters the frequency-curve of galactic latitudes has a distinct minimum at the galactic plane, whereas the corresponding curve for open clusters shows a pronounced maximum in low latitudes (Figs. 1 and 2 of this paper). The evidence suggests that the two kinds of clusters occupy regions of space that are mutually exclusive (Sections I and III of this paper). There is some indication that a transition from one kind to the other occurs along the outskirts of the equatorial galactic segment.

3. Numerous conditions suggest that the Galaxy is a heterogeneous assemblage of unequally organized parts. The remarkably

wide pairs of stars of common motion, the local moving groups, the open clusters, the clouds of the Milky Way, the star-streams[1]—all these combinations indicate that the galactic system may be largely composed of disintegrating clusters. The testimony of the distribution and radial motion of globular clusters indirectly supports this view, and suggests further that the system is now growing and has gradually grown throughout the past from a much less complicated state.

4. While both globular clusters and spiral nebulae appear to be mainly if not wholly outside the equatorial galactic segment, they occur in general in different parts of the sky. In the Southern Hemisphere the globular clusters crowd in close to the Milky Way and the bright spirals widely avoid it; in the Northern Hemisphere the spirals approach their nearest to the galactic circle and globular clusters are almost wholly absent.[2]

5. Globular clusters as a class appear to be rapidly approaching the galactic system; spiral nebulae as a class are receding with high velocities. The relation of the velocities of spirals to brightness and to position in the sky has been dealt with above in Section VII. Some of the reasons for not considering spiral nebulae to be separate galactic systems have been outlined in the introduction to the twelfth paper of this series. For the present we shall accept that the distances of globular clusters and spirals are of the same order, and that, with the possible exception of a few of the very brightest, none is within mid-galactic stellar regions.

The foregoing conditions, when considered in connection with previously accepted stellar and nebular results, suggest as a preliminary hypothesis that the discoidal galactic system originated

[1] Accepting the evidence presented in *Mt. Wilson Contr.* No. 157 (see also the ninth section of this contribution) bearing on the existence of a local cluster, we note that from material now available the circum-solar cluster appears to be both larger and more oblate than typical open or globular systems; it seems to be more like the Magellanic Clouds in dimensions. Possibly before it became a member of the galactic system it represented the discoidal combination of several smaller clusters, the residual nuclei of which are still shown in the Orion, Perseus, and Scorpio-Centaurus groups of B stars.

[2] See the last three paragraphs of Section VII of this paper.

from the combination of spheroidal star clusters and has long been growing into its present enormous size at their expense. The evidence further suggests that the galactic system now moves as a whole through space, driving the spiral nebulae before it and absorbing and disintegrating isolated stellar groups. Apparently the suggested interpretation requires that two types of sidereal organization prevail generally throughout extra-galactic space: spiral nebulae, and stars of known types assembled for the most part into globular clusters;[1] and while the globular clusters now known are, at least potentially, members of the galactic system, the spirals are not members, rather being general inhabitants of extra-galactic space. The hypothesis demands that gravitation be the ruling power of stars and star clusters,[2] and that a repulsive force, radiation pressure or an equivalent, predominate in the resultant behavior of spiral nebulae.

[1] The apparent limitation of the size and mass of a globular cluster (the first of the five conditions) suggests, for example, the narrow range in masses of stars, for which the limiting factor, according to Eddington's theory, is the balance of attractive and dispersive forces. Judging by the galactic system, and perhaps by the local cluster and the Magellanic Clouds, the discoidal form permits a greater mass; and if, as seems likely, the Magellanic Clouds recently passed through the galactic system (cf. p. 12, n. 1, of the twelfth paper) a stellar discoid possibly shows much greater stability than is possessed by the compact spheroidal distribution.

[2] Jeans has considered mathematically the effect of the encounter of two clusters, showing the transformation from the globular to the oblate form (*Monthly Notices*, **76**, 552, 1916). His analysis may direct the way to an understanding of the beginning of a flattened stellar system of growing mass.

If such a growth be theoretically possible, we may suppose that at first the combination of separate clusters would proceed very slowly to the formation of composite systems of increasing mass, the galactic system that finally results representing an advanced stage in the survival of the most massive and stable. Once an enormous mass had been acquired, subsequent accretions would be numerous and almost inevitable if ordinary unit spheroidal clusters were encountered rather than growing discoids of multiple mass. The distinctly limited galactic star clouds, some of which are not mid-galactic, and the rapidly receding Magellanic Clouds as well, might represent partially assimilated and controlled systems of greater mass. Undoubtedly some globular clusters would merely describe orbits around the growing Galaxy and outside the limits of the system would cross the galactic plane, but our observations do not show them; it may be the true "orbital" clusters are constrained to keep far from the galactic system, all near approaches ending in disorganization. (Compare the tentative hypothesis sketched in the seventh section of *Mt. Wilson Contr.* No. 147, 1918.)

Relative to spiral nebulae, two additional conditions that seem not to have been considered heretofore may be pointed out as significant in connection with the proposed hypothesis.

a) In the midst of a field of stars the effect of repulsive forces would be largely nullified by the symmetrical distribution of the sources of repulsion (assumed to be the stars); above or below a discoidal stellar field the action is of course wholly one-sided.

b) The extremely high velocities of recession indicate that, if the galactic system had remained stationary, most of the brighter spirals would have been among the stars in low galactic latitudes within recent cosmic times, for instance within the last twenty million years. That scarcely any spiral (on either side of the galactic plane) is approaching or is now among the stars in low galactic latitudes not only suggests that a repulsive force is predominant, but also indicates either (1) a movement of the entire Galaxy through space, or (2) recently accelerated motions of spirals, or (3) distances enormously greater than now seem at all probable.

The last supposition, though extending the time allotted, would not remove the difficulty of accounting for the absence of spirals from the equatorial segment at the present time. For example, four bright spirals in high galactic latitude are receding with a velocity in excess of 1/300 that of light. Suppose they are as much as two million light-years away—ten times the distance of the remotest globular cluster known. Then, with constant velocities throughout the past, they would have been at the galactic plane less than 600 million years ago—an interval of time so short in the life-history of a stellar system that in the case of the sun, for instance, it has not sufficed to show an appreciable change in radiation.[1]

It is more in keeping, however, with observations of apparent rotation, and with the luminosity of novae in spirals, to divide by one hundred the distance supposed above. Only a few million years are therefore involved in the problem, and it seems all the more necessary to accept, in view of the observed distribution and radial motion, either (1) that the Galaxy is moving or (2) that the velocities have been rapidly developed. The suggestion (2) that

[1] *Publications of the Astronomical Society of the Pacific*, **30**, 283, 1918.

the position of the Galaxy has remained stationary while the velocities of spirals have but recently become large does not commend itself for various reasons, including the consequent necessity of seeking the origin of spirals in the galactic regions where now they are not observed.

The remarkable progression of average velocity with distance from the "galactic apex," discussed in Section VII of this paper, is also simply explained by assuming that the galactic system moves in the direction and with the velocity required by these phenomena of distribution and motion. We conclude, therefore, that the most plausible interpretation of the present arrangement and motions of the brighter spirals implies repulsion by the moving discoidal galactic system;[1] but we keep in mind the alternative hypotheses.

IX. NOTE RELATIVE TO THE MORE DISTANT B STARS OF THE LOCAL SYSTEM

The stars of spectral type B, because of their relatively small dispersion in absolute magnitude and their tendency to clustering and to community motion, are particularly valuable in studying the form and position of the local star cluster whose existence and general properties have been discussed provisionally in the last part of

[1] Is it possible that the spirals represent the failure to form stars from the original condensing nebulosity through the presence of too much material? According to Eddington's theory of the structure of a giant star (*Monthly Notices*, **77**, 16. 596, 1917), the pressure of radiation nearly counterbalances gravitation if the condensing gaseous mass exceeds some 10^{35} grams. A mass 100 times that of the sun would incompletely condense, possibly with the result of a diffuse pseudo-stellar nucleus, grading off into the extensive, low-density envelopes of gas that are unable to fall into the center because of the balance of radiation. Such a body would not readily disintegrate (since the ratio of radiation pressure to gravitation cannot exceed unity), but would present to external repulsive forces a surface relatively large for the mass involved, and composed, it may be, of particles of molecular dimensions peculiarly susceptible to the pressure of such radiation as is emitted by the stars. The existence of powerful electrical fields in spiral nebulae would clarify the problem, and already Slipher has suggested (*Lowell Observatory Bulletin*, No. 80, 1917) that such may occur in Messier 77 and Messier 1. Spirals of greatest mass or density would be least repelled by the galactic system, and if their individual velocities directed them toward a radiant source they might not be easily turned aside. In his study of the rôle of rotation in cosmogony Jeans has shown the probable development of spiral arms when the condensing body greatly exceeds the sun in mass (*Monthly Notices*, **77**, 186, 1917; *Scientia*, **24**, 270, 1918).

the twelfth paper of this series. Although the overwhelming majority of brighter B stars treated in *Mount Wilson Contributions*, No. 157 belong to the cluster, a few appear from the evidence of space positions to be field stars, in corroboration of Kapteyn's result that occasionally B stars are found in the second stream.[1]

It has frequently been assumed, mainly on the basis of an extrapolation, that all B-type stars are included in the earlier catalogues of spectra—that fainter than the seventh or eighth visual magnitude no stars of that class of spectrum would be found. Such a condition would place a sharp limit to the extent of the local cluster, assuming it to be outlined by stars of type B. Extremely faint and distant blue stars, however, have been found in the galac-

[1] The complete identification of the local cluster with Stream I involves some difficulties that were not sufficiently appreciated when the earlier paper was written. The two most important obstacles in the way of the suggested interpretation, which made star-streaming simply the result of uniform motion of the local cluster through the galactic field, appear to be the high relative velocity of the two streams and the probably different stream velocities for different spectral types. There seems to be no doubt of the existence of a definitely organized local cluster, and it certainly contains practically all of the brighter B stars. Kapteyn's studies also leave little doubt that the B stars as a whole, and therefore this cluster, move toward an apex approximately the same as the apex of the first stream as determined from stars of other types. The moving cluster must give rise to a true stream-motion. It happens that the position of the sun with respect to the local cluster's center (the direction of which is shown by F-G-K stars as well as by the B's) is such that we may have, in addition to and hardly distinguishable in direction from the true stream-motion, a pseudo-stream-motion due to internal circulation (according to Strömberg's suggestion, for instance).

These two possible sources of the observed preferential drift of stars were recognized from the first, but in the provisional discussion the internal motion was considered relatively of minor importance. Now it appears that the two difficulties mentioned above, and other minor ones, may be avoided if necessary by inverting the relative importance of the two sources of stream-motion. The cluster accordingly contains not only all the stars of Stream I but a considerable part of those of Stream II, and the cluster's direction and speed of motion are best measured by the mean drift of the early B stars. The average internal velocity may of course vary with spectral type without disrupting the cluster. Professor Eddington suggests in a letter that the field should probably be identified with Halm's O-stream, which contains a great many second-type stars formerly assigned to the first and second streams.

The modified view sacrifices little of the simplicity of the former statement and at the same time uses in a rational manner the large local cluster, whose existence must play an important part in the problem of star-streaming. Except in this matter of motion and content, none of the earlier conclusions referring to the local cluster appears to require appreciable modification.

tic clouds far beyond the limits of the local cluster, and the data given below show that Class B is also represented in considerable numbers among stars from the seventh to the tenth apparent magnitude.

The first volume of the *Henry Draper Catalogue*,[1] which covers the first four hours of right ascension, contains nearly 26,000 stars, of which 832 belong to Class B. It contains 600 B stars fainter than the seventh magnitude, corresponding to about 3 per cent of the total for all types. For the naked-eye stars over the whole sky the percentage is of course much larger, and, because of the galactic concentration of B stars, it will likely be larger also for the faint stars in those instalments of the new catalogue that include larger portions of the Milky Way. Practically all the B stars of this first volume are in Cassiopeia and Perseus.

Stars of the subdivisions B8 and B9 are generally discussed in connection with A-type stars. In the present note we shall only call attention to the following tabulation of the total number of such stars brighter than successive half-magnitude limits:

Visual magnitude	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5
Total No. stars...	7	17	32	56	95	156	235	317	430	537:	591::
Ratio............		4.6	3.3	3.0	2.8	2.5	2.0	1.8	1.7:	1.4:	

The ratio N_{m+1}/N_m in the last line shows the decrease of density with distance.

The numbers of stars in the first five divisions of Class B are shown for different intervals of visual magnitude in Table VII. The limit to which the catalogue is complete is not specifically stated, but is probably in the neighborhood of visual magnitude 8.5. The 38 B stars with undetermined subdivision are more likely to belong in this table than with the stars of types B8 and B9.

Evidence discussed below indicates that the mean absolute magnitudes may be accepted as the same for all these faint stars as for the brighter B's, whose motions and parallaxes have led to the evaluation of their average intrinsic luminosities. Table VII indicates, therefore, that B stars are found continuously to a distance of possibly 1000 parsecs in the direction of Perseus and Cassiopeia.

[1] *Harvard Annals*, **91**, 1918.

TABLE VII

NUMBERS OF B-TYPE STARS IN *Harvard Annals*, 91

Spectral Type	Visual Magnitude							Total
	<7.0	7.0–7.5	7.5–8.0	8.0–8.5	8.5–9.0	9.0–9.5	>9.5	
B0.............	6	3	2	2	3	2	0	18
B1.............	4	2	1	0	1	0	0	8
B2.............	7	1	5	5	7	0	0	25
B3.............	26	11	7	5	7	1	2	59
B5.............	35	9	5	11	4	2	3	69
B0–B5.........	78	26	20	23	22	5	5	179
Undefined B....	0	0	4	12	8	10	4	

Table VIII affords evidence that the 101 stars fainter than the seventh magnitude and of spectral types B0–B5 are possibly in large part members of the local cluster. The galactic latitudes, derived graphically from charts prepared by Kapteyn, are tabulated in order of decreasing brightness, the horizontal lines in each column marking the magnitude intervals of Table VII. All stars in the table are north of the celestial equator except the eight with galactic latitudes in excess of 41°.

With respect to the galactic plane, the descending node of the central plane of the local cluster is in galactic longitude 70°, approximately, as is clearly shown by Fig. 4 of *Mount Wilson Contributions*, No. 157. The galactic longitudes of the stars involved in Table VIII are almost exclusively between 85° and 130°, and, therefore, if they belong to the cluster rather than to the general galactic field, their latitudes should be predominantly negative. This is seen to be the case, and in fact we are led to believe that most stars of types B0, B1, B2, brighter than the tenth magnitude, may be members of the local system.

The area of the sky between right ascension 0^h and 4^h is mainly in the southern galactic hemisphere. Hence, for a fair comparison of the number of stars in positive and negative galactic latitudes, we should consider only a narrow belt along the galactic circle—the region within $\beta \leqq 5°$, for instance. We have then the following indication that the distribution of the early B-type stars of Table VIII may have little or nothing to do with the galactic

TABLE VIII

GALACTIC LATITUDES OF B-TYPE STARS FAINTER THAN MAGNITUDE 7

B0	B1	B2	B3		B5	
+8°	−4°	+ 2°	− 3°	0°	− 8°	− 6°
+8	−3	− 3	− 4	+ 4	+ 2	+ 7
+1	−4	−12	+10	− 2	− 5	− 4
+6	−4	+ 1	−21	− 2	− 5	− 4
−2		− 5	−50	− 2	−12	− 6
−4		− 6	− 3	− 3	−23	+ 2
−4		− 2	− 2	− 5	−82	+ 4
−2		− 3	− 2	−42	− 3	+ 5
−4		−12	− 1	− 5	+ 1	− 5
−3		− 5	− 9	− 2	−12	−64
−4		− 5	−15	− 4	−16	− 6
−2		− 4	+ 2	− 5	+ 6	− 4
		− 3	+ 2	− 4	− 4	−63
		−19	+ 6	− 4	− 5	− 5
		− 4	− 3	− 6	−46	−54
		− 4	−54		− 7	− 6
		− 5	−16		+ 1	
		− 5	+ 2		+ 2	

plane—rather, these stars appear to be condensed to a circle that in this region of the sky is three or four degrees south of the galactic circle:

Spectrum		B0	B1	B2	B3	B5	B0–B5
Number of stars	β negative	8	4	12	18	10	52
	β positive	1	0	2	4	7	14

The maximum frequency of the galactic latitudes for the 101 faint stars of types B0–B5 is about -4° according to the following tabulation, which is based on Table VIII:

β	$\geqq+10^\circ$	$+9^\circ,+8^\circ$	$+7^\circ,+6^\circ$	$+5^\circ,+4^\circ$	$+3^\circ,+2^\circ$	$+1,0^\circ$	$-1^\circ,-2^\circ$
No. of stars	1	2	4	3	7	5	11

β	$-3^\circ,-4^\circ$	$-5^\circ,-6^\circ$	$-7^\circ,-8^\circ$	$-9^\circ,-10^\circ$	$-11^\circ,-12^\circ$	$-13^\circ,-14^\circ$	$<-14^\circ$
No. of stars	28	19	2	1	4	0	14

It is easy to show that the dip of the equatorial circle of the cluster, due to the sun's position to the north of its central plane, is less than -1° for stars at the mean distance of those concerned above.

Allowing for the dip, and assuming that in the mean these B-type stars are about 25° from the descending node, we may make a rough determination of the angle between the central plane of

the local cluster and that of the Galaxy. The result is 8°, in fair agreement with the earlier value of 12°, but naturally of much lower weight. Accordingly, we may conclude, as previously assumed, that the existence and inclination of the local cluster do not depend upon the accidental positions of a few groups of brighter B stars.

The most significant feature of the preceding tabulation, however, is the high concentration to the central plane. More than half of all these stars fall into the interval of latitude $-2°$ to $+6°$, and three-fourths of the B0, B1, B2 stars are within those limits. For B-type stars brighter than those considered here the dispersion is decidedly greater, both in this part of the sky and in general.

Three interesting conclusions may be drawn from the foregoing result: (1) The failure to find the fainter B stars heretofore may be due to the extremely narrow belts within which they are to be found—one belt, moreover, apparently standing well out of the lowest galactic latitudes for regions of small declination. (2) The local cluster is exceedingly flat, at least as far as the B-type stars are concerned. It may be more than five times as extended in its plane as at right angles. Table VIII shows how infrequent are the faint B stars in high galactic latitudes. (3) The greatly increased concentration for the fainter stars may be taken as proof that these objects, rather than peculiar B stars of abnormally low intrinsic brightness, are normal stars at a greater distance from the sun.

The completion of the *Henry Draper Catalogue* will afford a good basis for testing and extending the results outlined above.[1] With data for the whole sky we shall be able to define more accurately the position of the local cluster in the galactic system and perhaps determine its form completely.

Mount Wilson Solar Observatory
November 1918

[1] *Note added to proof:* Data derived from the second instalment of the *Henry Draper Catalogue* fully verify the existence of a secondary Galaxy as outlined by early B-type stars. The faintest B's, however, show a decided preference for the primary Galaxy, in contrast to the stars discussed above. Possibly a branching of the main Milky Way stream in Perseus and Cassiopeia, or the wide extent of the open clusters in Perseus, is largely responsible for the great preponderance of negative latitudes in the foregoing discussion; and accordingly these faint B's may be only in part members of the local cluster.

THE ECLIPSING BINARY RR VULPECULAE AND THE EVIDENCE OF DARKENING TOWARD THE LIMB

By MENTORE MAGGINI

A photometric light-curve of RR Vulpeculae has been published by me in an Appendix to *Pubblicazione di Arcetri*,[1] No. 34. This note will be devoted to the discussion of observations with a view to showing the evidence of absorption in the atmosphere and consequently of darkening toward the limb.

The light-elements derived from my observations were:

Primary minimum = J.D. $2420661.4749 + 5^{d}.051318$ E. (G.M.T.)

The individual 252 observations were collected into 42 normal points of equal weight. The first three columns of Table I contain the number of the normal, the phase referred to the epoch of primary minimum, and the mean magnitude.

From the observations we have:

Duration of light-change, $10^{h}50^{m} = 0^{d}.4514$
Duration of minimum light, $3^{h}20^{m} = 0^{d}.1389$
Stellar magnitude of constant light, $9^{M}.55$
Stellar magnitude of minimum, 10.87
No evidence of secondary minimum.

The range of variation at principal minimum, $1^{M}.32$, corresponds to a maximum loss of light of 0.7035 of the whole. The brighter star gives 0.7035 of the whole light of the system and, if isolated, would appear of magnitude $9^{M}.93$. If the eclipse were annular, the ratio of the stellar radii would be

$$k = \sqrt{\frac{1-\lambda_p}{\lambda_s}} = \sqrt{0.7035} = 0.845 .$$

[1] Osservazioni di tre stelle variabili—Appendice di M. Maggini. *Pubbl. del R. Osserv. di Arcetri*, Fasc. 34. Firenze, 1916.

TABLE I

NORMAL MAGNITUDES OF RR VULPECULAE

No.	Phase	Mag.	O−C$_u$	O−C$_d$
1	$-0^d.24169$	$9^M.55$	$0^M.00$	$0^M.00$
2	.23600	9.55	.00	.00
3	.23068	9.55	+ .02	.00
4	.20847	9.60	.00	.00
5	.19236	9.70	− .02	.00
6	.18072	9.74	− .03	− .01
7	.15883	9.88	− .04	− .02
8	.15265	9.96	− .04	− .03
9	.13622	10.16	− .03	− .01
10	.12648	10.29	.00	.00
11	.11590	10.44	+ .03	+ .02
12	.10625	10.57	+ .04	+ .02
13	.09444	10.73	+ .07	+ .02
14	.08663	10.80	+ .07	+ .02
15	.07441	10.85	+ .05	+ .01
16	.07200	10.87	+ .04	+ .01
17	.05578	10.87	.00	.00
18	.03467	10.87	.00	.00
19	.02154	10.87	.00	.00
20	−0.00732	10.87	.00	.00
21	+0.01185	10.87	.00	.00
22	.02500	10.87	.00	.00
23	.03199	10.87	.00	.00
24	.04933	10.87	.00	.00
25	.06255	10.87	+ .01	.00
26	.07368	10.84	+ .04	.00
27	.08462	10.78	+ .07	+ .02
28	.10340	10.57	+ .05	+ .01
29	.11107	10.46	+ .03	+ .01
30	.12628	10.25	− .02	.00
31	.13680	10.14	− .03	− .01
32	.15135	9.94	− .04	.00
33	.15639	9.91	− .04	.00
34	.16078	9.86	− .04	.00
35	.17010	9.82	− .03	.00
36	.18617	9.70	− .02	.00
37	.19862	9.64	.00	+ .01
38	.20423	9.61	+ .02	.00
39	.21800	9.58	+ .02	.00
40	.22835	9.56	+ .01	.00
41	.23810	9.55	.00	.00
42	+0.25359	9.55	0.00	0.00

But if t is the semi-duration of the whole-minimum and τ the semi-duration of the constant-minimum light, we have

$$k \leqq \frac{t-\tau}{t+\tau} \qquad k \leqq 0.555$$

This shows that the eclipse is total. The light of the brighter star is therefore $L_b=0.704$, and that of the large fainter companion $L_f=0.296$. The loss of light intensity,

$$1-l=0.7035\alpha,$$

corresponding to the various fractions α of the light of the brighter star, and the light remaining, expressed in stellar magnitude, are in the second and third columns of Table II.

A free-hand light-curve to represent the observed points is drawn, and the epochs t_1 and t_2 at which the magnitude computed is reached are read. Taking the period $P=5^d.051318$, the mean t of these times are transformed into orbital longitudes θ by the equation

$$\theta=\frac{2\pi}{P}t.$$

Using Russell's table[1] giving $\theta-\sin\theta$ we find $\sin\theta$ and $\sin^2\theta$. From the values of $\sin^2\theta$ we have:

$$A=\sin^2\theta_6=0.0286$$

$$B=\sin^2\theta_6-\sin^2\theta_9=0.0137$$

For every value of $\sin^2\theta$ we derive $\psi(\kappa, \alpha)=\dfrac{\sin^2\theta-A}{B}$. The k_u (uniform) and k_d (darkened) are obtained from Russell's Tables II and IIx[2] giving the ψ-function.

We see that the values of k_u are discordant, but those for k_d are in good agreement. We have for their mean values:

$$k_u=0.262$$

$$k_d=0.473$$

New values of constants A and B are now deduced from this mean k. From Russell's Table II and IIx we obtain the ψ-functions for $k_u=0.262$, $k_d=0.473$, and a least-squares solution of the equations

$$\sin^2\theta=A_u+B_u\psi(0.262, \alpha_1)$$

[1] *Astrophysical Journal*, **35**, 339, 1912, Table B.

[2] *Ibid.*, **35**, 335, 1912; **36**, 245, 1912.

TABLE II

a	$1-l$	Mag.	t_1	t_2	θ	$\sin^2\theta$	$\sin^2\theta - A$	$\psi(k_1a_1)$	k_u	k_d
0.0	0.0000	9^M550	-0^d2307	$+0^d2381$	0.2916	0.0824	+0.0538	+03.927	0.488	0.488
.1	.0703	9.629	.2022	.2004	.2504	.0615	.0329	2.402	.416	.524
.2	.1407	9.715	.1853	.1829	.2290	.0515	.0229	1.671	.364	.501
.3	.2111	9.807	.1720	.1692	.2122	.0445	.0159	1.160	.350	.513
.4	.2814	9.909	.1590	.1570	.1965	.0380	.0094	0.687	.227	.415
.5	.3518	10.021	.1475	.1445	.1832	.0332	+ .0040	+0.337	.235	.450
.6	.4221	10.145	.1396	.1338	.1700	.0286	.0000	....	....	....
.7	.4925	10.286	.1269	.1235	.1558	.0241	− .0045	−0.328	.273	.500
.8	.5628	10.448	.1150	.1118	.1411	.0198	.0088	0.642	.137	.400
.9	.6332	10.649	.1000	.0972	.1226	.0149	.0137	1.000	....	....
.95	.6683	10.748	.0917	.0880	.1117	.0124	.0162	1.183	.364	.533
.98	.6894	10.820	.0837	.0787	.1010	.0102	.0184	1.340	.220	.470
.99	.6965	10.845	.0775	.0720	.0929	.0086	.0200	1.460	.041	.420
1.00	0.7035	10.870	−0.0720	+0.0720	0.0833	0.0069	−0.0217	−1.584	0.030	0.457

and $$\sin^2 \theta = A_d + B_d \psi(0.473, \alpha_1^1)$$

furnish the new approximate values:

$$A_u = 0.02931 \qquad A_d = 0.02869$$
$$B_u = 0.01576 \qquad B_d = 0.01417$$

Two theoretical light-curves are computed with these constants. The results are given in Table III.

From the values t_1 and t_2 of Table II it appears that the epoch of mid-eclipse is 0^d0016 earlier than that of the light-elements.

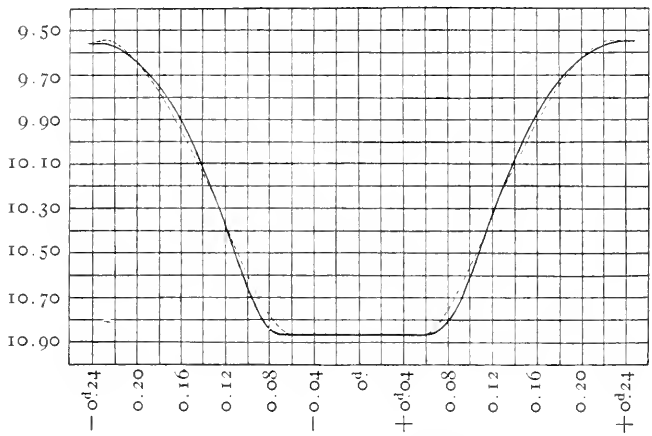

FIG. 1.—Light-curve of RR Vulpeculae
——— Observed and "darkened" light-curve
- - - - - "Uniform" light-curve

Plotting the magnitudes of Table II, in the first case against the epochs $-0^d0016 \pm t_u$, in the second case against the epochs $-0^d0016 \pm t_d$, we obtain two computed light-curves. They are shown in Fig. 1 and their residuals, $O - C_u$ and $O - C_d$, are in the last two columns of Table I.

The perfect agreement of the "darkened" solution with the observed curve is evident; the "uniform" solution is unsatisfactory.

Other elements were derived using the "darkened" solution. $\phi_1^1(k)$ and $\phi_2^1(k)$ were obtained from Russell's Table[1] II*ax*, and values of inclination i and radius of fainter companion r_1 calculated.

[1] *Astrophysical Journal*, **36**, 246, 1912.

TABLE III

UNIFORM AND DARKENED LIGHT-CURVE OF RR VULPECULAE

	Uniform $k_u=0.262$ $A_u=0.02931$ $B_u=0.01576$					Darkened $k_d=0.473$ $A_d=0.02869$ $B_d=0.01417$				
a	$\psi(k_u.a)$	$B_u\psi(k_u a)$	$\sin^2\theta$	θ	t_u	$\psi(k_d a_1^1)$	$B_d\psi(k_d a_1^1)$	$\sin^2\theta$	θ	t_d
0.0	+2.972	+0.04685	0.0762	0.2795	$0^d.2247$	+3.870	+0.05483	0.0835	0.2931	$0^d.2356$
.1	2.056	.03240	.0617	.2509	.2017	2.288	.03242	.0611	.2497	.2007
.2	1.532	.02415	.0535	.2332	.1875	1.637	.02319	.0519	.2298	.1847
.3	1.086	.01712	.0464	.2172	.1746	1.127	.01597	.0447	.2129	.1712
.4	0.704	.01110	.0404	.2023	.1626	0.708	.01003	.0387	.1979	.1591
.5	+0.342	+ .00539	.0347	.1873	.1506	+0.341	+ .00483	.0335	.1841	.1480
.6	0.000	.00000	.0293	.1720	.1382	0.000	.00000	.0287	.1702	.1368
.7	−0.327	− .00515	.0242	.1562	.1256	−0.325	− .00461	.0241	.1558	.1252
.8	0.654	.01031	.0190	.1383	.1112	0.651	.00922	.0195	.1400	.1125
.9	1.000	.01576	.0135	.1167	.0938	1.000	.01417	.0145	.1208	.0971
.95	1.195	.01883	.0105	.1026	.0825	1.192	.01689	.0118	.1088	.0875
.98	1.337	.02107	.0082	.0909	.0731	1.345	.01906	.0096	.0983	.0790
.99	1.401	.02208	.0072	.0851	.0684	1.422	.02015	.0085	.0925	.0744
1.00	−1.500	−0.02364	.0057	.0754	.0606	−1.575	−0.02232	.0064	.0799	.0642

Relative surface brightness $\frac{J_b}{J_f}$ and "equal-mass" densities $\bar{\rho}_b$, $\bar{\rho}_f$ are derived from the formulae:

$$\frac{J_b}{J_f}=\frac{L_b r_f^2}{L_f r_b^2}$$

$$\bar{\rho}_b=(5.29P^{\frac{2}{3}}r_b)^{-3} \qquad \bar{\rho}_f=(5.29P^{\frac{2}{3}}r_f)^{-3}$$

The quantities in parentheses are the radii $\bar{r}_b, \bar{r}_f$ of the stars relative to the sun.

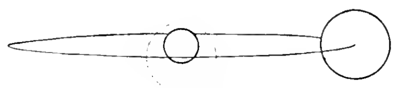

FIG. 2

Finally the hypothetical parallax was computed by Shapley's method.[1] Since the spectrum of RR Vulpeculae is unknown, it was estimated to be of type F from the consideration of hypothetical radius $\bar{r}_b > 1.2$. Definitive "darkened" elements of the systems are as follows.

RR VULPECULAE

Radius of bright component	r_b	0.096
Radius of faint component	r_f	0.202
Inclination of plane of orbit	i	86° 0′
Least apparent distance of centers	$\cos i$	0.070
Light of bright component	L_b	0.704
Light of faint component	L_f	0.296
Relative surface brightness	J_b/J_f	10.53
"Equal-mass" density of bright component . .	$\bar{\rho}_b$	0.304
"Equal-mass" density of faint component . . .	$\bar{\rho}_f$	0.032
Radius of bright component	$\bar{r}_b$	1.488
Radius of faint component	$\bar{r}_f$	3.146
Hypothetical parallax	$1000\,\pi''$	3.16

Fig. 2 shows the system of RR Vulpeculae as seen from the earth.

FLORENCE, ITALY
December 1918

[1] "A Study of the Orbits of Eclipsing Binaries," *Contributions from the Princeton University Observatory*, No. 3, pp. 117–18.

Minor Contributions and Notes

THE RESIDUAL RADIAL VELOCITIES OF THE CEPHEID VARIABLES AND THEIR BEARING ON A PULSATION THEORY

In an article in this number on the cause of Cepheid variation[1] I called attention to the fact that if this type of variation were due to pulsation effects, the radial velocities of the systems upon the hypothesis of orbital motion should show a considerable negative excess, which was not the case in the data then available. This seems to be a crucial test of such a theory and to merit a more careful examination of the data, which, although limited, is now somewhat increased, and sufficient to give confidence in the result.

For such an examination 21 stars are now available. The essential data are given in the table on page 149.

The solar motion was eliminated on the basis of 20 km per sec. toward $\alpha = 270^\circ$ and $\delta = +30^\circ$. V' is the corrected velocity.

The mean velocity of the 21 Cepheid stars given in the table, having regard to sign, is -4.4 km for all, or -2.0 km omitting RR Lyrae, whose velocity is 50 km. Regardless of sign it is 12.2 km, or 10.4 omitting RR Lyrae. The mean radial velocity of nearly two hundred stars of types F and G, with small proper motions, is 10.3 km. In good agreement with this are the values for the Cepheids upon the assumption of orbital motion. The average radial velocity of these Cepheids upon a pulsation theory would be 16.6 km, or 15.6 km omitting RR Lyrae.

The tendency appears now to be to admit the variations of radial velocity observed in the Cepheids as true Doppler-Fizeau effects, that the pulsations are radial translations of an outer shell. If such is the case and the mean algebraic velocity of the group of

[1] *Astrophysical Journal*, 49, 81, 1919.

stars is actually zero, then the mean velocity of the group derived in the usual way from the *mean* of the observed variable radial motions should be −16 km (*minus* K) instead of −4.4, or −2.0 km.

For it is clear that upon the assumption of pulsation the total observed variation of radial velocity is nothing more nor less than

TABLE I

CEPHEID VARIABLES

Star		1900.0		K	Obs. V	V'
		α	δ			
				km	km	km
	Polaris	1^h 22^m	+88° 46′	3	−15	− 5
SU	Cassiopeiae	2 43	+68 27	11	− 7	− 2
SZ	Tauri	4 31	+18 20	11	− 3	−16
RT	Aurigae	6 22	+30 34	18	+21	+12
ζ	Geminorum	6 58	+20 43	13	+ 7	− 5
S	Muscae	12 7	−69 36	18	+ 3	− 5
R	Triang. Austr.	15 10	−66 8	18	−20	−24
S	Triang. Austr.	15 52	−63 30	15	+ 1	− 2
S	Normae	16 10	−57 39	14	−10	−10
RV	Scorpii	16 51	−33 27	15	−28	−20
X	Sagittarii	17 41	−27 48	15	−14	− 3
Y	Ophiuchi	17 47	− 6 7	8	− 5	+11
W	Sagittarii	17 58	−29 35	20	−29	−19
Y	Sagittarii	18 15	−18 54	19	+ 4	+17
RR	Lyrae	19 22	+42 6	22	−69	−50
SU	Cygni	19 40	+29 1	25±	−33	−15
η	Aquilae	19 47	+ 0 45	21	−14	+ 2
S	Sagittae	19 51	+16 22	19±	−12	+ 5
T	Vulpeculae	20 47	+27 52	18	− 1	+15
β	Cephei	21 27	+70 7	17	− 5	+ 8
δ	Cephei	22 25	+57 54	20	0	+13
	Mean			16.1		− 4.4
	Mean omitting RR Lyrae					− 2.0

the amplitude of the pulsation, that the true radial motion of the star is not the *mean* of the curve, but entirely on the side of the greatest positive motion. The motion would be first all approach and then all recession, and by taking the mean of the velocity-curve we obtain, not the velocity of the star, but a quantity equal to *minus* K. In the case of the 21 Cepheids under discussion the mean V' is −4.4 km, or −2.0 km, which is much closer to the required zero for orbital motion than to the −16 km required by pulsations.

The magnitude of the difference between the residual velocity of the group, on the two assumptions, adds to the strength of the test, and pronounces definitely in favor of duplicity and against pulsation. This conclusion is further confirmed by the individual velocities, 8 of which are positive; on the assumption of pulsations practically all should be negative. In order to harmonize these results with pulsations it would be necessary to assume an average outward motion of from 12 to 14 km for these Cepheids, which seems highly improbable. Neither does a change of solar velocity from 20 km to 16 km, which is that derived from the stars of types F and G, greatly change the result. The use of 16 km gives a residual for the 21 stars of -5.7 km, or -3.2 km omitting RR Lyrae.

The objection to duplicity of the Cepheid stars appears to rest chiefly upon the small dimensions of their orbits in connection with their supposed giant size. The orbital dimensions are free from uncertainties of distance. The sizes of the Cepheids appear to rest finally upon distances derived from their small proper motions. I have already referred[1] to recent investigations at Mount Wilson and this Observatory which throw doubt on proper motion as a measure of distance, particularly of the stars with such small proper motions as the Cepheids. The work here is farther advanced than when the foregoing article was written, and the explanation of parallax for the discordances found is now strengthened. If the present suspicions as to parallax are confirmed the stars with small proper motions are in general much closer than has been assumed. In such a case the supposed giant sizes of the Cepheids are likely to be reduced to such an extent that the small orbits found for them would present no serious difficulty.

C. D. PERRINE

OBSERVATORIO NACIONAL ARGENTINO, CÓRDOBA
March 5, 1919

[1] *Astrophysical Journal*, 48, 296, 1918.

REVIEW OF RECENT WORK ON THE SERIES SPECTRA OF HELIUM AND OF HYDROGEN

At the time of the publication of Kayser's *Handbuch der Spektroscopie* it seemed that we knew everything about the spectrum of helium. Since then, however, several articles have appeared which throw much new light on the subject, and make it seem that there may even yet be more to learn. As these have been rather widely scattered, and have received less attention than they deserve, it may be worth while to review them briefly at this time.

Runge and Paschen[1] showed that the spectrum of He consists of two systems of series, a system of pairs, sometimes referred to as the He I spectrum, and a system of single lines, the He II spectrum, which was at one time referred to a pseudo-element parhelium—a hypothesis which was soon abandoned, though the term has survived. Each series system consists of four types of series, one principal, two subordinate (diffuse and sharp), and a fourth type which we may, following Hicks, refer to as the fundamental series. German writers frequently call it the "Bergmann" type, though the first series of this sort was published by Fowler, and the true nature of the type first discovered by Runge.

Fowler[2] and Bohr[3] have shown that there is also an "enhanced" or "spark" series system, which includes two series formerly supposed to belong to H. This system is emitted by an atom which has been more violently bombarded than is usual in ordinary sources. Stark would have us believe that it is emitted by a helium atom which has been deprived of two electrons. Such an atom would presumably be an alpha particle, and it is difficult to imagine in the light of present evidence how such a particle could emit any light-waves.[4] On various grounds it seems preferable to assume, as Bohr does, that the ordinary He spectrum is given by an atom which has, after having been deprived of one electron, just regained it; while the enhanced system comes from those atoms which,

[1] *Astrophysical Journal*, **3**, 4, 1896.

[2] *Monthly Notices*, **73**, 62, 1912.

[3] *Nature*, **92**, 231, 1913.

[4] It is here assumed that, even if we follow Rutherford's recent suggestion and think of the He nucleus as containing electrons, these could still furnish no ordinary light, as they would be too rapid in their motions.

having lost both electrons, have just regained one. Possibly Stark's own observations could be reconciled with such a view. Stark also suggests that the single-line and pair systems may be due each to the vibrations of one of the two electrons in the He atom; but it might be better to suppose that the atom is symmetrically constructed, and that the two types of vibration are executed in different directions with respect to the axis of symmetry.

The enhanced system consists of lines at first thought to be pairs, but Paschen[1] has shown in a very important article on "Bohr's Helium Lines" that they are complex, and there seems to be little hope that this complexity can be explained away by the assumption that it is due to Stark effect, or to any other disturbing cause. Sommerfeld's modification of Bohr's theory, on the assumption that the electronic orbits are elliptical rather than circular, furnishes a quantitative explanation of the complexities here discovered. This is one more strong argument in favor of the rotating atom, though one cannot lightly abandon the hope that the conception of an atom such as that of Lewis and of Langmuir,[2] which explains the chemical behavior of matter so much better, can be so enlarged as to fit the requirements of spectroscopy as well.

According to modern theories, the He atom, when deprived of one electron, should bear a strong resemblance to that of H. The enhanced series system of He bears out this idea perfectly, for it consists, as far as we now know it, of two series, which are related to one another in exactly the same manner as are the series in H. Further, they may be calculated by a formula of the Balmer type, in which $4N$ is used instead of N, though the value of N must be altered slightly from that necessary for H. One of these series is an exact copy of the other, merely shifted to another part of the spectrum. Stark has just shown that the electrical resolution of the main He enhanced series is identical, line for line, with that of the Balmer series of H. The main series of He, beginning with the famous 4686 line, lies largely in the ultra-violet, and includes what was formerly supposed to be the principal series of H. The other series, lying in the visible spectrum, includes, as every other line, the

[1] *Annalen der Physik*, **50**, 901, 1916.

[2] *Jour. Amer. Chem. Soc.*, **41**, 868, 1919.

Pickering series, also formerly supposed to belong to H. The alternate lines lie close to those of the Balmer series of H, but were successfully measured by Evans.[1] Other shifted series of this sort may also exist, and their positions can readily be predicted.

In a recent number of the *Annalen*[2] there are four articles by Stark and his pupils on the He spectrum, in which series of lines, largely new, are given, generated in that part of the source where the field is strongest. An earlier discovery of the same sort was that by Koch[3] of a "third subordinate series of He." This designation is misleading, but has been handed down from the time before the combination principle of Ritz came into general use. This series is a combination series of the principal rather than the subordinate type. Stark's series are also combination series, but he has applied new names to them, at variance with current custom, which will add confusion to the already somewhat mixed nomenclature of this subject. He calls all series principal series which run to the same limit as the principal series does. He thus gets "diffuse principal," "sharp principal," and even "near-sharp (fastscharfe) principal" series. Heretofore the nature of a series has always been supposed to be determined, not by the position of its limit, but by the spacing of its lines, their physical aspect, etc. That this is the proper procedure Stark himself has just shown, for he proves that the Stark effect for his "diffuse principal" series is identical with that of the diffuse series from which (by combination) it is derived. In order to help in translating these misleading designations back into the ordinary nomenclature, both sets are shown in the table below. It is a remarkable fact that these new series of Koch and of Stark appear in the strongest electrical fields only, and yet they belong, not to the enhanced system, as this would indicate, but to the other systems, whose lines are produced by a quite moderate degree of excitation. The same remark applies to the (erroneously) so-called "third subordinate series" of Li.

Recently Fowler[4] has found a remarkable set of bands in the He spectrum, some of which follow a line-series formula. They are

[1] *Philosophical Magazine*, **29**, 284, 1915.

[2] *Annalen der Physik*, **56**, 569–617, 1918.

[3] *Ibid.*, **48**, 98, 1915.

[4] *Proc. Roy. Soc.*, **91**, 208, 1915.

TABLE I

THE HELIUM SERIES SYSTEM

System	Name	Series "Formula"	First Lines	Remarks
System of single lines or He II or Parhelium	Principal	$(1S)-(mP)$*	20581, 5016, 3965	Stark's "near-sharp principal" series
	Sharp or second subordinate	$(1P)-(mS)$	7282, 5048, 4438	
	Diffuse or first subordinate	$(1P)-(mD)$	6678, 4922, 4388	
	Fundamental	$(1D)-(mF)$	18694, 12793	Not mentioned in Stark's summary of He spectrum.
Combinations in this system		$(1P)-(mP)$	6635, 4911, 4384	Lines observed by Merton also
		$(1S)-(mS)$	(5380) (4053) (3651) 3468	Bracketed lines not yet observed; Stark's "sharp principal" series
		$(1S)-(mD)$	(5043) 3974, 3617	Stark's "diffuse principal" series
		$(2P)-(mD)$	19096	One line, found by Paschen.
System of pairs	Principal	$(1s)-(mp)$	10830, 3889, 3188	Stark's "near-sharp principal" series
	Sharp	$(1p)-(ms)$	7066, 4713, 4121	
	Diffuse	$(1p)-(md)$	5876, 4472, 4026	
	Fundamental	$(1d)-(mf)$	18684, 12785	Found by Paschen
Combinations in this system		$(1p)-(mp)$	6060, 4518, 4046	Koch's "third subordinate" series
				Stark's "near-sharp subordinate" series
		$(1s)-(ms)$	(4277, (3285) 2986	Stark's "sharp principal" series
		$(1s)-(md)$	3809†, 3166, 2936	Stark's "diffuse principal" series
		$(2p)-(md)$	17008	Found by Paschen.
Enhanced system	Fowler's main series	$4N\left(\frac{1}{3^2}-\frac{1}{m^2}\right)$	4686, 3203, 2733	Includes series formerly called principal series of H
	Pickering	$4N\left(\frac{1}{4^2}-\frac{1}{m^2}\right)$	6560, 5411, 4859	Includes series formerly called the Pickering series of H.

* For an explanation of these symbols see *Astrophysical Journal*, 41, 324, 1915.

† The first line of this combination series was observed by Paschen and is given in the tables of Dunz.

not included in Table I, as their relationship to the other series is unknown.

It should be added that Merton[1] in the course of experiments on the Stark effect in He recognized at least one of the combination series which Stark (a few days later) published, but apparently did not definitely identify the combination which gives it.

For the sake of completeness the spectrum of the H atom, as far as it is now known, is appended. The second, or compound, spectrum of H is not here given, as it is now generally regarded as due to the H molecule, and it is not known to contain any line series.

TABLE II

Designation	Formula	First Lines
Ritz series.........	$N\left(\frac{1}{2^2}-\frac{1}{m^2}\right)$	1216, 1026
Balmer series......	$N\left(\frac{1}{3^2}-\frac{1}{m^2}\right)$	6563, 4861, 4340
Paschen series.....	$N\left(\frac{1}{4^2}-\frac{1}{m^2}\right)$	18751, 12817

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August 1919

THE ABSORPTION SPECTRUM OF CARBON DISULPHIDE VAPOR

The absorption spectrum of carbon disulphide vapor seems to have been investigated twice, viz., by Liveing and Dewar,[2] who found an absorption band between λλ 3400 and 3000 for very dense vapor and between λλ 3360 and 3020 for a less vapor-density, and by J. Pauer,[3] who resolved the band between λλ 3300

[1] *Proc. Roy. Soc.*, **95**, 30, 1918.

[2] *Ibid.*, **35**, 714, 1883.

[3] *Annalen der Physik*, **61**, 363, 1897.

and 3000 into lines and measured the wave-lengths of fourteen of these lines.

Pauer used as his source of illumination the light from a cadmium arc, which is not continuous in this spectral region.

In the present investigation the source of illumination was a spark between aluminium electrodes under water. The apparatus used was modified from a description given by H. E. Howe.[1] The spark was produced in a vessel of hard rubber and the light was allowed to emerge through a window of two quartz lenses, which gave a nearly parallel beam of light to the slit of the Féry quartz spectroscope. This spectroscope gave a spectrum about eighteen centimeters long in the ultra-violet.

The vapor was contained in a glass cylinder 9 cm long with quartz windows at the ends. The carbon disulphide was admitted through a side tube and allowed to evaporate in the vessel.

Photographs were made with some liquid in the bottom of the vessel at room temperature, and others after the liquid had all evaporated and much of the vapor had been driven out by warming the absorption vessel. The spectrum from a copper arc was used as a comparison spectrum from which to measure the wave-lengths of the lines. These measurements were made on a Hilger plate-measuring engine, and are probably accurate to one or two units in the fourth place.

With the saturated vapor-density at room temperature the absorption was complete throughout the greater part of the band, and it was only by using a very low vapor-density that the single lines could be made out in this part of the spectrum. In the case of even very dilute vapor the absorption was complete beyond λ 2230.

On account of the feeble illumination, long exposures were necessary to give a continuous spectrum to the limits set by the quartz apparatus. The exposures were accordingly about seven hours. The following lines were determined in the different photographs.

[1] *Physical Review*, 8, 574, 1916.

Pauly	Pauer
2919	
2931	
2939	
2944	
2949	
2956	
2960	
2967	
2973	
2975	
2982	2982
2996	
3000	
3010	3010
3013	
3016	
3018	
3020	
3024	3024
3028	3027
3034	
3039	
3043	3042
3046	3046
3054	
3057	
3065	3065
3067	
3070	3070

Pauly	Pauer
3074	
3081	
3084	
3090	3091
3093	
3103	
3110	3110
3119	
3133	3133
3135	
3138	
3142	3140
3150	3147
3154	
3156	
3158	
3165	3162
3174	3175
3183	3182
3190	
3206	3202
3215	3210
	3220
3227	3227
3235	
3246	3245
3252	3252

Pauly	Pauer
3259	
3264	
3268	3270
3275	
3284	
3291	
3302	3300
3306	
3309	
3322	
3326	
3331	
3348	
3356	
3368	
3383	
3388	
3391	
3406	
3421	
3438	
3453	
3471	
3499	
3508	
3524	
3547	
3554	
3577	
3589	

IRENE M. PAULY

STANFORD UNIVERSITY
June 18, 1919

We are pained to learn from a cable dispatch to an Italian paper published in New York that death has overtaken our esteemed collaborator

PROFESSOR ANNIBALE RICCÒ
Director of the Observatory of Catania

We shall publish later a sketch of the life and fruitful scientific activity of this distinguished astrophysicist.

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THE
ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME L — OCTOBER 1919 — NUMBER 3

THE RADIAL VELOCITIES OF 185 STARS OBSERVED AT THE CAPE

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In continuation of a paper on "The Radial Velocities of 119 Stars Observed at the Cape"[1] the radial velocities are here given for a further 185 stars of magnitudes 3.7 to 4.6 observed with a wider slit than was used for the brighter stars in the former list.

As before, the stars are divided into two lists, the first giving the results for those stars which appear to have fairly constant velocities and the second for those stars which are either known or suspected to be variable in velocity. The two lists, given in Tables I and II, contain 122 and 63 stars, respectively.

Owing to the more diffuse character of the spectra taken with the wider slit, a somewhat larger latitude for error has been allowed in assigning the stars to one list or the other. The dividing line has been drawn in an arbitrary manner, and it is probable that stars may have to be removed from one list to the other when further observations are available.

The first list includes all stars showing a range up to 7.7 km per second, or a difference, Lick *minus* Cape, of not more than

[1] *Astrophysical Journal*, **48**, 261, 1918.

TABLE I

Cape List No.	H.R. No.	Star	α (1900)	δ (1900)	Mag.	Type	Standard Plates Used	No. of Plates	Mean Epoch	Radial Velocities Km: Lick* (1)	Radial Velocities Km: Cape (2)	Range Km	Diff. (1)−(2) Km	D. O. Mills Expedition† Km
1	25	ε Phoenicis	$0^h\ 4^m.3$	−46°18′	3.94	K	*ae*	5	1911.62	− 9.1	− 9.0	3.6	−0.1	− 9.1
7	127	β² Toucani	27.0	−63 31	4.48	A2	*c*	1	1908.69	+12.0	+12.7	....	−0.7	....
14	402	θ Ceti	1 10.0	− 8 42	3.83	K	*ae*	8	1910.67	+17.9	+16.9	3.7	+1.0	....
17	440	δ Phoenicis	27.0	−49 35	3.96	K	*ae*	7	1911.78	− 6.0	− 7.1	5.0	+1.1	− 6.9
21	585	υ Ceti	55.3	−21 34	4.18	Ma	*b*	4	1910.63	+18.4	+18.3	3.9	+0.1	....
27	794	ι Eridani	2 36.7	−40 17	4.06	K	*ae*	6	1909.07	− 9.2	− 9.2	4.2	0.0	− 9.0
29	841	β Fornacis	44.9	−32 50	4.50	K	*ade*	4	1913.63	+17.5	+16.2	6.3	+1.3	+17.3
30	874	η Eridani	51.5	− 9 18	4.05	K	*ae*	4	1910.76	−20.0	−19.8	4.9	+0.2	....
33	963	α Fornacis	3 7.8	−29 23	3.95	F8	*ce*	7	1911.83	−20.6	−21.5	5.4	+0.9	−20.8
34	1003	τ⁴ Eridani	15.1	−22 7	3.95	Mb	*bd*	3	1912.85	+43.0	+39.7	7.0	+3.3	+42.7
35	1008	e Eridani	15.9	−43 27	4.30	G5	*ac*	3	1912.23	+87.6	+87.2	2.1	+0.4	....
38	1106	y Eridani	33.5	−40 36	4.58	K	*ae*	3	1912.62	+10.8	+13.6	5.5	−2.8	....
39	1173	τ⁶ Eridani	42.5	−23 33	4.33	F8	*c*	4	1911.15	+ 7.3	+ 3.8	5.7	+3.5	....
41	1195	g Eridani	45.7	−36 30	4.24	K	*ade*	3	1912.72	+ 3.6	+ 1.1	5.7	+2.5	....
44	1247	δ Reticuli	57.2	−61 41	4.41	Ma	*bd*	2	1913.79	+ 1.0	− 3.1	4.4	+4.1	....
47	1326	α Horologii	4 10.7	−42 32	3.83	K	*ade*	4	1911.47	+21.7	+22.6	5.2	−0.9	+21.4
49	1338	γ Doradus	13.4	−51 44	4.36	F5	*c*	4	1912.49	+27.0	+26.5	7.7	+0.5	....
50	1355	ε Reticuli	14.7	−59 32	4.42	K2	*d*	2	1916.12	+29.7	+31.3	1.9	−1.6	....
51	1373	δ Tauri	17.2	+17 18	3.93	K	*ade*	4	1914.21	+38.6	+37.1	4.2	+1.5	....
52	1393	d Eridani	20.3	−34 15	4.06	K5	*ab*	4	1913.55	+23.6	+26.2	2.4	−2.6	+22.8
55	1453	υ¹ Eridani	29.6	−29 57	4.59	K	*e*	3	1913.96	+21.6	+20.8	1.8	+0.8	....
57	1464	υ² Eridani	31.7	−30 46	3.88	K	*ae*	4	1911.58	− 3.2	− 4.4	3.4	+1.2	− 3.2
58	1481	53 Eridani	33.6	−14 30	3.98	K	*ae*	5	1911.29	....	+42.3	3.8	....	....
59	1496	54 Eridani	36.1	−19 52	4.54	Ma	*b*	3	1914.32	−33.7	−34.3	2.8	+0.6	....
63	1652	γ Caeli	5 0.8	−35 37	4.62	K	*ae*	3	1911.06	+12.4	+ 8.2	6.6	+4.2	....
69	1907	φ² Orionis	31.4	+ 9 15	4.39	K	*ae*	4	1911.02	+99.4	+98.4	7.2	+1.0	....
71	1983	γ Leporis‡	40.3	−22 29	3.80	F8	*ce*	5	1911.13	− 9.1	−11.4	6.8	+2.3	− 8.9
75	2035	δ Leporis	47.0	−20 53	3.90	K	*ae*	6	1910.65	+99.3	+99.0	7.1	+0.3	....
77	2042	γ Pictoris	48.0	−56 12	4.38	K	*ae*	3	1910.45	+18.9	+14.4	3.2	+4.5	....
79	2085	η Leporis	51.9	−14 11	3.77	F5	*c*	6	1910.13	+ 0.3	− 3.5	7.2	+3.8	....
80	2102	Doradus	53.4	−63 8	4.53	K	*de*	3	1914.73	+25.6	+25.8	5.6	−0.2	....
81	2120	η Columbae	56.1	−42 49	4.03	K	*ae*	5	1910.76	+17.9	+15.7	4.4	+2.2	+18.7
83	2256	κ Columbae	6 13.0	−35 6	4.51	K	*ae*	3	1912.08	+25.6	+24.0	5.2	+1.6	....
93	2580	o¹ Can. Maj.	6 49.9	−24 4	4.12	K2	*b*	4	1911.26	+37.9	+36.0	5.9	+1.9	....
95	2646	σ Can. Maj.	57.7	−27 47	3.68	K5	*b*	7	1911.04	+23.	+21.1	7.6	+1.9	+21.9
99	2740	l Puppis	7 9.7	−46 35	4.47	F	*c*	3	1914.50	− 2.3	− 0.4	5.9	−1.9	....
103	2803	δ Volantis	16.9	−67 46	4.02	F5	*c*	3	1912.13	+22.7	+21.7	1.5	+1.0	+22.7
104	2821	ι Geminorum	19.5	+28 0	3.89	K	*dc*	3	1914.83	+ 7.8	+11.2	1.6	−3.4	....
106	2906	Puppis	29.8	−22 5	4.52	F8	*c*	3	1911.82	+60.8	+61.2	2.1	−0.4	....
107	2905	υ Geminorum	29.8	+27 7	4.22	K5	*d*	2	1916.16	−20.	−19.3	1.5	−0.7	....
109	2970	α Monocerotis	36.5	− 9 19	4.07	K	*ade*	3	1912.79	+11.6	+ 9.4	4.4	+2.2	....
114	3017	c Puppis	41.7	−37 44	3.72	K5	*b*	4	1910.84	+18.3	+14.8	3.6	+3.5	....
115	3024	ζ Volantis	43.0	−72 22	3.89	K	*de*	3	1911.78	+49.2	+46.7	1.5	+2.5	+49.0
118	3102	j Puppis	52.6	−22 37	4.35	F8	*c*	3	1910.41	+14.7	+12.2	3.9	+2.5	....
122	3249	β Cancri	8 11.1	+ 9 30	3.76	K2	*ae*	3	1912.09	+22.4	+23.3	2.5	−0.9	....
126	3340	θ Chamaeleontis	23.6	−77 10	4.26	K	*ae*	3	1912.22	+23.7	+21.3	4.7	+2.4	....
128	3426	e Velorum	34.2	−42 38	4.13	A5	*c*	3	1913.10	+21.2	+17.1	2.9	+4.1	....

129....	3438	β Pyxidis......	36.2	−34 57	4.04	G5	*ae*	3	1911.45	−15.0	−15.4	1.4	+0.4	−15.3
131....	3475	ι Cancri......	40.6	+29 8	4.20	G5	*e*	3	1914.91	+15.4	+18.1	1.8	−2.7	....
132....	3477	d Velorum.....	40.8	−42 17	4.12	G5	*ae*	3	1912.47	− 1.9	− 3.1	5.3	+1.2	− 1.6
134....	3518	γ Pyxidis......	40.3	−27 21	4.19	K2	*b*	4	1913.27	+26.4	+24.2	5.4	+2.2	....
137....	3614	c Velorum.....	9 0.7	−46 42	3.69	K	*ae*	5	1911.42	+24.3	+24.6	2.9	−0.3	+24.2
141....	3696	g Carinae.....	13.4	−57 7	4.18	K5	*b*	4	1913.03	− 4.9	− 6.6	2.4	+1.7	− 5.3
147....	3845	ι Hydrae.....	34.7	− 0 41	4.10	K	*de*	3	1914.50	+24.6	+20.3	5.4	+4.3	....
151....	3905	μ Leonis......	47.1	+26 29	4.10	K	*ae*	3	1912.57	+15.5	+13.6	0.9	+1.9	....
156....	4050	q Carinae.....	10 13.7	−60 50	3.44	K5	*ab*	3	1911.23	+ 8.3	+10.1	2.0	−1.8	+ 8.1
158....	4063	− Velorum.....	15.8	−54 32	4.58	K	*ade*	3	1913.32	+16.2	+12.9	0.1	+3.3	....
159....	4094	μ Hydrae......	21.3	−16 20	4.06	K5	*ade*	3	1912.44	+39.8	+39.5	3.3	+0.3	....
162....	4114	s Carinae.....	24.2	−58 14	4.08	F	*c*	3	1912.54	+10.1	+ 6.3	2.3	+3.8	+10.4
165....	4174	γ Chamaeleontis	34.3	−78 6	4.10	Ma	*b*	3	1914.92	−22.7	−21.2	0.7	−1.5	−22.6
166....	4180	x Velorum.....	35.3	−55 5	4.37	G	*de*	3	1915.24	+20.9	+17.3	0.5	+3.6	+21.0
170....	4257	u Carinae.....	49.4	−58 19	3.88	K	*ae*	3	1912.58	+ 8.	+ 8.9	3.0	−0.9	+ 8.2
171....	4287	α Crateris	54.9	−17 46	4.20	K	*ade*	5	1913.39	+47.9	+46.2	4.2	+1.7	....
176....	4399	ι Leonis......	11 18.7	+11 5	4.03	F5	*c*	3	1909.95	− 9.9	− 7.3	5.2	−2.6	....
180....	4522	− Centauri....	41.7	−60 37	4.22	G	*e*	3	1913.86	− 3.5	− 4.5	4.6	+1.0	− 3.5
184....	4608	o Virginis.....	12 0.1	+ 9 18	4.24	G5	*e*	3	1914.50	−20.4	−28.9	5.3	−0.5	....
186....	4623	α Corvi.......	3.3	−24 10	4.18	F2	*c*	3	1911.63	+ 4.2	+ 3.1	4.3	+1.1	....
189....	4700	ε Crucis......	15.9	−59 51	3.57	K2	*b*	3	1910.01	− 4.5	− 3.6	3.8	−0.9	− 4.6
197....	4910	δ Virginis.....	50.6	+ 3 56	3.66	Ma	*b*	3	1913.54	−17.6	−17.1	0.6	−0.5	....
200....	4983	43 β Com. Ber...	13 7.2	+28 23	4.32	G	*e*	3	1916.41	+ 5.8	+ 6.2	7.2	−0.4	....
203....	5041	m Centauri.....	13 17.2	−64 1	4.50	G	*ae*	3	1912.74	+11.6	+10.7	5.2	+0.9	....
204....	5089	d Centauri....	25.2	−38 54	3.96	K	*e*	3	1912.32	− 3.0	− 1.6	5.8	−1.4	− 3.5
207....	5192	g Centauri....	43.6	+33 58	4.40	Mb	*b*	1	1912.52	+42.7	+40.2	....	+2.5	....
208....	5200	υ Boötis.......	44.6	+16 17	4.28	K5	*ad*	3	1914.07	− 6.2	− 3.7	0.7	−2.5	....
213....	5339	δ Octantis.....	14 10.9	−83 13	4.14	K2	*b*	3	1913.85	+ 5.1	+ 2.4	2.7	+2.7	+ 5.0
219....	5470	α Apodis......	35.4	−78 37	3.81	K5	*b*	4	1914.02	0.0	− 0.6	1.7	+0.6	+ 0.4
220....	5485	c¹ Centauri....	37.5	−34 44	4.13	K	*ae*	4	1910.91	−38.7	−39.7	2.8	+1.0	−38.7
228....	5771	ε Triang. Aust.	15 27.6	−65 59	4.11	K	*e*	4	1913.80	−14.1	−17.0	5.1	+2.9	....
229....	5787	γ Librae......	29.9	−14 27	4.02	K	*ae*	4	1912.93	−26.9	−28.0	4.8	+1.1	....
230....	5794	υ Librae......	30.9	−27 48	3.78	K2	*b*	3	1912.66	−24.5	−25.1	2.4	+0.6	−25.0
232....	5820	ψ¹ Lupi.......	33.5	−34 5	4.63	K	*ae*	3	1912.45	−20.5	−24.1	4.2	+3.6	....
236....	5879	κ Serpentis....	44.2	+18 27	4.28	K5	*de*	3	1915.52	−38.1	−37.9	5.7	−0.2	....
239....	5947	ε Coronae Bor..	53.4	+27 10	4.22	K	*d*	3	1916.52	−31.1	−29.6	2.6	−1.5	....
240....	{5977, 5978}	ξ Scorpii......	58.9	−11 6	{5.07, 4.77}	F8	*e*	4	1913.21	−32.9	−32.9	5.5	0.0	....
241....	5997	ω² Scorpii......	16 1.6	−20 36	4.58	G	*e*	3	1914.47	− 5.0	− 6.5	3.8	+1.5	....
242....	6030	δ Triang. Aust.	6.4	−63 26	4.03	G	*ae*	3	1911.71	− 5.2	− 4.3	4.8	−0.9	− 5.1
244....	6072	γ² Normae.....	12.4	−49 55	4.14	K	*ae*	4	1912.78	−27.8	−29.5	5.1	+1.7	....
249....	6163	β Apodis......	28.8	−77 18	4.16	K	*e*	3	1914.41	−27.7	−31.9	4.0	+4.2	....
250....	6166	H Scorpii......	29.8	−35 3	4.30	Ma	*b*	3	1914.27	− 1.2	− 2.5	5.8	+1.3	....
252....	6229	η Arae........	41.1	−58 52	3.68	K5	*b*	3	1914.21	+ 9.7	+ 7.1	1.8	+2.6	+ 9.9
254....	6271	ζ² Scorpii......	47.6	−42 12	3.75	K5	*b*	3	1914.53	−18.4	−20.7	1.2	+2.3	−19.2
256....	6295	ε¹ Arae........	51.6	−53 0	4.15	K2	*b*	3	1914.63	+24.4	+21.7	4.6	+2.7	....
264....	6402	d Ophiuchi	17 21.0	−29 47	4.37	F5	*ce*	3	1915.19	+37.1	+36.8	7.7	+0.3	....
269....	6582	η Pavonis.....	35.9	−64 41	3.58	K	*de*	3	1915.07	− 8.6	− 6.0	1.3	−2.6	− 8.5
275....	6703	ξ Herculis.....	53.9	+29 16	3.82	K	*d*	3	1916.70	− 1.5	+ 0.5	2.0	−2.0	....
276....	6745	π Pavonis.....	58.9	−63 40	4.44	A5	*c*	3	1914.50	−19.6	−18.2	0.9	−1.4	−19.5
280....	6783	ε Telescopii....	18 3.8	−45 58	4.60	K	*de*	3	1916.32	−25.8	−25.9	1.0	+0.1	....

* *Lick Observatory Bulletin*, **7**, 19–113. † *Publications of the Lick Observatory*, **9**, 329–332. ‡ One plate, October 21, 1908, gives +3.1 km, not included.

TABLE I—*Continued*

Cape List No.	H.R. No.	Star	α (1900)	δ (1900)	Mag.	Type	Standard Plates Used	No. of Plates	Mean Epoch	Radial Velocities Km: Lick* (1)	Radial Velocities Km: Cape (2)	Range Km	Diff. (1)−(2) Km	D. O. Mills Expedition† Km
285	6805	109 Herculis	18h 19m.4	+21° 43′	3.92	K	*d*	3	1916.72	−58.0	−56.3	1.9	−1.7	....
286	6905	ζ Telescopii	21.1	−49 7	4.14	K	*e*	3	1913.89	−30.1	−30.6	2.3	+0.5	−30.1
289	6982	ζ Pavonis	31.4	−71 31	4.10	K	*e*	3	1914.17	−17.0	−16.9	3.0	−0.1	−17.0
290	7061	110 Herculis	41.4	+20 27	4.26	F5	*ce*	3	1916.03	+22.6	+26.2	1.7	−3.6	....
293	7217	39 Sagittarii	58.7	−21 53	3.90	K	*ae*	4	1910.74	+26.2	+24.7	6.5	+1.5	....
294	7226, 7227	γ Coron. Aust.	59.7	−37 12	5.01, 5.01	F8	*c*	3	1915.02	−51.0	−53.7	1.2	+2.7	....
308	7581	ι Sagittarii	19 48.4	−42 8	4.21	K	*e*	4	1913.09	+36.2	+36.3	5.5	−0.1	+35.4
309	7602	β Aquilae	50.4	+ 6 9	3.90	K	*ae*	3	1912.37	−39.6	−39.4	2.7	−0.2	....
312	7747	α¹ Capricorni	20 12.1	−12 49	4.55	G	*ae*	3	1913.28	−25.5	−26.9	3.0	+1.4	....
319	7948	γ Delphini	42.0	+15 46	4.49	G5	*de*	3	1916.12	− 6.1	− 4.7	3.8	−1.4	....
324	8181	γ Pavonis	21 18.2	−65 49	4.30	F8	*c*	3	1913.41	−31.1	−31.5	3.4	+0.4	−30.0
326	8213	b Capricorni	23.0	−22 15	4.59	G5	*ae*	4	1910.65	−21.7	−20.9	3.9	−0.8	....
334	8411	λ Gruis	22 0.1	−40 2	4.60	K2	*d*	3	1916.85	+40.8	+37.5	0.8	+3.3	....
338	8556	δ¹ Gruis	23.3	−44 0	4.02	G5	*e*	3	1914.36	+ 5.7	+ 4.6	5.2	+1.1	+ 5.8
341	8630	β Octantis	35.8	−81 54	4.34	F	*c*	2	1913.81	....	+19.6	6.7	....	....
344	8665	ξ Pegasi	41.6	+11 40	4.31	F5	*cd*	3	1911.64	− 4.6	− 4.2	4.6	−0.4	....
345	8667	λ Pegasi	41.7	+23 2	4.14	K	*d*	3	1916.50	− 3.6	− 4.3	4.3	+0.7	....
347	8679	τ Aquarii	44.3	−14 7	4.21	K5	*ad*	3	1914.06	+ 1.4	+ 0.5	6.0	+0.9	....
348	8684	μ Pegasi	45.2	+24 4	3.67	K	*de*	3	1914.81	+14.2	+15.7	4.0	−1.5	....
349	8698	λ Aquarii	47.4	− 8 7	3.84	Ma	*bd*	3	1915.52	− 8.8	− 7.0	2.4	−1.8	....
351	8720	δ Piscis Aust.	50.4	−33 4	4.33	K	*ade*	3	1913.41	−12.0	−11.0	2.8	−1.0	−12.0
359	8852	γ Piscium	23 12.0	+ 2 44	3.85	K	*ae*	6	1910.82	−13.6	−12.9	5.7	−0.7	....
360	8863	γ Sculptoris	13.4	−33 5	4.51	K	*ae*	3	1910.52	+18.5	+14.3	4.2	+4.2	....
361	8892	b¹ Aquarii	17.7	−20 39	4.20	K	*c*	3	1912.70	− 5.3	− 6.4	2.9	+1.1	....
362	8906	b³ Aquarii	20.8	−21 12	4.52	K5	*bd*	3	1914.85	+16.2	+17.2	5.0	−1.0	....
		122 stars						422			Means..	3.8 Range	+0.7 Lick−Cape	+ 0.5 Chile−Cape

* *Lick Observatory Bulletin*, **7**, 19–113.

† *Publications of the Lick Observatory*, **9**, 329–332.

Standard Plates Used

a = solar (daylight) plates

b = α Tauri plate 2885. Adopted shift $\begin{cases} +50.28 \pm 0.13 \text{ km Simpson.} \\ +49.81 \pm 0.15 \text{ km Jackson.} \end{cases}$

c = α Can. Min. plate 2914. Adopted shift $\begin{cases} -21.49 \pm 0.14 \text{ km Simpson.} \\ -21.40 \pm 0.11 \text{ km Jackson.} \end{cases}$

d = α Tauri plate 3408. Adopted shift +75.06 Halm.

e = α₂ Centauri plate 4005. Adopted shift -47.73 ± 0.14 km Jackson.

f = α Can. Min. plate 2147. Adopted shift -1.22 ± 0.14 km Lunt.

TABLE II

Cape List No.	H.R. No.	Star	α (1900)	δ (1900)	Mag.	Type	Standard Plates Used	No. of Plates	Radial Velocities Km: Lick* (1)	Radial Velocities Km: Cape (2)	Range Km: Cape	Range Km: Published	Diff. (1)-(2) Km	*Lick Observatory Bulletin* Reference
3	77	ζ Toucani†	0^h14^m9	−65° 28′	4.34	F8	*ce*	4	+ 9.3	+ 9.7	8.0	......	−0.4	....
10	215	ζ Andromedae	42.0	+23 43	4.30	K	*e*	4	var.	var.	47.1	35.6	....	**6**, 141
11	224	δ Piscium†	43.5	+ 7 2	4.55	K5	*ad*	3	+32.6	+32.2	8.4	......	+0.4	....
15	429	γ Phoenicis	1 24.0	−43 50	3.40	K5	*b*	6	var.	var.	23.2	34.5	....	**9**, 116
19	555	ψ Phoenicis†	49.0	−46 48	4.41	Mb	*bd*	3	+ 8.9	+ 3.8	4.2	......	+5.1	....
28	813	μ Ceti†	2 39.5	+ 9 42	4.36	A5	*c*	3	+26.0	+30.2	9.0	......	−4.2	....
36	1030	o Tauri	3 19.4	+ 8 41	3.80	G5	*ae*	6	V_o−20.0	var.	9.4	10.0	....	**4**, 161
40	1175	β Reticuli	42.9	−65 7	3.80	K	*ae*	6	V_o+50.0	var.	6.6	7.4	....	**4**, 97
53	1411	θ¹ Tauri†	4 22.8	+15 44	4.04	K	*de*	3	+37.5	+46.2	8.0	......	−8.7	....
60	1502	α Caeli	37.3	−42 3	4.52	F2	*ce*	3	V_o± 0.0	var.	10.5	14.5	....	**6**, 143
67	1862	ε Columbae	5 27.7	−35 33	3.92	K	*ae*	5	V_o− 4	var.	5.1	7.0	....	**6**, 153
70	1922	β Doradus	32.8	−62 33	3.81	F5	*c*	4	V_o+12.	var.	20.2	27.1	....	3, 3
82	2216	η Geminorum	6 8.8	+22 32	var.	Ma	*b*	3	var.	var.	20.5	11.0	....	1, 158
85	2296	δ Columbae	18.4	−33 23	3.98	G5	*ae*	8	var.	var.	13.3	16.0	....	3, 110
92	2554	A Carinae	47.6	−53 31	4.38	G5	*ae*	4	var.	var.	23.9	45.4	....	3, 111
96	2650	ζ Geminorum	58.2	+20 43	var.	G	*ae*	3	V_o+ 6.8	var.	19.6	30.9	....	[13, 91 *Ap.J.*]
110	2973	σ Geminorum	7 37.0	+29 7	4.26	K	*de*	4	V_o+45.8	var.	60.7	75.8	....	[5, 201 *J.R.A.S.C.*]
113	2996	1 Puppis	39.8	−28 43	4.10	A2p	*c*	5	var.	+25.4	17.4	10.6	....	6, 145
120	3225	h¹ Puppis	8 7.8	−39 19	4.43	K5	*bd*	4	var.	+13.7	4.7	13.2	....	6, 55
121	3243	h² Puppis	10.5	−40 2	4.43	K	*ade*	6	var.	var.	22.1	16.5	....	6, 55
136	3591	w Velorum	56.3	−40 52	4.42	F8	*ce*	5	V_o+ 9.	var.	34.2	10.7	....	3, 3
138	3615	α Volantis	9 0.9	−66 0	4.18	A5	*cf*	4	V_o+ 6.	var.	24.7	108.	....	3, 111
140	3643	G Carinae†	4.9	−72 12	4.50	F5	*c*	3	+21.0	+22.4	8.7	......	−1.4	....
148	3852	14 Leonis‡	35.8	+10 21	3.76	F5p	*c*	3	V_o+27.1	var.	36.2	{113.2 / 129.8}	....	5, 21
152	3912	m Velorum	47.9	−46 5	4.56	G5	*ade*	4	var.	var.	23.8	16.4	....	4, 97
153	3994	λ Hydrae	10 5.7	−11 52	3.83	K	*ade*	9	V_o+20.	+18.8	6.4	9.0	....	4, 68
160	4102	I Carinae†	22.4	−73 32	4.08	F5	*c*	3	+ 3.	− 2.6	4.1	......	+5.6	....
161	4104	α Antliae†	22.6	−30 34	4.42	K5	*b*	3	+16.	+10.7	8.1	......	+5.3	....
164	4167	p Velorum	33.1	−47 42	4.06	F2	*ce*	6	V_o+17.	var.	67.0	50.0	....	3, 111
167	4200	w Carinae†	39.7	−60 3	4.49	K5	*bd*	6	+12.8	+ 7.9	8.4	......	+4.9	....
172	4337	x Carinae	11 4.4	−58 26	4.02	F8p	*c*	5	V_o+ 8.0	+ 6.1	4.8	14.1	....	4, 161
175	4382	δ Crateris†	14.3	−14 14	3.82	K	*ade*	6	− 4.7	− 7.4	10.8	......	+2.7	....
183	4599	θ¹ Crucis	58.0	−62 45	4.48	A3	*c*	3	var.	− 7.2	3.5	24.6	....	6, 56
185	4616	η Crucis†	12 1.7	−64 3	4.30	F	*c*	3	+12.0	+11.3	12.5	......	+0.7	....
188	4671	ε Muscae	12.1	−67 24	4.16	Mb	*b*	3	V_o+ 6.	+ 7.2	8.4	12.3	....	6, 145
191	4775	η Corvi	12 26.9	−15 38	4.42	F	*c*	4	var.	var.	13.2	30.8	....	**5**, 175

TABLE II—*Continued*

Cape List No.	H.R. No.	Star	α (1900)	δ (1900)	Mag.	Type	Standard Plates Used	No. of Plates	Radial Velocities Km: Lick* (1)	Radial Velocities Km: Cape (2)	Range Km: Cape	Range Km: Published	Diff. (1)−(2) Km	*Lick Observatory Bulletin* Reference
195	4888	e Centauri†	12h47m5	−48°24′	4.35	K2	*b*	3	+ 0.9	− 4.9	6.0		+5.8	
198	4923	δ Muscae†	55.4	−71 1	3.63	K2	*b*	3	+31.7	var.	17.2			
200	5168	i Centauri†	13 40.0	−32 33	4.36	F5	*ce*	3	−14.6	var.	28.0			
210	5260	υ² Centauri	55.4	−45 7	4.30	F5	*c*	4	var.	var.	12.2	13.2		4, 97
215	5396	τ² Lupi†	14 19.8	−44 56	4.49	F5	*c*	3	+ 7.6	var.	22.5			
227	5747	β Coronae Bor.	15 23.7	+29 27	3.72	Fp	*c*	3	V_0−21.3	var.	10.8	6.3		[6, 343 *J.R.A.S.C.*]
231	5797	ω Lupi†	31.3	−42 14	4.27	K5	*bd*	3	− 3.4	− 9.1	10.1		+5.7	
238	5933	γ Serpentis†	51.8	+15 59	3.86	F8	*ae*	3	+ 7.3	+ 4.4	10.1		+2.9	
240	6102	γ Apodis	16 18 1	−78 40	3.90	K	*ae*	3	V_0+ 5.	+ 6.0	3.9	6.3		5, 177
265	6546	Q Scorpii	17 29 6	−38 34	4.34	K	*de*	3	V_0−47.	−51.3	4.8	10.5		7, 97
278	6752	70 Ophiuchi	18 0.4	+ 2 31	4.07	K	*ae*	3	V_0− 7.4	− 6.9	1.9	4.0		5, 63
282	6855	ξ Pavonis	11.0	−61 33	4.25	K2	*d*	3	var.	+25.0	5.6	36.6		6, 152
288	6951	θ Coron. Aust.†	26.4	−42 23	4.69	G5	*e*	3	+ 3.3	− 5.7	3.5		+9.0	
296	7242	δ Coron. Aust.†	19 1.3	−40 39	4.66	K	*ade*	3	+23.2	+17.6	2.1		+5.6	
298	7259	β Coron. Aust.†	3.1	−39 30	4.16	G5	*ae*	3	+ 3.3	+ 2.8	8.6		+0.5	
305	7536	δ Sagittae	42.9	+18 17	3.78	Map	*bd*	3	V_0+ 3.	− 2.2	2.0	15.0		4, 96
307	7570	η Aquilae	47.4	+ 0 45	var.	G	*ae*	3	V_0−14.2	−17.4	6.8	42.0		[9, 66 *Ap.J.*]
318	7936	ψ Capricorni†	20 40.2	−25 38	4.26	F8	*c*	4	+26.5	var.	12.3			
320	7980	ω Capricorni†	45.9	−27 18	4.24	Ma	*bd*	3	+12.	+ 5.1	6.2		+6.9	
323	8131	α Equulei	21 10.8	+ 4 50	4.14	A8p	*ae*	3	var.	−12.0	6.9	24.0		**1**, 159
328	8254	ν Octantis	30.4	−77 50	3.74	K	*ae*	4	V_0+31.	+37.6	11.5	12.0		**5**, 61
331	8315	κ Pegasi	40.1	+25 11	4.27	F5	*ce*	3	V_0− 5.	var.	57.1	95.1		**9**, 120
336	8430	ι Pegasi	22 2.4	+24 51	3.96	F5	*ce*	3	V_0− 4.1	var.	73.9	95.4		**2**, 169
340	8560	δ² Gruis†	23.8	−44 15	4.31	Mb	*bd*	4	+ 5.	+ 0.7	8.1		+4.3	
353	8747	ζ Gruis	55.0	−53 17	4.18	G5	*ae*	3	V_0− 4.	− 4.4	8.9			
357	8820	ι Gruis	23 4.7	−45 47	4.10	K	*ae*	3	var.	var.	13.2	16.5		4, 161
363	9072	ω Piscium†	54.2	+ 6 19	4.03	F5	*c*	5	− 2.4	+ 9.9	10.1		−12.3	
		63 stars						244						

* *Lick Observatory Bulletin*, **7**, 19, 113.

† Either obviously variable in velocity or suspected.

‡ Measures refer to first component.

V_0 = velocity of the center of mass of the system (estimated or determined).

4.5 km per second. The stars in this list are divided as shown in the following table:

Range km km	Stars	Diff. Lick *minus* Cape km km	Stars
0.0 to 3.0	47	0.0 to 1.0	49
3.1 to 5.0	35	1.1 to 2.0	32
5.1 to 7.0	31	2.1 to 3.0	21
7.1 to 7.7	7	3.1 to 4.0	11
1 plate only	2	4.1 to 4.5	7
	122		120
Mean range, 3.8 km		Mean difference, +0.7 km	

Two-thirds of the stars therefore differ 2.0 km or less from the Lick values, and 85 per cent differ 3.0 km or less; the mean difference, Lick *minus* Cape, is +0.7 km per second.

Forty-one stars common to this list and that published by the D. O. Mills Expedition to Chile[1] show a systematic difference, Chile *minus* Cape, +0.5 km per second. In the previous paper the difference Chile *minus* Cape for 33 stars was −0.3 km per second.

Of the 63 stars in Table II, 39 have already been recorded as variable in velocity; the remaining 24, marked with a dagger, are either obviously variable or belong to the suspected list. For 11 of these latter stars the difference, Lick *minus* Cape, is between 4.9 and 12.3 km, and for the remaining 13 stars the observed range is between 8.0 and 28.0 km.

The results for the individual plates of the stars in Table II are given in Table III.

In some cases in which orbits have been published the measures have been compared with the theoretical velocities by a rough calculation and found to agree satisfactorily

Twenty-eight stars of types A to F5 were rejected, the lines in their spectra being too diffuse or too few to yield satisfactory results with the dispersion employed, and these are given in Table IV.

The measures were made on the Hartmann spectrocomparator by Mr. J. W. Jackson, who measured more than half; the earlier

[1] *Publications of the Lick Observatory*, **9**, 329–332.

TABLE III

H.R. No.	Plate No.	Star and Date	Sid. Time and R.A.	Rad. Vel. km	Measured by	H.R. No.	Plate No.	Star and Date	Sid. Time and R.A.	Rad. Vel. km	Measured by
77		*ζ Toucani*	$0^h14^m.9$			1030	2513	1909 Sept. 22	$3^h\ 6^m$	−23.1	S
	2470	1909 Sept. 1	1 0	+12.5	S		3824	1912 Sept. 29	3 33	−18.2	J
	4151	1913 Oct. 21	23 5	+5.2	J		3834	Oct. 4	3 48	−17.4	J
	4152	Oct. 23	22 36	+13.2	J		3923	Dec. 18	2 40	−19.5	J
	4430	1914 Nov. 7	1 35	+7.9	J		4170	1913 Dec. 19	2 29	−26.8	J
215		*ζ Andromedae*	$0^h42^m.0$			1175		*β Reticuli*	$3^h42^m.9$		
	4165	1913 Dec. 1	1 45	+6.2	J		2541	1909 Oct. 20	4 38	+47.5	S
	4437	1914 Nov. 4	23 27	−2.2	J		2816	1910 Sept. 13	2 48	+49.6	S
	4440	Nov. 11	0 46	−40.9	J		3338	1911 Oct. 31	4 32	+47.1	J
	4446	Nov. 18	0 46	−28.4	J		3360	Nov. 17	4 30	+51.4	J
224		*δ Piscium*	$0^h43^m.5$				3364	Nov. 29	1 35	+53.7	J
	1929	1908 Oct. 3	1 52	+36.0	S		3804	1912 Sept. 10	4 30	+52.8	J
	2013	Nov. 16	1 5	+27.7	H	1411		*θ^1 Tauri*	$4^h22^m.8$		
	4665	1915 Dec. 4	2 22	+33.1	H		4169	1913 Dec. 8	4 6	+41.5	J
429		*γ Phoenicis*	$1^h24^m.0$				4184	Dec. 27	3 52	+49.5	J
	2034	1908 Nov. 27	2 1	+8.3	S		4693	1916 Jan. 28	5 34	+47.7	J
	2592	1909 Nov. 25	2 46	+18.6	S	1502		*α Caeli*	$4^h37^m.3$		
	3390	1911 Dec. 11	2 16	+31.6	J		2052	1908 Dec. 2	5 20	−3.8	S
	3396	Dec. 18	2 57	+29.5	J		4460	1914 Dec. 14	2 45	+6.7	J
	3809	1912 Sept. 12	2 28	+12.7	J		4690	1916 Jan. 25	5 32	−0.9	H
	4448	1914 Nov. 19	2 8	+20.3	J	1862		*ε Columbae*	$5^h27^m.7$		
555		*ψ Phoenicis*	$1^h49^m.6$				2218	1909 Feb. 11	7 34	−8.1	S
	4464	1914 Dec. 22	3 35	+6.2	J		2222	Feb. 12	6 4	−9.1	S
	4670	1915 Dec. 18	3 24	+1.9	H		3320	1911 Oct. 24	6 10	−4.0	J
	4672	Dec. 22	3 20	+3.3	H		3330	Oct. 27	4 29	−5.5	J
813		*μ Ceti*	$2^h39^m.5$				3392	Dec. 14	4 6	−7.3	J
	3929	1912 Dec. 19	3 0	+31.6	J	1922		*β Doradus*	$5^h32^m.8$		
	3932	Dec. 21	3 3	+34.0	J		2136	1909 Jan. 18	6 42	+0.0	S
	3935	Dec. 23	3 26	+25.0	J		2254	Mar. 1	6 44	−8.5	S
1030		*o Tauri*	$3^h19^m.4$				3859	1912 Oct. 22	4 0	+0.7	J
	1921	1908 Sept. 30	2 58	−21.2	S		4183	1913 Dec. 23	4 1	+11.7	J

2216	......	*η Geminorum*	$6^h\ 8^m.8$	....	......
	2162	1909 Jan. 25	7 17	+42.0	S
	3077	1913 Feb. 17	6 31	+23.9	J
	4006	Mar. 12	7 42	+21.6	J
2296	......	*δ Columbae*	$6^h 18^m.4$	....	......
	2096	1908 Dec. 23	6 37	−11.8	S
	2674	1910 Feb. 14	7 54	− 2.7	S
	2699	Mar. 2	7 39	− 3.0	S
	3334	1911 Oct. 30	4 36	− 5.9	J
	3419	1912 Jan. 19	4 47	− 2.1	J
	3478	Feb. 16	7 17	− 1.9	J
	3891	Nov. 13	7 6	+ 1.6	J
	4504	1915 Feb. 23	7 3	− 3.4	J
2554	......	*A Carinae*	$6^h 47^m.6$	....	......
	2137	1909 Jan. 18	8 37	+19.0	S
	2710	Mar. 24	8 22	+ 4.4	S
	3401	1911 Dec. 19	7 48	+27.0	J
	4474	1915 Jan. 16	5 22	+ 3.1	J
2650	......	*ζ Geminorum*	$6^h 58^m.2$	....	......
	2223	1909 Feb. 12	7 39	+13.7	S
	4209	1914 Jan. 23	5 48	+ 7.9	J
	4695	1916 Feb. 1	6 46	− 5.9	H
2973	......	*σ Geminorum*	$7^h 37^m.0$	....	......
	4503	1915 Feb. 22	7 48	+81.6	J
	4507	Feb. 26	8 0	+48.0	J
	4720	1916 Mar. 11	8 13	+20.9	H
	4721	Mar. 13	8 16	+32.2	H
2996	......	*l Puppis*	$7^h 39^m.8$	....	......
	2702	1910 Mar. 4	8 30	+22.3	S
	4185	1913 Dec. 29	8 15	+31.1	J
	4221	1914 Feb. 4	6 33	+24.1	J
	4746	1916 Apr. 14	9 9	+16.1	H
	4750	Apr. 19	9 31	+33.4	H
3225	......	*h¹ Puppis*	$8^h\ 7^m.8$	....	......
	4484	1915 Jan. 27	6 29	+14.7	J
	4485	Jan. 28	6 50	+14.8	J

3225	4542	1915 Apr. 1	$9^h 14^m$	+15.0	J
	4762	1916 May 4	11 30	+10.3	H
3243	......	*h² Puppis*	$8^h 10^m.5$	....	......
	2316	1909 Apr. 14	9 51	+13.1	S
	3067	1911 Mar. 20	9 14	+ 0.8	S
	4481	1915 Jan. 25	6 33	+21.3	J
	4743	1916 Apr. 11	9 59	+ 3.7	H
	4748	Apr. 15	9 12	+ 5.7	H
	4752	Apr. 27	10 57	− 0.8	H
3501	......	*w Velorum*	$8^h 56^m.3$	....	......
	4261	1914 Mar. 16	10 11	− 5.0	J
	4274	Mar. 31	10 1	−23.2	J
	4723	1916 Mar. 15	7 40	+ 8.1	H
	4744	Apr. 12	10 32	−20.1	H
	4756	Apr. 20	9 39	−11.7	H
3615	......	*α Volantis*	$9^h\ 0^m.9$	....	......
	4201	1914 Jan. 14	6 17	+ 5.2	J
	4227	Feb. 11	8 2	+ 6.7	J
	4245	Mar. 6	10 12	+25.2	J
	4757	1916 Apr. 29	11 20	+ 0.5	H
3643	......	*G Carinae*	$9^h\ 4^m.9$	....	......
	2326	1909 Apr. 19	10 53	+25.3	S
	4292	1914 Apr. 24	10 26	+16.7	J*
	4294	Apr. 25	10 11	+25.3	J
3852	......	*14 Leonis*	$9^h 35^m.8$	....	......
	2227	1909 Feb. 15	10 37	+25.8	S
	3409	1912 Jan. 15	8 22	+57.2	J
	4014	1913 Mar. 10	10 10	+21.0	J
3912	......	*m Velorum*	$9^h 47^m.9$	....	......
	2164	1909 Jan. 25	10 29	+26.1	S
	3045	1911 Mar. 10	8 7	+ 2.3	S
	4282	1914 Apr. 9	10 18	+ 9.5	J
	4772	1916 May 16	11 37	+20.1	H
3994	......	*λ Hydrae*	$10^h\ 5^m.7$	....	......
	2143	1909 Jan. 20	10 34	+17.4	S
	2148	Jan. 21	9 40	+15.5	S

* Very poor plate.

TABLE III—*Continued*

H.R. No.	Plate No.	Star and Date	Sid. Time and R.A.	Rad. Vel. km	Measured by
3004	2155	1909 Jan. 22	$0^h 52^m$	+17.7	S
	2902	1910 Dec. 2	8 10	+20.9	S
	3030	1911 Mar. 2	9 1	+20.5	S
	3531	1912 Mar. 11	9 24	+21.8	J
	3926	Dec. 18	8 34	+15.8	J
	4766	1916 May 10	10 57	+19.7	H
	4770	May 15	10 34	+20.0	H
4102	...	*I Carinae*	**$10^h 22^m.4$**		
	2351	1909 May 14	11 21	− 4.0	S
	4265	1914 Mar. 23	8 17	+ 0.2	J
	4267	Mar. 26	8 36	− 3.8	J
4104	.....	*a Antliae*	**$10^h 22^m.6$**		
	2357	1909 May 17	11 52	+14.9	S
	4269	1914 Mar. 27	8 34	+10.4	J
	4304	May 5	11 35	+ 6.8	J
4167	.....	*p Velorum*	**$10^h 33^m.1$**		
	2239	1909 Feb. 22	11 31	+16.6	S
	4510	1915 Mar. 5	8 1	+39.8	J
	4527	Mar. 25	9 37	+39.9	J
	4537	Mar. 31	11 16	+32.5	J
	4758	1916 May 1	11 2	+ 2.5	H
	4774	May 20	11 14	−27.1	H
4200	......	*w Carinae*	**$10^h 39^m.7$**		
	4545	1915 Apr. 8	9 33	+ 9.0	J
	4568	May 29	12 54	+ 2.9	J
	4729	1916 Mar. 20	9 8	{+10.7 {+11.3	H H
	4731	Mar. 23	8 43	+ 3.3	H
	4733	Mar. 25	8 52	+10.1	H
4337	......	*x Carinae*	**$11^h 4^m.4$**		
	2270	1909 Mar. 8	12 43	+ 8.2	S

R.H. No.	Plate No.	Star and Date	Sid. Time and R.A.	Rad. Vel. km	Measured by
4337	4302	1914 May 2	$11^h 50^m$	+ 8.6	J
	4307	May 8	12 49	+ 4.6	J
	4759	1916 May 1	12 32	+ 3.8	H
	4764	May 8	12 6	+ 5.2	H
4382	......	*δ Crateris*	**$11^h 14^m.3$**		
	2156	1909 Jan. 22	11 11	− 9.9	S
	2286	Mar. 21	12 5	− 7.1	S
	3467	1912 Feb. 9	12 3	− 0.1	J
	4765	1916 May 9	11 51	−10.8	H
	4773	May 17	12 8	− 8.3	H
	4775	May 20	12 43	− 8.2	H
4599	......	*θ¹ Crucis*	**$11^h 58^m.0$**		
	4198	1914 Jan. 11	10 52	− 7.3	J
	4266	Mar. 23	10 4	− 5.4	J
	4268	Mar. 26	10 15	− 8.9	J
4616	......	*η Crucis*	**$12^h 1^m.7$**		
	3117	1911 Apr. 28	13 30	+16.1	J
	4019	1913 Mar. 26	10 38	+14.3	J
	4270	1914 Mar. 27	10 51	+ 3.6	J
4671	......	*ε Muscae*	**$12^h 12^m.1$**		
	4063	1913 June 20	14 45	+ 4.2	J
	4345	1914 June 30	14 28	+ 4.7	J
	4544	1915 Apr. 7	10 17	+12.6	J
4775	......	*η Corvi*	**$12^h 26^m.9$**		
	2383	1909 June 14	13 21	− 1.3	S
	4276	1914 Apr. 2	11 0	− 2.3	J
	4570	1915 June 3	14 2	+ 0.9	J
	4763	1916 May 6	11 54	−12.3	H
4888	......	*e Centauri*	**$12^h 47^m.5$**		
	4078	1913 July 9	14 12	− 8.2	J
	4343	1914 June 29	14 13	− 4.4	J

4888	4347	1914 July 1	14^h38^m	− 2.2	J
4923		*δ Muscae*	12^h45^m4		
	2277	1909 Mar. 14	11 32	+47.6	S
	4075	1913 June 30	14 12	+30.4	J
	4248	1914 Mar. 8	14 3	+37.8	J
5168		*i Centauri*	13^h40^m0		
	3147	1911 May 22	12 37	− 4.2	J
	4080	1913 July 10	14 44	− 9.8	J
	4549	1915 Apr. 10	11 17	−32.2	H
5260		*υ² Centauri*	13^h55^m4		
	4022	1913 Mar. 30	15 36	− 5.8	J
	4067	June 21	15 44	− 9.7	J
	4351	1914 July 3	15 2	+ 0.8	J
	4360	July 14	15 35	+ 2.5	J
5396		*τ² Lupi*	14^h19^m8		
	2311	1909 Apr. 9	13 3	+ 9.0	S
	3709	1912 July 15	15 13	−12.0	J
	4086	1913 July 17	15 23	− 2.8	J
5747		*β Coronae*	15^h23^m7		
	3980	1913 Feb. 19	14 33	−15.7	J
	4015	Mar. 19	14 46	−22.3	J
	4785	1916 June 16	14 7	−26.5	H
5797		*ω Lupi*	15^h31^m3		
	2270	1909 Mar. 15	13 58	− 3.1	S
	4387	1914 Aug. 12	17 9	13.2	J
	4788	1916 June 30	14 2	11.0	H
5933		*γ Serpentis*	15^h51^m8		
	2424	1909 July 17	16 25	+ 4.8	S
	3684	1912 July 2	16 13	+ 9.2	J
	4374	1914 July 23	14 52	− 0.9	J
6102		*γ Apodis*	16^h18^m1		
	2441	1909 July 30	17 15	+ 6.1	S
	4259	1914 Mar. 15	16 31	+ 7.9	J
	4600	1915 Aug. 21	18 13	+ 3.9	J
6546		*Q Scorpii*	17^h29^m6		
	4102	1913 Aug. 6	19 2	−48.5	J
	4352	1914 July 4	16 5	−53.3	J

6546	4798	1916 Aug. 3	16^h25^m	−52.1	H
6752		*70 Ophiuchi*	$18^h\ 0^m4$		
	2469	1909 Aug. 27	18 31	− 6.1	S
	4388	1914 Aug. 12	19 3	− 8.0	J
	4392	Aug. 19	18 12	− 6.7	J
6855		*ξ Pavonis*	18^h14^m0		
	4800	1916 Aug. 8	16 29	+22.6	H
	4805	Aug. 10	16 54	+24.3	H
	4806	Aug. 15	16 36	+28.2	H
6951		*θ Coron. Aust.*	18^h26^m4		
	4404	1914 Sept. 14	19 26	− 7.7	J
	4609	1915 Sept. 11	19 58	− 5.3	J
	4613	Sept. 17	20 12	− 4.2	J
7242		*δ Coron. Aust.*	$19^h\ 1^m3$		
	2794	1910 Aug. 19	20 2	+18.0	S
	4616	1915 Sept. 18	20 43	+18.5	J
	4830	1916 Sept. 27	20 23	+16.4	H
7259		*β Coron. Aust.*	$19^h\ 3^m1$		
	2828	1910 Sept. 22	20 13	+ 7.6	S
	4393	1914 Aug. 19	20 7	+ 1.6	J
	4604	1915 Sept. 3	21 7	− 0.9	J
7536		*δ Sagittae*	19^h42^m9		
	4617	1915 Sept. 22	19 55	− 1.0	J
	4826	1916 Sept. 25	20 21	− 3.1	H
	4831	Sept. 29	21 7	− 2.5	H
7570		*η Aquilae*	19^h47^m4		
	2801	1910 Aug. 22	21 6	−13.9	S
	3269	1911 Sept. 6	19 5	−20.7	J
	3775	1912 Aug. 23	19 58	−17.5	J
7936		*ψ Capricorni*	20^h40^m2		
	2403	1909 July 5	22 32	+31.2	S
	2820	1910 Sept. 16	20 7	+23.4	S
	3135	1911 May 11	20 8	+18.9	J
	3197	June 26	21 36	+25.6	J
7980		*ω Capricorni*	20^h45^m9		
	4625	1915 Oct. 8	22 22	+ 8.4	J
	4628	Oct. 15	22 18	+ 2.2	H

TABLE III—*Continued*

H.R. No.	Plate No.	Star and Date	Sid. Time and R.A.	Rad. Vel. km	Measured by	H.R. No.	Plate No.	Star and Date	Sid. Time and R.A.	Rad. Vel. km	Measured by
7980	4632	1915 Oct. 19	21^h48^m	+ 4.6	H	8560	4451	1914 Nov. 24	1^h15^m	− 1.8	J
8131	......	α *Equulei*	**$21^h10^m.8$**	......	......		4645	1915 Nov. 3	0 32	− 0.5	H
	2552	1909 Oct. 29	22 36	− 8.1	S		4850	1916 Nov. 14	1 28	+ 6.3	H
	2819	1910 Sept. 15	20 5	−15.0	S		4851	Nov. 16	0 43	− 1.1	H
	3840	1912 Oct. 5	22 34	−13.0	J	8747	......	ζ *Gruis*	**$22^h55^m.0$**	......	......
8254	......	ν *Octantis*	**$21^h30^m.4$**	......	......		2463	1909 Aug. 25	0 27	− 9.7	S
	2410	1909 July 12	22 39	+39.7	S		4434	1914 Oct. 28	23 54	− 2.6	J
	2827	1910 Sept. 21	22 18	+30.3	S		4447	Nov. 19	0 35	− 0.9	J
	3301	1911 Oct. 7	22 36	+41.8	J	8820	......	ι *Gruis*	**$23^h\ 4^m.7$**	......	......
	4411	1914 Sept. 18	22 46	+38.7	J		1991	1908 Nov. 6	0 1	+ 2.3	S
8315	......	κ *Pegasi*	**$21^h40^m.1$**	......	......		3902	1912 Nov. 19	0 28	−10.9	J
	4635	1915 Oct. 25	23 5	−36.6	H		4175	1913 Dec. 12	1 48	− 7.3	J
	4640	Oct. 28	22 57	+16.8	H	9072	......	ω *Piscium*	**$23^h54^m.2$**	......	......
	4838	1916 Oct. 19	22 17	+20.5	H		1965	1908 Oct. 21	1 8	+13.3	S
8430	......	ι *Pegasi*	**$22^h\ 2^m.4$**	......	......		2872	1910 Nov. 14	23 52	+ 3.3	S
	4608	1915 Sept. 8	22 14	+14.8	J		3912	1912 Dec. 2	1 17	+ 6.3	J
	4626	Oct. 11	23 4	−43.0	J		4435	1914 Oct. 29	1 42	+13.2	J
	4636	Oct. 26	22 50	+30.9	H		4436	Oct. 31	23 57	+13.4	J
8560	......	δ² *Gruis*	**$22^h23^m.8$**	......	......						

TABLE IV

Cape List No.	H. R. No.	Star	R. A.	Dec.	Mag.	Type
6	100	κ Phoenicis	0^h21^m3	$-44°14'$	3.90	A3
23	596	α Piscium	1 56.9	+ 2 17	4.33	A2p
26	705	δ Hydri	2 20.0	−69 7	4.26	A2
32	919	τ^3 Eridani	2 58.0	−24 1	4.16	A3
46	1298	o^1 Eridani	4 7.0	− 7 6	4.14	F5
62	1560	ω Eridani	4 48.0	− 5 37	4.45	A5
73	2015	δ Doradus	5 44.6	−65 46	4.52	A5
74	2020	β Pictoris	5 44.9	−51 6	3.94	A3
94	2590	π Can. Maj	6 51.3	−20 1	4.62	F5
123	3270	q Puppis	8 14.8	−36 21	4.43	A5
125	3318	α Chamaeleontis	8 21.1	−76 36	4.08	F5
146	3836	M Velorum	9 33.3	−48 55	4.49	A5
154	4023	q Velorum	10 10.5	−41 38	4.09	A2
173	4343	β Crateris	11 6.7	−22 17	4.52	A2
177	4405	γ Crateris	11 19.9	−17 8	4.14	A2
179	4520	λ Muscae	11 40.9	−66 10	3.80	A5
193	4802	τ Centauri	12 32.3	−47 59	4.02	A2
196	4889	n Centauri	12 47.9	−39 38	4.34	A5
225	5670	β Circini	15 9.6	−58 26	4.16	A3
233	5825	g Lupi	15 34.3	−44 20	4.69	F
235	5867	β Serpentis	15 41.6	+15 44	3.74	A2
263	6486	b Ophiuchi	17 20.3	−24 5	4.28	F
279	6771	72 Ophiuchi	18 2.6	+ 9 33	3.73	A2
297	7254	α Coron. Aust	19 2.7	−38 4	4.12	A2
300	7340	ρ^1 Sagittarii	19 15.9	−18 2	3.95	F
301	7343	β^2 Sagittarii	19 16.0	−44 59	4.51	F
333	8368	δ Indi	21 51.1	−55 28	4.56	F
358	8848	γ Toucani	23 11.6	−58 47	4.10	F2

plates were measured by Mr. J. A. Simpson, and the later ones by Dr. Halm. In Table III the names of the measurers are indicated by their initials.

Miss M. K. Stephens assisted in compiling the tabular matter.

ROYAL OBSERVATORY, CAPE OF GOOD HOPE
June 20, 1918

THE COLOR-CHANGES OF CERTAIN VARIABLE STARS OF SHORT PERIOD[1]

By F. C. JORDAN

The determination of the colors of stars has occupied an increasingly important place in astronomical investigations of recent years. They have been studied partly for the mere purpose of ascribing to each star its exact position in the color-scale; but the investigation becomes much more important when we consider the intimate connection between color and spectral type, color and temperature, and the place which colored stars occupy in the scheme of stellar evolution. Slow changes in the colors of some stars have been suspected, but not thoroughly proved.

It is well known that the long-period variable stars, all of which are reddish in color, become more strongly tinted as they decrease in brightness. As the mere decrease in brightness would make the tint appear less intense, the cause must lie in the star itself, and indicate a change in the absorption, and hence in the distribution, of energy in the spectrum. The cause of this is entirely unknown. The same phenomenon is found in short-period variables of certain types; but though there is a clue here to the cause, many points remain obscure, and much further investigation will be necessary in order to arrive at a satisfactory explanation.

By the study of star-colors combined with spectroscopic investigation we shall ultimately increase our knowledge of stellar evolution, and consequently of the development of the universe.

COLOR-DETERMINATIONS AND COLOR-SCALES

In the determination of the exact grade of color in a star various methods have been suggested and used, the most obvious one being that of eye-estimates. Various more or less fantastic names have been given to stellar tints in a general description of them for the

[1] Submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy at the University of Chicago, 1914.

use of the amateur observer. A few of these are: "red lilac," "pale gray," "flushed purple." It is probable that all star-colors are comprehended within the limits white to red, through the various shades of yellow and orange, with a possible trace of blue in some; though the latter color may usually be explained as the effect of contrast.

In indicating star-colors various scales have been suggested and used, such as the well-known numerical one ranging from 0, pure white, to 10, pure red. Müller and Kempf use the very simple method of naming the colors: white, yellowish white, yellow, and so on. Whatever the nomenclature may be, the object of all is the same, namely, that of locating the star as accurately as possible on the color-scale. None of these give place for any colors other than the various shades of yellow, orange, and red; and no others are needed.

DIFFICULTIES OF EYE-ESTIMATES

In nearly all the work which has been done in this line of research the eye has been the sole determining factor. It is true that various kinds of colorimeters have played some part in these investigations, but here also the eye is the final resort, and the result depends upon what the individual eye sees and records. In studying the results given by various observers curious anomalies are occasionally found, and such can hardly fail to be the case. Different observers have different color-perceptions; this perception may change as the observer becomes older; the same observer with larger or smaller aperture obtains varying results; stars of different brightness have a different physiological color-effect on the eye. These and other causes conspire to make the eye a rather faulty instrument in the determination of colors, and also in estimating the magnitudes of other than white stars.

It is well known that, in general, color increases with advance in spectral type; indeed, it is possible that there is no exception to this rule. But in catalogues of colored stars the order will sometimes be reversed, a star of more advanced type being credited with less color than one preceding it in the spectral scale. These anomalies are undoubtedly due to the effect of personal equation in determining the colors.

THE PHOTOGRAPHIC METHOD OF DETERMINING COLORS

Since the eye, because of its limitations as above mentioned, is unsatisfactory in this line of research, other methods have been proposed which seek to eliminate as far as possible any dependence upon the eye. It was Schwarzschild who first suggested the photographic method of determining colors: that is, the difference between the photographic and visual magnitudes of a star may be taken as the indication of its color. This effect is called by him *Farbentönung*. The method is becoming more and more extensively used in one form or another. As originally proposed, it meant merely the substitution of determinations of stellar magnitudes for the estimates of color, the visual magnitudes being found as usual by the eye, the others from the photographic plate. Thus were eliminated only partially the difficulties before mentioned in regard to colored stars, for there still remains the problem of comparing, for example, a deep yellow star with a neighboring white one, or with the artificial one of the photometer. This is always a difficult task and gives rise to decided differences in determinations of magnitude by different observers.

METHOD USED IN THIS WORK

In the present paper the Schwarzschild definition of color is used with the designation "Color-Index"; but the method of obtaining it is entirely photographic. The process is fully described in a paper by Professor J. A. Parkhurst and the writer, "The Photographic Determination of Star-Colors and Their Relation to Spectral Type."[1]

I shall give only an outline of the parts necessary for the work of this paper.

The instrument.—The telescope used was the two-foot reflector of the Yerkes Observatory. Since with the full aperture the field of good definition is very limited, it was always stopped down to an aperture of twelve inches, or a ratio of 1 to 7.8, which makes the effect of curvature of the field very much less. Even with this aperture it is necessary to make a correction for magnitude, depend-

[1] *Astrophysical Journal*, **27**, 169, 1908.

ing upon the distance of the star's image from the optical axis, as described later.

The plates.—All plates were taken in the primary focus of the instrument. For the photographic magnitudes Seed 27 plates were employed; for the photo-visual, Cramer Trichromatic and Wallace "Pan-Iso"[1] with a special color-filter constructed for this work by Mr. R. J. Wallace.[2] The Trichromatic and Pan-Iso plates with the color-filter have the same effect on colored stars and can be used interchangeably. The spectral luminosity-curves of the two, though somewhat different in shape, and also in the position of the maxima, give practically the same integrated effect, as can be seen in Figs. 1 and 2 (reprinted from this *Journal*, **27**, 171, 173, 1908). The actual working out of the results with the filter and Trichromatic plates is shown (Plate XI, *Astrophysical Journal*, **27** [opposite p. 170], 1908) in the reproduction of the photographs of the region of the intensely red star U Cygni, when its visual magnitude, as far as could be judged by the eye, was practically the same as that of its white companion. On the Seed 27 plate the difference of magnitude, or color-index, is 5.6, while on the Trichromatic plate the two images are equal. This is perhaps as severe a test as could be applied to the red-sensitive plate and filter, but the combination fully stands the test.

It is well known that Müller and Kempf are probably as accurate in their color-estimates as any other observers; therefore a comparison of their results with those obtained by the use of the visual-luminosity filter and color-sensitive plates will furnish further evidence as to the validity of the photographic method of determining star-colors. In Table IV of the paper "The Photographic Determination of Star-Colors and Their Relation to Spectral Type" will be found the comparison.[3] While in most of the individual visual groups there is a considerable range in the photographic color, they agree in general. In the comparison of spectral type and photographic color, the probable error of the mean color-index is but ±0.05 magnitude, part of this being due to determinations of color, part to errors in estimating spectral type. This shows that

[1] *Astrophysical Journal*, **26**, 299, 1907.

[2] *Ibid.*, **24**, 268, 1906.

[3] *Ibid.*, **27**, 169, 1908.

the visual-luminosity filter with properly sensitized plates is true to its name and really gives visual magnitudes.

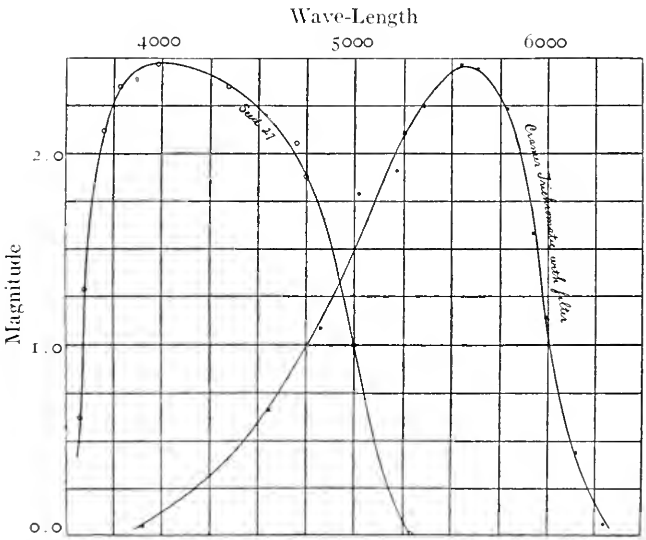


FIG. 1.—Spectral intensity-curves. Seed, and Trichromatic with filter

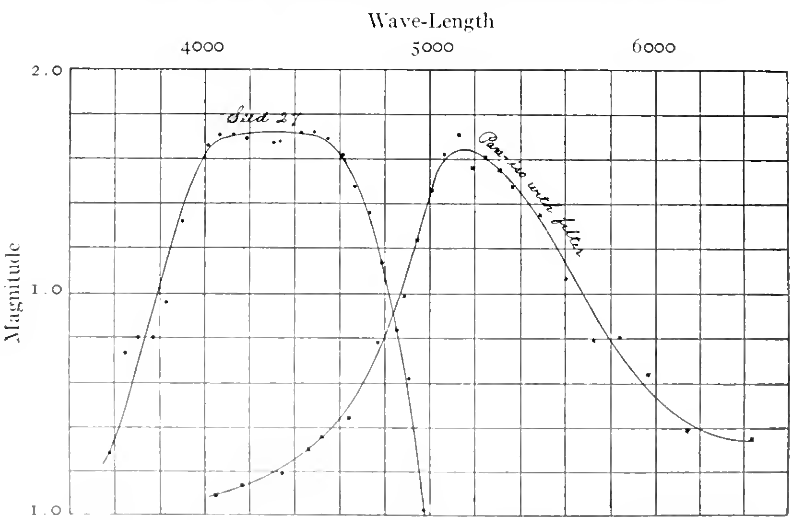


FIG. 2.—Spectral intensity-curves. Seed, and Pan-Iso with filter

Thus in the determination of color the eye is entirely eliminated as far as actual work on the stars is concerned, and in its place is

substituted the measuring engine and star images, in the measurement of whose diameters the only possible source of error is that of setting the measuring wires on their more or less diffuse edges. Since for the results here given all the measuring was done by the writer, and since the images of stars of all colors are exactly similar, they would all be affected alike. Hence this would introduce no error in the derived magnitudes. The errors remaining, therefore, are the accidental ones of the plate itself, which cannot be eliminated, but which can be reduced by taking a sufficient number of plates.

THE MAGNITUDE-FORMULA

I have hitherto tacitly assumed that we have a satisfactory formula for translating star diameters into stellar magnitudes. Before going further it will be necessary to prove that this is the case. Various formulae have been suggested and used, all of them empirical, though some of them employ in one form or another the light-ratio for one magnitude, a number whose logarithm is 0.4. However, the action of light on the sensitive film and the cause of the growth of a star image with increase of exposure are so imperfectly known that it will suffice to select that formula which most nearly satisfies the results obtained with the particular instrument and plates with which the observations are made.

In the earlier work of photographic photometry with the two-foot reflector, Charlier's well-known formula, $m=a-b \log D$, was used,[1] but it was found on further investigation that this did not exactly suit, and the formula $m=a-b\sqrt{D}$ was substituted. A graphical representation of the two formulae is given by Mr. J. A. Parkhurst in the "Yerkes Actinometry,"[2] and is here reproduced (Fig. 3) by permission of Mr. Parkhurst. Although it was drawn from data furnished by the six-inch Zeiss doublet of the Yerkes Observatory the results are of the same character for the two-foot reflector.

APPLICABILITY OF THE FORMULA

I now proceed to show that the formula satisfies the observations. Suppose a group of white stars be photographed. Then plot the stars with magnitudes as abscissae and square roots of

[1] *Astrophysical Journal*, **23**, 79, 1906. [2] *Ibid.*, **36**, 185, 1912.

diameters of the images as ordinates. If these plotted points lie on a straight line the square-root formula applies. The Pleiades is a group eminently suited for this because of the spectral type of its stars and the careful determinations which have been made of their magnitudes. In pursuance of this plan a number of photographs of the group were taken with the two-foot reflector diaphragmed

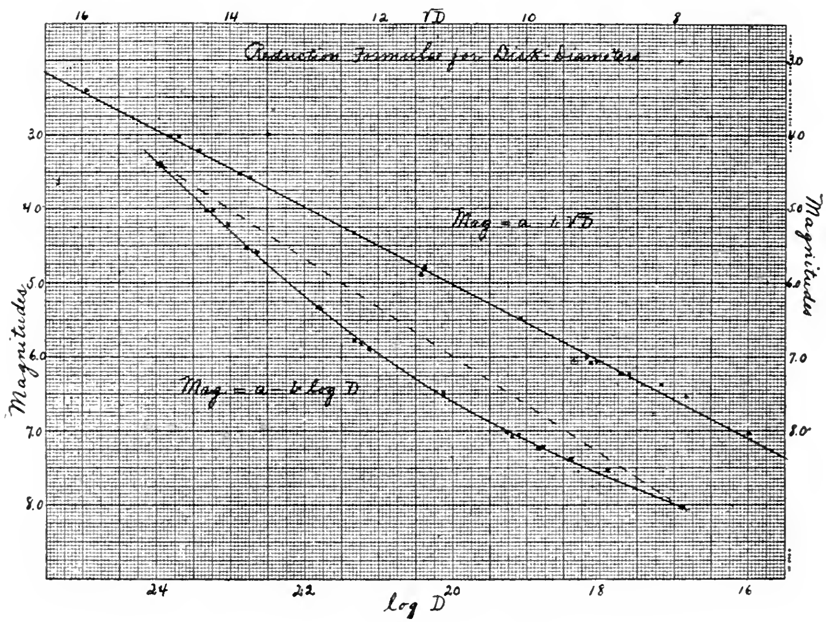


FIG. 3.—Reduction formulae for focal plates (by J. A. Parkhurst)

to twelve inches. The plates used were Seed 27, Cramer Trichromatic, and Wallace Pan-Iso. The diameters of the images were measured on a Gaertner measuring engine to 0.001 mm. The magnitudes of the stars were determined by Mr. Parkhurst by the extra-focal method, and are certainly as accurate as, or even a little better than, other determinations of magnitude in this much-studied group. The basis of the extra-focal determinations is given in "An Absolute Scale of Photographic Magnitudes of Stars."[1]

[1] *Astrophysical Journal*, 26, 244, 1907.

To illustrate, I have selected one each of the different kinds of plates for the construction of the diagrams Figs. 4, 5, and 6, and give the data for them in Table I.

TABLE I

DATA FOR CONSTRUCTION OF MAGNITUDE-CURVES

Star	Mag.	Plate 532, Seed 27			Plate 613, Trich.			Plate 1501, Pan-Iso		
		Exp.	*D*	$\sqrt{D}$	Exp.	*D*	$\sqrt{D}$	Exp.	*D*	$\sqrt{D}$
		Sec.			Min.			Min.		
		...	...	...	1	204.5	14.30	1	202.0	14.21
η......	3.32	...	...	...	2	229.5	15.15	2	228.5	15.12
		...	...	...	4	264.0	16.25	4	265.5	16.29
		8	183.0	13.53	1	171.5	13.10	1	172.0	13.11
b...	4.03	15	200.0	14.14	2	196.5	14.02	2	197.0	14.04
		...	...	...	4	224.0	14.97	4	226.0	15.03
		8	173.5	13.17	1	163.0	12.77	1	170.5	13.06
c...	4.20	15	194.5	13.95	2	188.5	13.73	2	191.5	13.84
		30	212.5	14.58	4	215.0	14.66	5	221.5	14.88
		8	168.0	12.96	1	159.0	12.61	1	156.0	12.49
d...	4.52	15	189.0	13.75	2	180.0	13.42	2	179.0	13.40
		30	204.5	14.30	4	207.5	14.40	5	206.5	14.37
		8	168.5	12.98	1	153.0	12.37	1	159.0	12.61
e.....	4.57	15	189.0	13.75	2	181.0	13.45	2	183.5	13.55
		30	203.0	14.25	4	205.0	14.32	5	207.5	14.40
		8	134.0	11.58	1	117.5	10.84	1	125.5	11.20
g.....	5.77	15	150.5	12.27	2	142.5	11.94	2	150.0	12.25
		30	162.5	12.75	4	161.5	12.71	5	170.5	13.06
		8	104.0	10.20	1	83.5	9.14	1	91.5	9.57
24...	7.07	15	123.0	11.09	2	107.0	10.34	2	107.0	10.34
		30	135.0	11.62	4	124.5	11.16	5	131.0	11.45
		...	...	...	...	...	...	1	82.5	9.08
19.	7.21	...	...	...	...	...	...	2	101.0	10.05
		...	...	...	...	...	...	5	122.5	11.07
		8	92.5	9.62	...	...	...	1	76.5	8.75
10	7.54	15	107.0	10.34	...	...	...	2	91.0	9.54
		30	116.0	10.77	4	112.0	10.59	5	113.5	10.65

The positions of points in Figs. 4, 5, and 6 are affected by accidental errors in the plates, errors in the assumed magnitudes, and in the measured diameters. Within the consequent allowable limits of errors the points in every case lie on a straight line, the magnitude-curve of the plate. This shows that the formula is applicable for

white stars without any reference to the values of a and b, since it depends solely upon the measured diameters of the star images. The value of a varies from plate to plate and depends upon the effective exposure. In the work of this paper the value of b has been determined once for all for each kind of plate, and the fact that in each diagram the curves for the different exposures are practically parallel shows that this value does not depend upon the time of exposure. I have tested this matter for the three kinds of plates with a range of exposure-time from one to forty on any one plate,

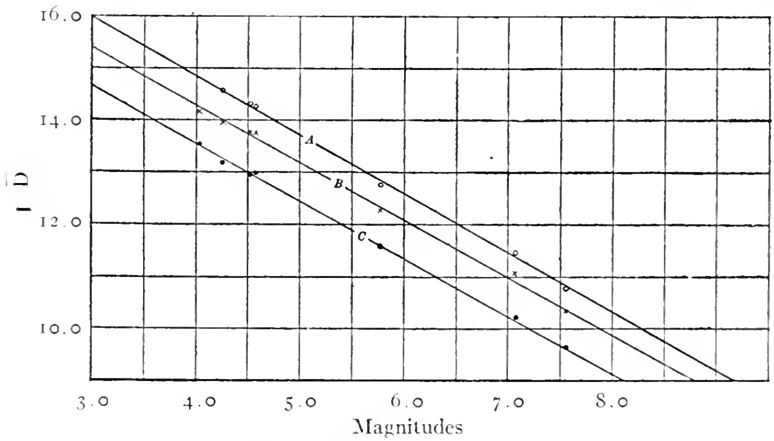


FIG. 4.—Magnitude-curve for Seed 27 plate
A, 30-second exposure
B, 15-second exposure
C, 8-second exposure

and find that even this extreme difference does not necessitate any change in the value of b. It probably varies somewhat from plate to plate because of changes in the seeing, but unless a standard field be photographed on the plate, in addition to the field containing the stars whose magnitudes are to be obtained, this change cannot be determined. This was not done for the plates whose results are to be given later; hence the value of b is considered as fixed.

METHODS OF DETERMINING THE VALUE OF b

From known magnitudes of certain Pleiades stars.—The data for the first method are given in the table and the lines drawn in the

diagrams through the plotted points. Each line furnishes two equations of the form $m=a-b\sqrt{D}$. For example, in Fig. 4, for plate 532, Seed 27, the line representing the exposure of eight seconds intersects the line for magnitude 3.0 at ordinate 14.66, and for 8.0 magnitude at ordinate 9.11; hence the two equations are:

$$3.0=a-b\ (14.66)$$
$$8.0=a-b\ (9.11)$$

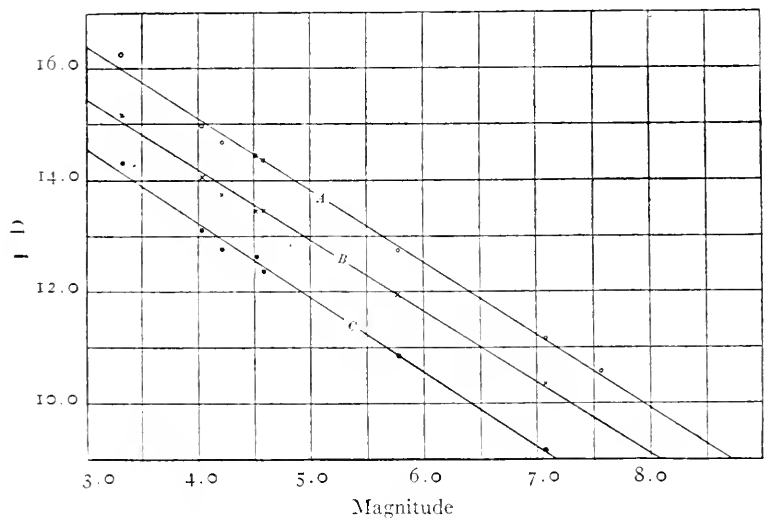


FIG. 5.—Magnitude-curve for Trichromatic plate
A, 4-minute exposure
B, 2-minute exposure
C, 1-minute exposure

The solution of these gives $b=0.901$. Similarly for the exposures of fifteen seconds and of thirty seconds the values of b are respectively 0.904 and 0.882. A number of other plates were reduced in the same way, and from the results 0.90 was adopted as the value of b for the Seed 27 plates. The probable error of this determination is ± 0.01.

The same method applied to the Trichromatic and Pan-Iso plates yields, for the former, the values 0.755, 0.790, and 0.777; and for the latter, 0.789, 0.773, and 0.794. From these and other

plates the value of b adopted for both the Trichromatic and Pan-Iso plates is 0.77 ± 0.01.

A least-squares solution is applied to the observations in the following manner. Each measured diameter of an image together with the magnitude of the star gives an observation equation. From these are formed normal equations in the usual way. Below is given in detail the solution for the exposure of plate 532 of eight seconds.

$$\begin{aligned}
a-b(13.53)&=4.03\\
a-b(13.17)&=4.20\\
a-b(12.96)&=4.52\\
a-b(12.98)&=4.57\\
a-b(11.58)&=5.77\\
a-b(10.20)&=7.07\\
a-b(\ 9.62)&=7.54
\end{aligned}$$

Normal Equations

$$\begin{aligned}
7a-84.04b&=+\ 37.70\\
-84.04a+1023.63b&=-439.20\\
b&=0.914
\end{aligned}$$

The exposures of fifteen and of thirty seconds give the values respectively 0.881 and 0.886. Observation equations for the Trichromatic plate 613 give the values 0.743, 0.802, and 0.764 for the respective exposures. Pan-Iso plate 1501 yields in the same manner the values 0.789, 0.781, and 0.787.

The grating method.—A second and entirely independent way of determining the value of b is offered by the use of the so-called *Halb-Gitter*. In his "Plan of Selected Areas" Kapteyn suggested the use of an absorbing plate over half of the field of the camera, by which the magnitudes of the corresponding stars could be diminished by a known amount, and hence compared with the same images obtained without the absorbing medium. Schwarzschild used in place of the absorbing plate a "*gitter*" of fine wire, as described in *Astronomische Nachrichten* (**183**, 297, 1910).

In pursuance of this plan a grating was used in connection with the reflector. The data for the grating are as follows: Mean mesh, center to center, 0.125 mm; diameter of wire (b) 0.0433 mm; free mesh (a) 0.0817 mm. Placed a short distance (about 75 mm) in front of the sensitive plate, this forms a central image surrounded by four sets of diffraction images arranged at intervals of 90°. The central image is exactly similar in appearance to the ordinary star image, and can be measured with equal facility.

The reduction in magnitude with this grating is 1.878 magnitudes as determined by the photometer. Theoretically the fraction of the incident light thrown into the central image is given by the formula $\left(\frac{a}{a+b}\right)^4$. This gives a reduction in magnitude of 1.628. One advantage of this method is the fact that we need know nothing

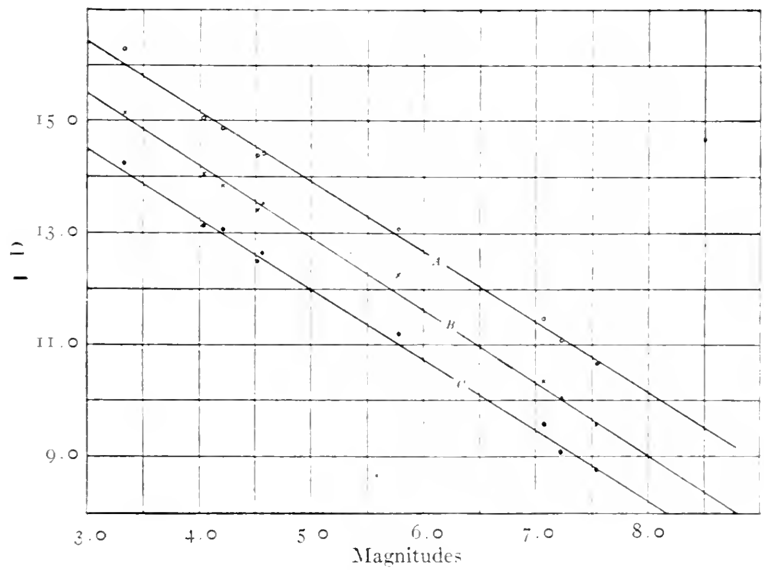


FIG. 6.—Magnitude-curve for Pan-Iso plate

A, 4-minute exposure
B, 2-minute exposure
C, 1-minute exposure

about the magnitudes of the stars used, either absolute or relative. We are concerned solely with the square roots of the measured diameters.

EXPOSURES WITH THE GRATING

Two methods of exposure were used: (1) A plate is taken of a region, say the Pleiades, without the grating; then on the same or a different plate another exposure is made with half the plate covered by the grating. The effective exposures need not be the same, and if different plates are used it is not necessary that they be obtained at the same time or even under the same conditions. (2) An

exposure is made with half the plate covered by the grating. The grating is then reversed so as to cover the other half of the plate and another exposure made.

In the first method, suppose that one plate is used: we then have two groups of stars, one of them normal on both exposures, the other normal on one and through the grating on the other. Let d^1, d^2, d^3, and d^4 be the mean square roots of the measured diameters in the respective cases. Let $d^2-d^1=\Delta^1$, $d^4-d^3=\Delta^2$, then $\Delta^1-\Delta^2$ is the change produced by the grating in the square-root factor; therefore $\frac{1.878}{\Delta^1-\Delta^2}$ is the value of b.

Using the same notation in the second method, $\Delta^1+\Delta^2$ is double the absorption effect of the grating. Table II gives the details of the measurement of two plates.

Other plates taken with the grating lead to an average value for b of about 0.90. This constant has also been determined in both ways by Dr. C. H. Gingrich, who finds the values 0.90 and 0.91 from given magnitudes and from exposures with grating, respectively.[1] In the work which follows I have therefore assumed 0.90 as the definitive value of b for Seed 27 plates. Trichromatic plates and filter were also tested with the grating, giving results such that 0.77 is adopted as the definitive value of b for these plates.

APPLICATION TO SHORT-PERIOD VARIABLE STARS

The magnitude-formula having been established and the values of b in this formula having been found for the various kinds of plates, it remains now to give the methods of work and results for individual stars. All those selected for investigation are stars of the type of variation of δ Cephei, or at least of a similar spectral type. The instrument and plates used were as described in the earlier part of this paper.

Each complete observation consists of two exposures, one on a Seed 27 plate, the other on a Trichromatic or Pan-Iso plate with visual-luminosity filter, which was placed in contact with the sensitive film of the plate. Usually the Seed 27 plate was taken first, then the other with an interval of only about a minute necessary

[1] *Astrophysical Journal*, 36, 171, 1912.

for the changing of plate-holders. The Trichromatic plates with filter required nine times the exposure of the Seed 27 plates to give

TABLE II

GRATING MEASUREMENTS

FIRST METHOD (Pleiades Plate 2511, Seed 27)

Stars	First Exposure $\sqrt{D}$ Normal	Second Exposure $\sqrt{D}$ Normal
e	15.83	16.93
g	14.27	15.73
k	13.89	15.28
1	13.32	14.19
4	12.24	13.46
72	10.42	11.09
101	10.39	11.77
Means,	$12.91=d^1$	$14.19=d^2$
	Normal	**Grating**
f	16.37	15.42
h	15.07	14.00
32	13.41	12.51
s	13.20	12.57
p	12.93	12.07
29	12.59	11.81
19	12.56	11.72
24	12.53	11.79
31	12.32	11.23
22	12.29	11.63
17	12.17	11.42
33	11.78	10.84
23	11.75	10.96
27	11.03	10.08
18	10.63	10.01
13	10.63	9.89
15	9.75	8.88
Means,	$12.41=d^3$	$11.58=d^4$

$d^2-d^1=+1.28=\Delta^1$
$d^4-d^3=-0.83=\Delta^2$

$\Delta^1-\Delta^2=+2.11$

$1.878\div 2.11=0.89=b$

SECOND METHOD (Pleiades Plate 2520, Seed 27)

Stars	First Exposure $\sqrt{D}$ Normal	Second Exposure $\sqrt{D}$ Grating
31	14.42	11.98
32	15.43	12.93
s	15.30	13.00
p	14.95	12.68
19	14.76	12.40
29	14.66	12.29
24	14.63	12.16
22	14.44	12.08
31	14.42	11.98
17	14.32	11.98
23	14.03	11.37
33	13.82	11.47
18	12.92	10.42
27	12.89	10.46
13	12.72	10.28
Means,	$14.23=d^1$	$11.82=d^2$
	Grating	**Normal**
4	12.25	13.81
7	11.39	13.14
1	11.15	12.94
51	11.11	12.84
101	10.49	12.24
72	10.37	12.45
Means,	$11.13=d^3$	$12.90=d^4$

$d^1-d^2=\Delta^1=2.41$
$d^4-d^3=\Delta^2=1.77$

$$\frac{\Delta^1+\Delta^2}{2}=2.09$$

$1.878\div 2.09=0.90=b$

images of about the same size; with the Pan-Iso plates used in the later observations, this ratio was only five to one, thus affecting a

considerable saving of time at the telescope. The maximum exposure for any field with the Seed 27 plates was three minutes and increased correspondingly for the other plates. All were developed with hydroquinone for just ten minutes. It is not necessary that the same care should be taken to have a constant temperature of developer as in the case of extra-focal plates, but the temperature was usually between 15° and 20° C.

Position of the stars on the plate.—The variable and comparison stars were located as symmetrically as possible with reference to the optical center of the plate, and a suitable star was selected for guiding. The double-slide plate-holder is furnished with three scales: right ascension, declination, and guiding eyepiece. The latter can be moved through a range of about two inches, the others considerably less. Care was taken in each exposure on a given field to set all these scales at the readings noted in the first exposure; hence the stars have always the same position on the plate, and consequently the matter of magnitude-corrections for distance from the center is much simplified.

I am indebted to Mr. Parkhurst for a table of corrections, the necessary portion of which is given in Table III with his explanation of its use.

TABLE III

REDUCTION TO THE CENTER FOR REFLECTOR PLATES (APERTURE, TWELVE INCHES)

Distance from Center	Correction		Distance from Center	Correction	
	Seed 27	Trich.		Seed 27	Trich.
7′	−0.03	−0.03	19′	−0.24	−0.18
8	− .04	− .04	20	− .26	− .20
9	− .06	− .05	21	− .28	− .22
10	− .07	− .06	22	− .30	− .24
11	− .09	− .07	23	− .32	− .28
12	− .10	− .08	24	− .35	− .30
13	− .11	− .09	25	− .37	− .32
14	− .13	− .10	26	− .40	− .34
15	− .15	− .12	27	− .43	− .37
16	− .17	− .14	28	− .46	− .40
17	− .19	− .16	29	− .49	− .42
18	−0.21	−0.17	30	−0.53	−0.45

"The size of the image and the corrections are expressed in terms of the square root of *D*, in thousandths of a millimeter. The corrections are proportional to the values of the square root of *D*. Therefore, since the reductions are tabulated for a value of 10.0, to find the correction for an image of any size multiply the tabular correction by one-tenth of the square root of *D*."

The corrections are a function both of the distance from the center and the square root of the measured diameters of the images. The distance from the center is always the same because of the arrangement of the field on the plate, but the size of the image of any one star varies somewhat from plate to plate because of variations in the effective exposures. However, it seemed sufficiently accurate to make the correction once for all for the comparison stars by taking the mean square roots of the diameters from all the plates. The corrections applied were made, not to the center, but to the star which was used on each plate as the standard of magnitude. As the stars are so arranged on the plate that differences are in general only a few minutes of arc, the corrections are but a few hundredths of a magnitude. One extreme case requires a change of 0.26 magnitude, while the variation in square roots of diameters of the images is 2.1. Therefore the extreme error caused by using the mean is only 0.027 magnitude, a quantity which is completely masked by the accidental errors of the plate itself.

DERIVATION OF THE MAGNITUDES OF THE COMPARISON STARS

One star of the first-type spectrum was selected in each field, its magnitude assumed, and then from it were derived the magnitudes of the comparison stars on all plates of that field. These varied considerably from plate to plate because of accidental errors, but the means of all were used in obtaining the magnitude of the variable. Using the mean from each plate would be equivalent to rejecting all comparison stars but the one selected as a standard.

The method of using the comparison stars in deriving the magnitude of the variable may be best explained by giving the details for a single plate. All plates have been reduced by this method.

X CYGNI, PLATE 1311, SEED 27

Stars	D	$\sqrt{D}$	$b\sqrt{D}$
X.	152.0	12.33	11.10
1.	162.5	12.75	11.48
2.	127.5	11.29	10.16
3.	108.5	10.42	9.38
4.	127.5	11.29	10.16

Mean $b\sqrt{D}$ of comparison stars $=10.30$.
Mean of magnitudes from all plates $=8.12$.
Δ Mag. (X$-$comparison stars) $=+0.80$.
$8.12-0.80-0.01$ (correction) $=7.31=$ magnitude of X.

ABSOLUTE AND RELATIVE VALUES

Since the principal object of this research is the determination of the difference between the visual and photographic magnitudes of the stars at various phases of the light-curves, no special effort has been made to have the magnitudes conform to those of any light-curves which may have been derived by other observers. From the fact that the magnitude-formula and the values of *b* have been definitively determined, the range of magnitude will also be correct. The assumption of a different magnitude for the standard star would have no effect upon the shape or relative position of the two curves, visual and photographic. Furthermore, the limited number of observations of each star does not justify a statement that the derived curves are definitive either as to accurate shape or the exact phase of maximum light.

Some of these Cepheid variables have been investigated spectroscopically,[1] and it has been shown that they have all the characteristics of spectroscopic binaries whose orbital period is the same as that of the variation in light, and whose maximum and minimum light correspond in time nearly to maximum velocity of approach and recession, respectively. The same is undoubtedly true of all other variables of the same class. Perhaps the best suggestion of the cause as well as the working out of the details of this relation of light and orbital velocity has been given by Dr. F. H. Loud.[2]

An alternative theory has been advanced, however, which seeks to explain their real variation in light and apparent variation in radial velocity as due to the pulsations of a single body.[3] This theory has many points in its favor, but it is not the purpose here to discuss the relative merits of the two hypotheses.

The tables.—In the tables of observations for individual stars, in the second column, *S* indicates Seed 27, *T*, Trichromatic, *P*, Pan-Iso plates. All other columns are self-explanatory. The residuals in the sixth and eighth columns are to be added algebraically to the observed magnitudes to produce the magnitudes of the smooth curve.

X CYGNI

This star, B.D. $+35°4234$ ($\alpha = 20^h 30^m$, $\delta = +35° 13'.6$), was discovered to be a variable star by Chandler in 1886.[4] From observa-

[1] *Lick Observatory Bulletin*, 4, 130, 1907.

[2] *Astrophysical Journal*, 26, 369, 1907.

[3] *Ibid.*, 40, 448, 1914.

[4] *Astronomical Journal*, 7, 32, 1886.

tions made at the Yerkes Observatory, it has been found to have a variable radial velocity, with a range of at least 50 km.[1]

In deriving the phases the following elements are used:

$$\text{J.D. } 2410190.684 + 16^{d}.38543\ E$$
$$= 1886,\ \text{Oct. } 10,\ 16^{h}25^{m} + 16^{d}9^{h}15^{m}1^{s}.2\ E \text{ (Luizet).}^{2}$$

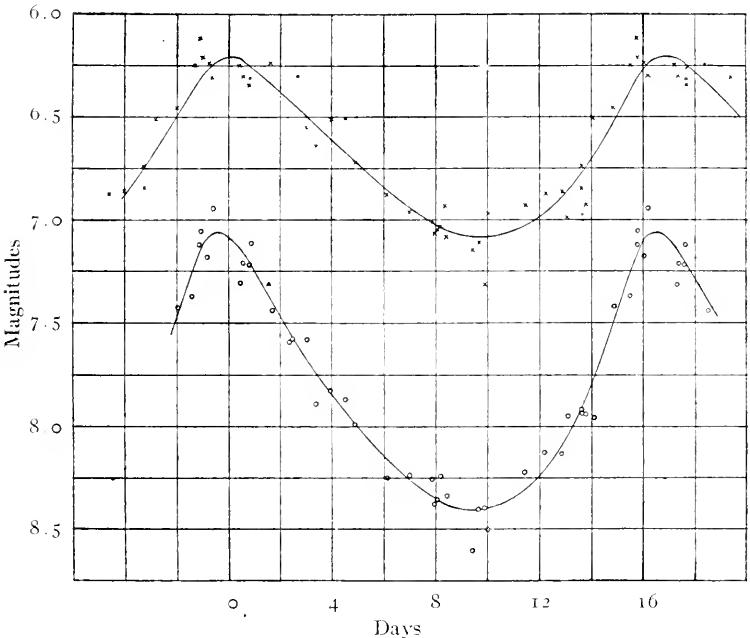


FIG. 7.—Light-curves of X Cygni

Data for the variable and comparison stars are:

STARS	B.D.	B.D. MAG.	ADOPTED MAGNITUDES		DISTANCE FROM CENTER
			Photographic	Photo-visual	
X.	+35° 4234	Var.			11′
1.	34 4127	7.5	6.96	6.96	11
2.	35 4219	7.5	8.20	7.56	12
3.	35 4221	8.3	9.12	8.51	14
4.	35 4232	8.5	8.20	8.12	10

From the smooth curves drawn through the individual observations, it can easily be seen (Fig. 7) that the maximum is less

[1] *Astrophysical Journal*, **25**, 60, 1907.

[2] *Bulletin de la Société Belge d'Astronomie*, No. 12, 1913.

TABLE IV

X Cygni: Observations

No. of Plate	Kind of Plate	Date, G.M.T.	Phase	Mags. and Residuals				Color-Index
				Photographic		Photo-visual		
				Mag.	Res.	Mag.	Res.	
		1906	Days					
714	S	Oct. 27d 15h 00m	13.04	7.91	+0.08			
715	T	27 15 13	13.05			6.98	−0.13	0.93
718	S	28 15 18	14.05	7.96	−0.20			
719	T	28 15 26	14.06			6.82	+0.17	1.14
726	S	30 14 38	16.02	7.17	−0.07			
727	T	30 14 40	16.03			6.23	−0.03	0.94
747	S	31 13 20	0.59	7.21	−0.05			
748	T	31 13 43	0.60			6.30	−0.06	0.91
775	S	Nov. 1 13 30	1.60	7.44	−0.03			
776	T	1 13 50	1.61			6.22	+0.09	1.22
796	S	9 14 13	9.62	8.40	0.00			
797	T	9 14 35	9.64			7.11	−0.03	1.29
809	S	13 14 36	13.64	7.92	0.00			
810	T	13 14 47	13.65			6.74	+0.03	1.18
818	S	22 12 40	6.17	8.24	−0.09			
819	T	22 12 51	6.18			6.88	−0.03	1.36
845	S	23 14 17	7.24	8.23	+0.01			
846	T	23 14 28	7.25			6.96	−0.02	1.27
857	S	24 13 28	8.20	8.26	+0.12			
858	T	24 13 30	8.21			7.03	0.00	1.23
868	S	28 13 20	12.20	8.12	+0.09			
869	T	28 13 37	12.21			6.87	+0.08	1.25
874	S	Dec. 3 13 32	0.82	7.22	0.00			
875	T	3 13 43	0.83			6.33	−0.09	0.89
898	S	18 12 22	15.77	7.13	+0.04			
899	T	18 12 35	15.78			6.11	+0.10	1.02
		1907						
952	T	Jan. 8 11 37	3.07			6.59	+0.09	
953	S	8 11 54	3.08	7.82	+0.01			1.23
1049	S	May 8 21 17	7.99	8.40	−0.03			
1050	T	8 21 29	8.00			7.07	−0.05	1.33
1099	S	12 21 10	13.07	7.94	−0.01			
1100	T	12 21 21	13.08			6.84	−0.07	1.10

TABLE IV—*Continued*

No. of Plate	Kind of Plate	Date, G.M.T.	Phase	Mags. and Residuals				Color-Index
				Photographic		Photo-visual		
				Mag.	Res.	Mag.	Res.	
		1907	Days					
1111.....	S	May $17^d 19^h 40^m$	2.23	7.58	0.00			
1112.....	T	17 20 00	2.24			6.30	+0.15	1.28
1126.....	S	19 20 42	4.27	7.87	+0.06			
1127.....	T	19 20 53	4.28			6.52	+0.15	1.35
1176.....	S	31 19 07	16.20	6.94	+0.13			
1177.....	T	31 19 22	16.21			6.30	−0.06	0.64
1189.....	S	June 1 20 41	0.88	7.11	+0.10			
1190.....	T	1 20 52	0.89			6.31	−0.05	0.80
1209.....	S	5 20 24	4.87	7.98	0.00			
1210.....	T	5 20 35	4.88			6.72	−0.01	1.26
1226.....	S	8 20 05	7.85	8.26	+0.08			
1227.....	T	8 20 16	7.86			7.01	0.00	1.25
1236.....	S	10 20 15	9.86	8.39	+0.01			
1237.....	T	10 20 36	9.88			7.31	−0.23	1.08
1252.....	S	13 19 28	12.83	8.13	−0.04			
1253.....	T	13 19 40	12.84			6.85	+0.02	1.28
1262.....	S	14 18 19	13.78	7.94	−0.06			
1263.....	T	14 18 30	13.79			6.92	−0.17	1.02
1283.....	S	15 19 54	14.85	7.43	+0.05			
1284.....	T	15 20 05	14.85			6.45	+0.07	0.98
1298.....	S	16 18 43	15.80	7.05	+0.11			
1299.....	T	16 18 54	15.81			6.21	+0.10	0.84
1311.....	S	17 20 13	0.48	7.31	−0.17			
1312.....	T	17 20 24	0.48			6.25	−0.02	1.06
1325.....	S	19 18 38	2.41	7.59	−0.05			
1326.....	T	19 18 49	2.42			6.58	−0.16	1.01
1335.....	S	20 15 58	3.30	7.89	−0.13			
1346.....	T	20 16 09	3.31			6.64	−0.09	1.25
1356.....	S	25 18 49	8.42	8.33	+0.04			
1357.....	T	25 19 00	8.43			7.08	−0.04	1.25
1358.....	P	25 19.24	8.44			6.93	+0.11	1.40
1367.....	S	26 18 42	9.41	8.50	−0.20			
1368.....	P	26 18 53	9.42			7.14	−0.06	1.46

TABLE IV—*Continued*

No. of Plate	Kind of Plate	Date, G.M.T.	Phase	Mags. and Residuals				Color-Index
				Photographic		Photo-visual		
				Mag.	Res.	Mag.	Res.	
		1907	Days					
1387.....	S	June $28^d 19^h 34^m$	11.45	8.22	+0.08			
1388.....	T	28 19 46	11.46			6.92	+0.10	1.30
1401.....	S	July 2 20 21	15.48	7.37	−0.10			
1402.....	P	2 20 31	15.49			6.25	+0.12	1.12
1411.....	S	6 18 48	3.03	7.58	+0.09			
1412.....	P	6 18 58	3.04			6.55	−0.04	1.03
1425.....	S	11 19 08	8.04	8.35	0.00			
1426.....	P	11 18 18	8.05			7.04	−0.02	1.31
1433.....	S	13 18 19	10.01	8.50	−0.10			
1434.....	P	13 18 29	10.02			6.97	+0.11	1.53

accurately determined than the minimum. As the curves are drawn, the photo-visual maximum and minimum are respectively 6.21 and 7.08, giving a range of 0.87 magnitude; the photographic are 7.06 and 8.40, or a range of 1.34 magnitudes. The color-index at maximum is 0.85, at minimum 1.32. The photographic range is therefore 1.54 times the photo-visual.

These results differ quite decidedly from those given by Wilkens in his "Photographische-Photometrische Untersuchungen," where he gives the visual range as 1.0 and the photographic as 1.80 magnitudes.[1] These, however, are the extremes from individual plates, and cannot be interpreted as meaning that the actual range of a smooth light-curve would be as much. Interpreted in the same way, my observations would give as the visual and photographic ranges 1.20 and 1.64 magnitudes, respectively.

S SAGITTAE

This variable, B.D. $+16°4067$ ($\alpha = 19^h 52^m$, $\delta = +16° 22'$), was discovered by Gore in 1885.[2] It was found by R. H. Curtiss, from the measure of plates obtained at the Lick Observatory, to have a

[1] *Astronomische Nachrichten*, **172**, 305, 1906.

[2] *Monthly Notices*, **46**, 106, 1886.

variable radial velocity with a range of at least 36 km.[1] Phases are derived from the formula

$$\text{J.D. } 2409860.36+8^{d}.38209 \text{ E.} = 1886, \text{ Nov. } 14, 8^{h}\,38^{m}+8^{d}\,9^{h}\,10^{m}\,9^{s} \text{ E.}$$

Data for the comparison stars are as follows:

Stars	B.D.	B.D. Mag.	Adopted Magnitudes		Distance from Center
			Photographic	Photo-visual	
S..........	+16° 4067	Var.			15′
1..........	16 4081	5.8	5.67	5.67	17
2..........	16 4085	7.0	6.77	6.70	18
3..........	16 4073	8.4	8.11	8.29	11

The light-curve of this star is peculiar, inasmuch as all published ones show a slight depression where maximum light would be expected. This peculiarity is also made manifest by the results of

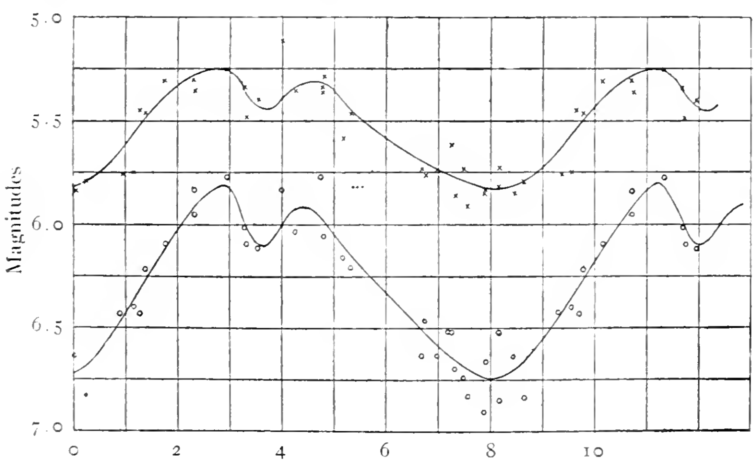


FIG. 8.—Light-curves of S Sagittae

the observations here given, though the curve as drawn at that phase could be considerably varied in shape and still satisfy the observed magnitudes equally well. The photo-visual curve has a range from 5.25 to 5.83 magnitudes; the photographic from 5.81

[1] *Lick Observatory Bulletins*, 3, 40, 1904.

TABLE V

S SAGITTAE: OBSERVATIONS

No. of Plate	Kind of Plate	Date, G.M.T.	Phase	Mags. and Residuals				Color-Index
				Photographic		Photo-visual		
				Mag.	Res.	Mag.	Res.	
		1906	Days					
710.....	S	Oct. $27^d 13^h 37^m$	6.74	6.47	+0.06			
711.....	T	27 13 50	6.75			5.73	−0.06	0.70
724.....	S	30 13 56	1.37	6.22	+0.05			
725.....	T	30 14 09	1.38			5.46	+0.02	0.76
743.....	S	31 12 13	2.30	5.83	+0.07			
744.....	T	31 12 31	2.31			5.31	−0.02	0.52
773.....	S	Nov. 1 13 11	3.34	6.09	−0.08			
774.....	T	1 13 20	3.35			5.48	−0.13	0.61
783.....	S	5 12 29	7.31	6.70	−0.05			
784.....	T	5 12 40	7.32			5.86	−0.08	0.84
792.....	S	9 12 48	2.94	5.77	+0.05			
793.....	T	9 12 55	2.95			5.25	0.00	0.52
805.....	S	13 13 13	6.96	6.64	−0.06			
806.....	T	13 13 22	6.97			5.73	0.00	0.91
816.....	S	22 12 10	7.54	6.83	−0.13			
817.....	T	22 12 18	7.54			5.86	−0.11	0.97
843.....	S	23 13 49	0.22	7.83	−0.15			
844.....	T	23 13 56	0.23			5.84	0.00	0.99
853.....	S	24 12 07	1.15	6.40	−0.03			
854.....	T	24 12 15	1.16			5.75	−0.20	0.65
864.....	S	28 12 09	5.15	6.16	−0.06			
865.....	T	28 12 17	5.16			5.28	−0.17	0.88
896.....	S	Dec. 18 12 35	0.02	6.63	+0.07			
897.....	T	18 12 44	0.03			5.84	−0.03	0.67
916.....	S	22 12 01	4.00	5.88	+0.16			
917.....	T	22 12 10	4.01			5.11	+0.28	0.77
		1907						
1047.....	S	May 8 20 40	7.25	6.52	+0.12			
1048.....	T	8 20 58	7.26			5.62	+0.15	0.90
1058.....	S	9 18 58	8.18	6.85	−0.12			
1059.....	T	9 19 10	8.19			5.82	+0.02	1.03
1082.....	S	10 21 11	0.89	6.44	+0.02			
1083.....	T	10 21 20	0.89			5.76	−0.13	0.68
1115.....	S	17 21 04	7.88	6.91	−0.17			
1116.....	T	17 21 13	7.89			5.85	−0.03	1.06

TABLE V—*Continued*

No. of Plate	Kind of Plate	Date, G.M.T.	Phase	Mags. and Residuals				Color-Index
				Photographic		Photo-visual		
				Mag.	Res.	Mag.	Res.	
		1907	Days					
1211.....	S	June $5^d 20^h 57^m$	1.73	6.09	+0.01			
1212.....	T	5 21 06	1.74			5.31	+0.08	0.78
1228.....	S	8 20 38	4.72	5.77	+0.20			
1229.....	T	8 20 47	4.72			5.37	−0.05	0.40
1234.....	S	10 19 46	6.68	6.63	−0.12			
1235.....	T	10 19 55	6.69			5.74	−0.05	0.89
1250.....	S	13 19 00	1.27	6.43	−0.13			
1251.....	T	13 19 10	1.27			5.45	+0.07	0.98
1268.....	S	14 20 00	2.31	5.95	−0.04			
1269.....	T	14 20 09	2.31			5.36	−0.07	0.59
1281.....	S	15 19 24	3.28	6.02	−0.03			
1282.....	T	15 19 33	3.29			5.29	+0.02	0.73
1296.....	S	16 18 19	4.24	6.04	−0.11			
1297.....	T	16 18 26	4.24			5.36	−0.03	0.68
1309.....	S	17 19 44	5.30	6.21	−0.06			
1310.....	T	17 19 53	5.30			5.47	−0.02	0.74
1319.....	S	19 16 55	7.18	6.52	+0.11			
1337.....	S	20 16 31	8.16	6.52	+0.22			
1338.....	T	20 16 40	8.17			5.73	+0.11	0.79
1348.....	S	25 16 27	4.78	6.06	−0.07			
1349.....	T	25 16 36	4.79			5.29	+0.03	0.77
1350.....	P	25 16 55	4.81			5.34	−0.02	0.72
1385.....	S	28 19 06	7.89	6.66	+0.08			
1386.....	P	28 19 14	7.90			5.83	−0.01	0.83
1399.....	S	July 2 19 36	3.53	6.12	−0.02			
1400.....	P	2 20 03	3.55			5.45	+0.04	0.67
1409.....	S	6 18 23	7.48	6.74	−0.06			
1410.....	P	6 18 30	7.48			5.73	+0.05	1.01

to 6.75, or a loss at minimum of respectively 0.58 and 0.94 magnitude. The color-indices at maximum and minimum phases are respectively 0.56 and 0.92. The ratio of photographic to photo-visual range is 1.62.

TT AQUILAE

This star, B.D. $+1^\circ 3899$ ($\alpha = 19^h 03^m$, $\delta = +1^\circ 09'$), was discovered to be a variable by Miss Annie J. Cannon from photographs taken at the Harvard College Observatory.[1] The statement is made that the spectrum seems also to be variable, classified as K at minimum and G at maximum. This variation in spectrum has since been found to be true in a number of Cepheid variables which have been observed spectroscopically, and is probably true in all variables of this type. Such a variation would be expected from the known change in color of these stars at the two epochs.

The elements used in determining the phases are those given by Ichinohe from early observations at the Yerkes Observatory, and from later ones made at Tokyo.[2] The formula is:

J.D. $2411873.865 + 13^d.753$ E, or, 1891, April 21, $20^h 46^m + 13^d 18^h 4^m 19^s.2$ E.

The data for the comparison stars follow:

Stars	B.D.	B.D. Mag.	Adopted Magnitudes		Distance from Center
			Photographic	Photo-visual	
TT	$+1^\circ 3899$	7.5	....	....	13′
1	1 3905	8.0	7.44	7.44	20
2	1 3896	9.0	8.76	8.72	15
3	1 3889	7.7	8.46	8.16	17
4	1 3898	8.1	7.75	7.90	18

The photo-visual range is from 6.70 to 7.53, or 0.83 magnitude; the photographic range is from 7.0 to 8.37, or 1.37 magnitudes. The latter is almost exactly the range given by Miss Cannon, 1.40 magnitudes, although the actual magnitudes differ by 0.6, the Harvard values being numerically the greater. The color-index at maximum is 0.30, at minimum, 0.84, a ratio of 2.8; greater than that of any of the other stars whose results are given here. These values are only approximate because of the comparatively few plates from which the curves are drawn.

[1] *Harvard College Observatory Circular*, No. 120.

[2] *Astronomische Nachrichten*, **187**, 299, 1911.

TABLE VI

TT AQUILAE: OBSERVATIONS

No. of Plate	Kind of Plate	Date, G.M.T.	Phase	Mags. and Residuals				Color-Index
				Photographic		Photo-visual		
				Mag.	Res.	Mag.	Res.	
		1907	Days					
1182.....	S	June $1^d 18^h 31^m$	9.88	8.04	+0.04			
1183.....	T	1 18 43	9.89			7.50	−0.07	0.54
1203.....	S	5 18 36	0.13	7.07	−0.03			
1204.....	T	5 18 47	0.14			6.76	−0.02	0.31
1220.....	S	8 18 29	3.13	7.56	+0.01			
1221.....	T	8 18 47	3.14			7.04	+0.04	0.52
1230.....	S	10 18 35	5.13	7.89	+0.11			
1231.....	T	10 18 51	5.14			7.23	+0.03	0.66
1246.....	S	13 17 47	8.10	8.30	+0.05			
1247.....	T	13 18 03	8.11			7.53	−0.02	0.77
1264.....	S	14 18 54	9.15	8.14	+0.07			
1265.....	T	14 19 10	9.16			7.55	−0.03	0.59
1277.....	S	15 18 16	10.12	7.89	+0.07			
1278.....	T	15 18 32	10.13			7.35	+0.05	0.54
1292.....	S	16 17 20	11.08	8.03	−0.25			
1293.....	T	16 17 31	11.09			7.30	−0.11	0.73
1305.....	S	17 18 40	12.14	7.52	+0.05			
1306.....	T	17 18 56	12.15			7.12	−0.13	0.30
1321.....	S	19 17 29	0.33	7.07	0.00			
1322.....	T	19 17 46	0.34			6.82	−0.05	0.25
1339.....	S	20 17 05	1.32	7.36	−0.12			
1340.....	T	20 17 22	1.33			6.90	−0.02	0.46
1351.....	S	25 17 19	6.33	8.33	−0.12			
1352.....	T	25 17 36	6.34			7.41	−0.02	0.92
1374.....	S	27 17 10	8.32	8.30	+0.05			
1375.....	P	27 17 27	8.33			7.54	−0.02	0.76
1381.....	S	28 18 04	9.36	8.22	−0.05			
1382.....	P	28 18 18	9.37			7.48	+0.02	0.74
1392.....	S	29 15 25	10.25	7.90	+0.07			
1393.....	P	29 15 35	10.25			7.23	+0.14	0.67
1396.....	S	July 2 17 27	13.33	6.97	+0.13			
1397.....	P	2 17 41	13.34			6.64	+0.08	0.33
1421.....	S	11 17 37	8.58	8.38	−0.07			
1422.....	P	11 17 50	8.59			7.48	+0.04	0.90
1439.....	S	13 17 40	10.59	7.88	+0.10			
1440.....	P	13 17 54	10.60			7.25	+0.05	0.63
1466.....	S	19 17 37	2.83	7.39	+0.12			
1467.....	P	19 17 58	2.85			7.10	−0.05	0.29

δ CEPHEI

This well-known variable, B.D. $+57°2548$ ($\alpha = 22^h\ 24^m$, $\delta = +57°\ 40'$), was discovered by Goodricke in 1784. In 1894 Belopolsky found it to be a spectrocsopic binary with the orbital period the same as the light-period.[1]

The latest elements of the light-variation as given in the *Vierteljahrsschrift* are:

Maximum, 1840, Sept. 26, $9^h57^m.8 + 5^d8^h47^m45^s.00\ E - 0^s.00075\ E^2 - 0^s.00000062\ E^3$.

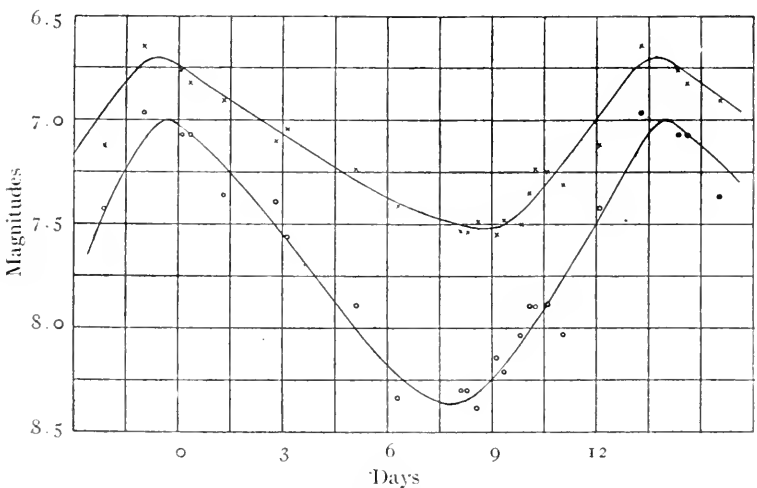


FIG. 9.—Light-curves of TT Aquilae

Because of the brightness of the star and the very limited field of the reflector, only one comparison star is available, the fainter, white component of the double, whose magnitude is here assumed to be 6.61.

Usually four exposures were made on the Seed 27 plates, ranging from ten to fifty seconds. The adopted magnitude for each plate is the mean of those from the different exposures. With the filter and the other two kinds of plates three exposures were made, ranging from one to five minutes.

[1] *Astronomische Nachrichten*, **136**, 281, 1894.

There is unfortunately a lack of exposures just at the maximum, making the curves somewhat uncertain at this phase, though the probable error in the location of the curve at this point can scarcely be as much as 0.05 magnitude.

The range obtained for the photo-visual curve as drawn is from 4.02 to 4.73, or 0.73 magnitude; that of the photographic curve

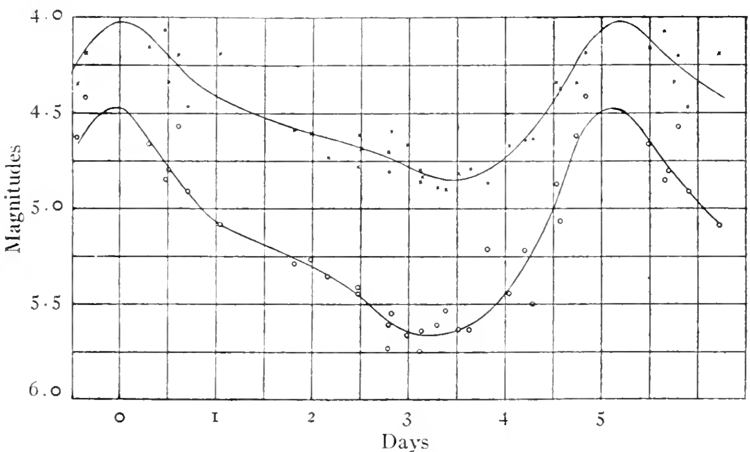


FIG. 10.—Light-curves of δ Cephei

from 4.47 to 5.67, or 1.20 magnitudes. The ratio of photographic to photo-visual range is 1.64. The color-indices at maximum and minimum are respectively 0.45 and 0.92.

SUMMARY

The object of this paper, the determination of stellar magnitudes, both photo-visual and photographic, by means of the photographic plate alone, and the application of this to short-period variables has necessitated the following investigations:

1. It has been shown that a properly sensitized photographic plate used with a suitable color-filter does actually give magnitudes of stars of all colors on a visual scale, and hence the expressions "photo-visual magnitudes" and "photo-visual light-curve" can be properly applied to such results.

TABLE VII

δ CEPHEI: OBSERVATIONS

No. of Plate	Kind of Plate	Date, G.M.T.	Phase	Photographic Mag.	Photographic Res.	Photo-visual Mag.	Photo-visual Res.	Color-Index
		1906	Days					
693.....	S	Oct. $21^d 16^h 34^m$	2.78	5.73	−0.14			
694.....	T	21 16 40	2.79			4.70	+0.03	1.03
730.....	S	30 15 56	1.02	5.08	0.00			
731.....	T	30 16 00	1.03			4.19	+0.22	0.89
751.....	S	31 14 50	1.98	5.23	+0.03			
752.....	T	31 14 56	1.98			4.61	0.00	0.66
779.....	S	Nov. 1 15 05	2.99	5.67	−0.02			
780.....	T	1 15 10	2.99			4.67	+0.11	1.00
788.....	S	8 13 56	4.58	5.07	−0.16			
789.....	T	8 14 03	4.58			4.37	+0.01	0.70
800.....	S	9 15 50	0.29	4.67	−0.02			
801.....	T	9 15 55	0.29			4.15	−0.05	0.52
813.....	S	13 15 51	4.29	5.50	−0.27			
814.....	T	13 15 58	4.29			4.63	−0.05	0.87
822.....	S	22 13 47	2.47	5.45	0.00			
823.....	T	22 14 00	2.48			4.78	−0.10	0.67
847.....	S	23 14 51	3.51	5.63	+0.01	4.82	+0.03	0.81
848.....	T	23 15 01	3.52					
872.....	S	28 14 46	3.14	5.64	+0.03			
873.....	T	28 14 54	3.15			4.83	−0.02	0.81
886.....	S	Dec. 17 13 25	0.62	4.57	+0.28			
887.....	T	17 13 34	0.63			4.20	+0.05	0.37
		1907						
938.....	T	Jan. 4 12 59	2.50			4.68	0.00	
1123.....	S	May 19 19 55	3.63	5.63	−0.03			
1124.....	T	19 20 00	3.64			4.79	+0.04	0.84
1186.....	S	June 1 19 56	0.53	4.80	−0.01			
1187.....	T	1 20 00	0.54			4.34	−0.13	0.46
1207.....	S	5 19 58	4.54	4.87	+0.08			
1208.....	T	5 20 05	4.54			4.34	+0.07	0.53
1224.....	S	8 19 46	2.16	5.36	0.00			
1225.....	T	8 19 52	2.17			4.73	−0.11	0.63

TABLE VII—*Continued*

No. of Plate	Kind of Plate	Date G.M.T.	Phase	Mags. and Residuals				Color-Index
				Photographic		Photo-visual		
				Mag.	Res.	Mag.	Res.	
		1907	Days					
1238.....	S	June 10d 20h 49m	4.20	5.22	+0.08			
1239.....	T	10 20 54	4.21			4.63	−0.01	0.59
1254.....	S	13 20 01	1.80	5.29	−0.03			
1255.....	T	13 20 06	1.81			4.64	−0.01	0.65
1270.....	S	14 20 27	2.82	5.55	+0.06			
1271.....	T	14 20 32	2.83			4.59	+0.15	0.96
1285.....	S	15 20 25	3.82	5.21	+0.33			
1286.....	T	15 20 32	3.82			4.86	−0.07	0.35
1301.....	S	16 20 43	4.83	4.41	+0.16			
1302.....	T	16 20 48	4.84			4.18	−0.03	0.23
1313.....	S	17 20 44	0.47	4.85	−0.08			
1314.....	T	17 20 49	0.47			4.06	+0.11	0.79
1331.....	S	19 20 44	2.47	5.42	+0.04			
1332.....	T	19 20 50	2.47			4.61	+0.06	0.81
1341.....	S	20 18 43	3.38	5.53	+0.12			
1342.....	T	20 18 48	3.39			4.90	−0.05	0.63
1359.....	S	25 19 44	3.06	5.75	−0.08			
1360.....	T	25 19 50	3.07			4.80	+0.01	0.95
1361.....	P	25 20 00	3.08			4.86	−0.05	0.89
1369.....	S	26 19 14	4.04	5.45	−0.02			
1370.....	P	26 19 20	4.04			4.67	+0.04	0.78
1387.....	S	28 20 03	0.71	4.91	−0.01			
1388.....	T	28 20 08	0.71			4.47	−0.16	0.44
1401.....	S	July 2 20 42	4.73	4.62	+0.05			
1402.....	P	2 20 52	4.74			4.34	−0.11	0.28
1411.....	S	6 19 16	3.31	5.61	+0.05			
1412.....	P	6 19 22	3.31			4.89	−0.05	0.72
1519.....	S	Aug. 12 20 24	2.79	5.61	−0.02			
1520.....	T	12 20 28	2.79			4.80	−0.07	0.81

This is shown by a comparison of these results with those of competent visual observers, by the curves of spectral intensity from the plates, and by an actual photograph of a region containing a strongly colored star.

2. It has been shown that the magnitude formula $m = a - b\sqrt{D}$ represents within the limits of error the results with the reflector and the three kinds of plates used; also that the value of b in the formula, or the slope of the magnitude-curve, does not change perceptibly with the varying length of exposure used.

3. The value of b has been determined in two entirely independent ways: first, from the photographs of the Pleiades reduced with known magnitudes derived from extra-focal images, and an "absolute scale" of magnitudes; second, from grating exposures

TABLE VIII

		X Cygni	S Sagittae	TT Aquilae	δ Cephei
α		$20^h 39^m$	$19^h 52^m$	$19^h 03^m$	$22^h 24^m$
δ		+35° 14′	+16° 22′	+1° 09′	+57° 40′
Galactic latitude		−5°	−7°	−4°	−1°
Spectrum		F8 to G	G	G to K	G
No. of Plates	Photographic	37	31	19	31
	Photo-visual	38	31	19	33
Magnitudes at Max.	Photographic	7.06	5.81	7.00	4.47
	Photo-visual	6.21	5.25	6.70	4.02
Magnitudes at Min.	Photographic	8.40	6.75	8.37	5.67
	Photo-visual	7.08	5.83	7.53	4.73
Color-Index at	Max.	0.85	0.56	0.30	0.45
	Min.	1.32	0.92	0.84	0.92
Ratio $\frac{\text{Color at Min.}}{\text{Color at Max.}}$		1.55	1.64	2.80	2.04

on the Pleiades, in which it is not necessary to assume any magnitudes of the stars used, since the reductions and the final value depend only upon the measured diameters of the images and the known magnitude-absorption of the grating.

The adopted values of b are: 0.90 for Seed 27; 0.77 for Trichromatic and Pan-Iso plates.

4. The selected fields, variable and comparison stars, have been arranged as symmetrically as possible with reference to the optical center of the plate, and care has been taken to have them

always in the same position on the plate. Corrections have then been applied to reduce their magnitudes to the center, or rather to one star as a basis. This fundamental star is of the first spectral type, and upon its assumed magnitude depends the position of the light-curves in the scale of magnitudes.

5. It has been shown that in each case the photographic range is greater than the photo-visual; or, in other words, that the star becomes redder as it becomes fainter, indicating some change in the spectrum.

The color of the star, or color-index, is expressed in magnitudes, and is a perfectly definite quantity, depending in no way upon the personal equation of the observer.

The results may be duplicated by any person using similar instruments and plates.

ACKNOWLEDGMENTS

My thanks are due to Director E. B. Frost, who placed at my disposal the two-foot reflector and all other instruments and supplies necessary for carrying out this investigation; to Mr. R. J. Wallace, whose skill and success in constructing a "visual-luminosity" filter made possible this method of carrying on the work; and especially to Professor J. A. Parkhurst, to whom is due the inception of the work, and by whose general oversight and helpful suggestions its completion was made possible.

ALLEGHENY OBSERVATORY
PITTSBURGH, PA.
February 1919

THE GREAT ERUPTIVE PROMINENCES OF MAY 29 AND JULY 15, 1919

By EDISON PETTIT

The spring and summer of the present year have been remarkable for the quantity and variety of solar phenomena; for the number and size of the spots and prominences. By far the most interesting, however, were the eruptive prominences of May 29 and July 15, notable for their dimensions and height of ascent. Weather conditions permitting, a quite complete history of both prominences was obtained with the Rumford spectroheliograph attached to the 40-inch telescope.

With this instrumental equipment a spectroheliogram of either the entire disk of the sun or of all the prominences on one-half its circumference may be photographed in 99 seconds in the H_3 line (calcium), on account of the great light-collecting power of the objective. The scale of the plates obtained is 179 mm (7 inches) for the sun's diameter. Thus it is possible to make successive exposures of the entire limb at intervals less than 10 minutes apart.[1] Early in the spring a new dark-room was fitted up on the first floor of the observatory for solar work, which greatly facilitated the developing of the plates as the exposures were made. During the eruption of a prominence it is quite essential that each photograph be developed as soon as taken so that the progress of the prominence above the limb of the sun may not carry it beyond the field of the instrument.

I had some time ago planned to take as many exposures as practicable during the progress of an eruption in order to determine if possible the law of vertical motion of the prominence. For this purpose it is necessary to take the plates not only frequently but at as nearly regular intervals as possible. This will make the plotted curve equally well determined in all parts and not throw doubt on any peculiarities it might present. This procedure seems

[1] With two observers operating the instrument, this time may be reduced to about 3 minutes.

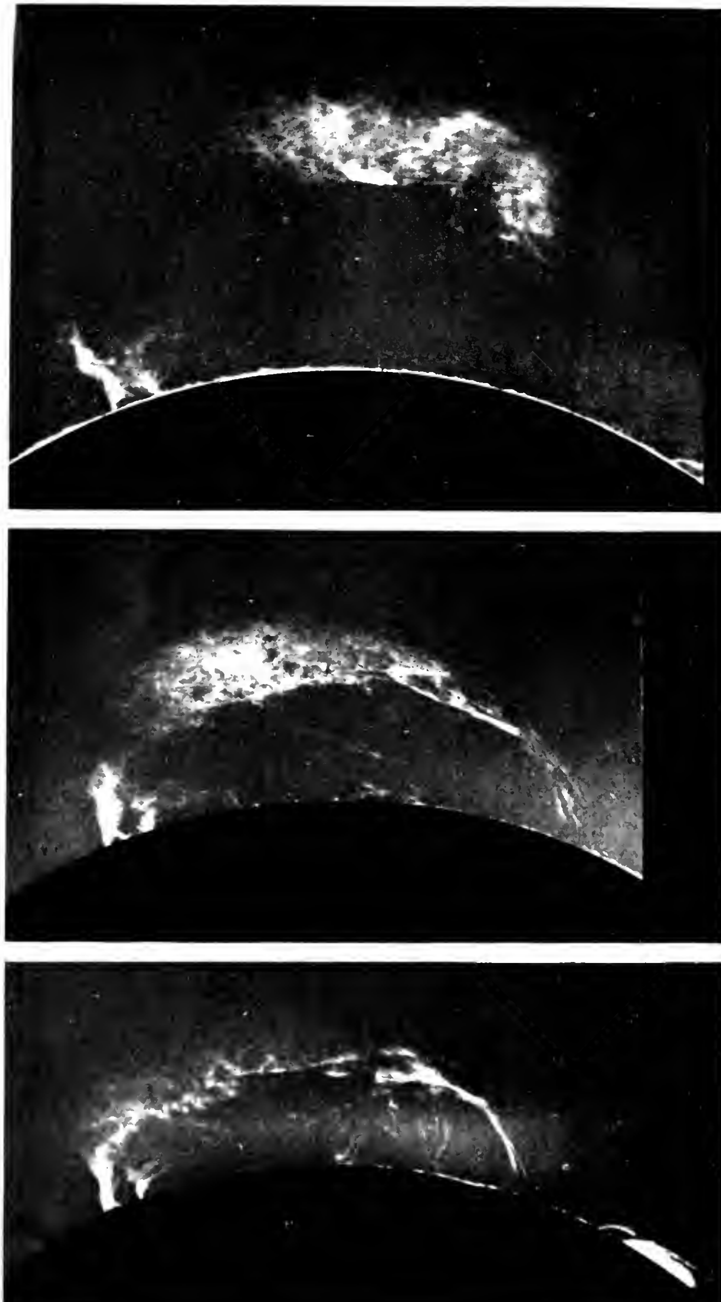
2669

PLATE IV

c
G.M.T.
$5^h 32^m 41^s$

b
$2^h 50^m 56^s$

a
$1^h 41^m 46^s$



THE GREAT PROMINENCE OF MAY 29, 1919
Scale: for *a*, 1 mm = 9320 km; for *b* and *c*, 1 mm = 8416 km

quite essential in cases of this kind. Unfortunately, prominences affording material for studies of this nature are very rare, and in what measure success was attained in discovering the law of vertical motion of eruptive prominences will be seen in what follows.

THE PROMINENCE OF MAY 29

This prominence seems to have first made its appearance March 22, on the east limb at latitude $-35°$, extending northward $13°$. The sky of the early spring was very hazy, hence these photographs are very weak in contrast. Each return of the prominence to the limb was observed (excepting the return of April 5, when no observing was had for a period of 6 days). It gradually grew in intensity and height and generally extended along the limb $10°$ or $15°$ northward.

On May 27 it appeared as the crest of a high prominence coming over the east limb, having an apparent height of about $1'.5$ and extending from $-44°$ to $-3°$ at a nearly uniform height. A heavy haze so obscured the prominences on this day that only one exposure shows it plainly. On the twenty-eighth the prominence appeared as an enormous body of interlacing streamers, having a height of $2'.7$ and maintaining the same position and extent on the limb, appearing to rise from two columns at $-37°$ and $-41°$, the whole body lying parallel to the limb. The sky had cleared considerably so that 16 exposures were made during the day. No general changes took place in the form or dimensions of the prominence on this day. The interlacing streamers of which the body was made, on the other hand, seemed to be continuously shifting. The height remained practically constant.

On the morning of the twenty-ninth a special effort was made to begin photography as early as possible on account of the prominence and also of its connection with the total solar eclipse near the equator on that day. The first exposure was taken at 1^h17^m Greenwich Mean Time (hour angle 5^h East) and showed that the entire form of the prominence had changed into a great arc $4'.5$ high and extending from $-42°$ to $+6°$. Plate IV*a* shows the appearance of the prominence near this stage of its development. It had broken away from the northern column and was

connected by long streamers with a spot at latitude +6.6°. The longitude of this spot determined from measures of later disk plates is 9°.6, which was also practically the longitude of the limb (determined from the ephemeris) at the time of the eruption of the prominence.

That the prominence was on the limb can be argued in a different manner than its connection with the spot. The two columns at the southern extremity from which the prominence arises show a rootlike or clawlike structure at the base. This structure is partially obscured by the occulting disk in Fig. *a* but shows better in Fig. *b*. Even there it was slightly overset, though on many of the plates where the sierra is well exposed this clawlike structure stands like a tripod on a table. Unpublished data obtained from a study of the Rumford plates made here shows that many types of prominences when coming over the limb through a period of several days exhibit, at the middle of the series, this clawlike structure at the base, whence it seems that this is further evidence that the prominence was on the limb during the eruption and oriented nearly in a solar meridian of longitude 10°.

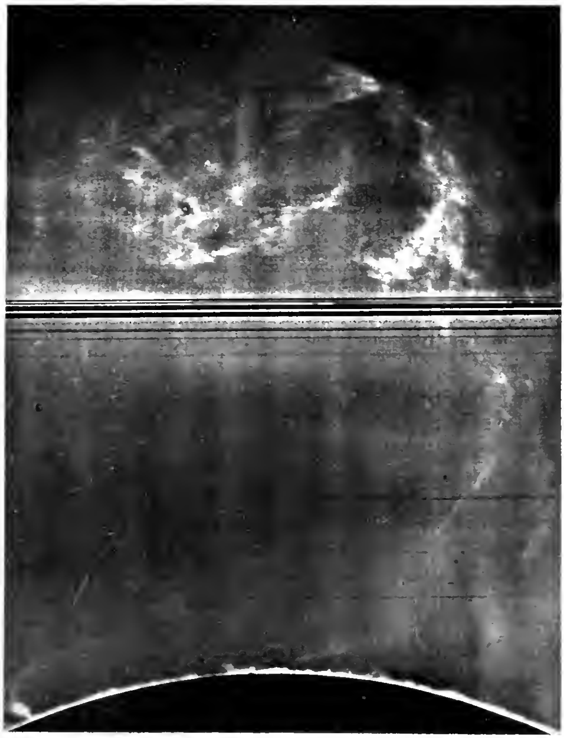
The possibility that the prominence might become eruptive being recognized, a campaign was inaugurated along the lines previously stated. One observer (Mrs. Pettit) developed all of the plates while another (myself) made all of the exposures.

At 2^h50^m G.M.T. the prominence began breaking away from the remaining (southern) stem and by 3^h10^m had entirely parted. Plate IV, Fig. *c*, shows it at this stage. At 6^h42^m it began to show a spiral structure, as if the whole body were twisted into a giant spring. This formation endured till the object disappeared. Plate V, Figs. *d* and *e*, show the prominence at this stage. The last plate taken shows it at greatest height, 760,000 km (17′) above the solar surface. Although it broke away from its stems at 3^h10^m, as indicated above, the prominence was always connected to the spot by streamers on every plate taken on this day. Even on the last plate reproduced (Plate V, Fig. *e*) they are seen descending in great sheets.

Toward afternoon an unfortunate occurrence made it difficult to use the spectroheliograph on the east side of the pier, so that the

PLATE V

c
G.M.T.
$7^h57^m22^s$



d
$7^h10^m31^s$

THE GREAT PROMINENCE OF MAY 29, 1910
Scale: 1 mm = 8410 km

telescope was not reversed till nearly 2 hours after noon. Near the close of the series we ran out of Seed 30 plates and were compelled to use Graflex plates. At 8^h30^m the prominence ceased to appear on the plates; the exposures, however, were carried on to 10^h30^m G.M.T. A thickening sky probably helped to obscure the prominence at this epoch. Fortunately the sky was quite clear throughout the day and only hazy toward late afternoon. Twenty-six plates show the prominence with a wealth of detail. It had risen from 200,000 km to 760,000 km in a period of 6^h40^m.

The following table gives the data of the exposures, together with the measurements made by means of a scale graduated on glass directly to units of 10,000 km.

TABLE I

MEASURES OF THE GREAT PROMINENCE OF MAY 29, 1919
(In Thousands of Kilometers)

Series No.	G.M.T.	Mean Height	Least Height	Greatest Height	Length	Remarks
7269...	$1^h 17^m 09^s$	150	100	200	520	
70...	36 25	150	100	200	530	
71...	41 16	150	100	200	540	*Correction of +10,000 km applied
72...	51 49	152.5	100	205	560	
73...	2 35 21	162.5	120	205	560	
74...	41 44	170	120	220	580	
75...	49 39	170	120	220	585	Prom. breaking away from stem
76...	56 56	175	130	220	500	
78...	4 09 40	195	140	250	560	Prom. parted
79...	19 32	200	140	260	500	
80...	31 03	210	150	270	480	
81...	38 12	220	160	280	520	
82...	46 49	225	160	290	440	
83...	5 13 59	250	180	320	480	
84...	32 41	265	210	320	380	
85...	40 39	275	210	340	440	
86...	56 36	285	220	350	450	
87...	6 14 12	305	232.5	377.5	560	*Correction of +12,500 km applied
88...	23 34	310	240	380	420	
89...	32 18	322	245	400	400	
90...	42 00	335	260	410	450	Spiral form
91...	53 22	360	280	440	450	" "
92...	7 08 49	387.5	280	495	460	" "
93...	19 31	410	300	520	420	" "
94...	57 22	545	430	660	420	" "
7295...	8 23 29	640	520	760	500	" "

* Correction applied on account of oversetting of the occulting disk of the spectroheliograph.

The two columns from which the prominence arose at latitudes $-37°$ and $-41°$ were observed at each return to the limb until August 5, when they had disappeared, giving a duration of 4 months for this prominence.

THE PROMINENCE OF JULY 15

This prominence, unlike that of May 29, had little to herald its coming. A small prominence occupied the position of its southern extremity on July 1. On the fourteenth it appeared as a low cloud coming over the limb; nothing extraordinary, however. The early morning of the fifteenth was cloudy, but the sky cleared suddenly about 3^h G.M.T.

The first exposure on this limb was taken at 3^h8^m (Plate VI, Fig. *f*), when it appeared to be already well on its ascent, having a height of 6′ and extending in a long elliptical arc from $+11°$ to $-18°$, thence turning toward and terminating in a spot at $-13°.6$. The longitude of this spot as determined from later plates was $117°.2$, but at the time the prominence appeared it was wholly visible at the limb. The longitude of the limb was 103° (from the ephemeris), whence it appears that this prominence lay at a considerable angle to its meridian.

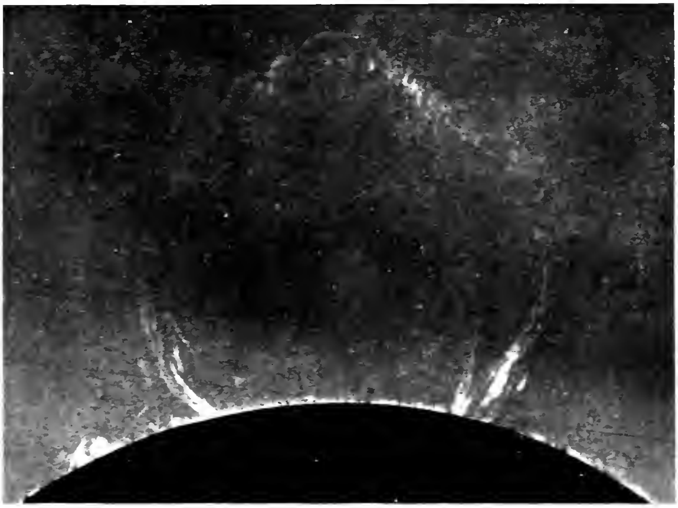
Succeeding exposures showed the prominence to be in rapid vertical motion. Clouds interrupted the work only occasionally, so that a total of 10 plates was obtained at quite regular intervals before the prominence disappeared. The maximum height attained was 16′ (720,000 km) and the period of ascent from first exposure only 1^h26^m. Professor Biefeld (of Denison University) developed all the plates and Mrs. Pettit assisted at the telescope.

In this prominence a ropelike structure endured throughout, the remaining matter fading away after the first four exposures, so that the maximum and minimum values are practically identical. Table II gives data of the exposures and measurements.

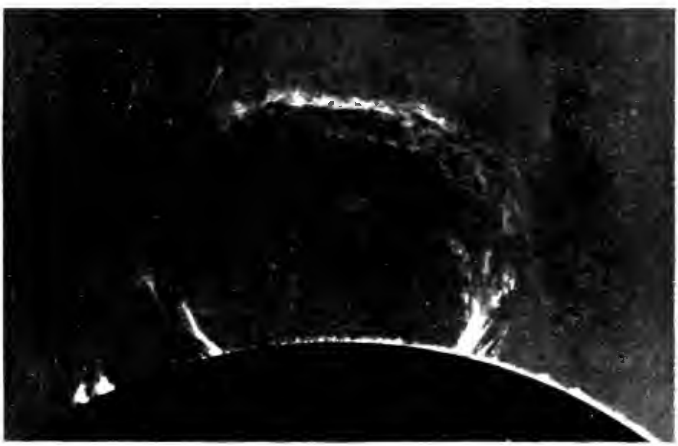
This prominence was peculiar in that it did not seem to break suddenly away and float upward, but stretched like an elastic band, the center expanding upward and the sides becoming more nearly straight, the general outline approximating an isosceles triangle. Plate VI, Figs. *g* and *h*, represent it in this stage. It first appeared

PLATE VI

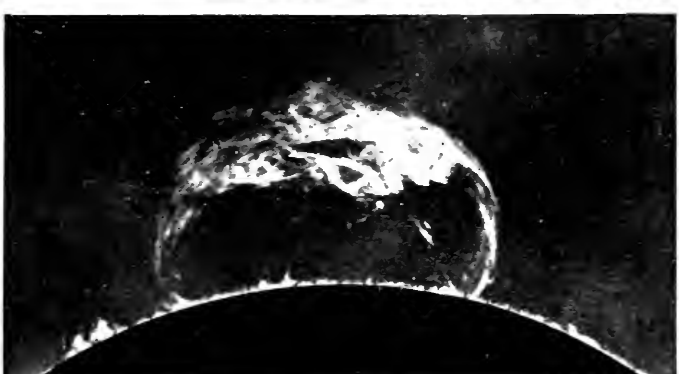
h
G.M.T.
$4^{h}\,7^{m}\,10^{s}$



g
$3^{h}\,51^{m}\,56^{s}$



f
$3^{h}\,8^{m}\,2^{s}$



THE PROMINENCE OF JULY 15, 1919
Scale: 1 mm = 0572 km

July 1 as two small prominences occupying the terminal positions of that of July 15, and these were seen last on July 28.

TABLE II

MEASURES OF THE GREAT PROMINENCE OF JULY 15, 1919
(In Thousands of Kilometers)

Series No.	G.M.T.	Mean Height	Least Height	Greatest Height	Remarks
7524...	$3^h 08^m 02^s$	200	140	260	
26...	28 48	250	190	290	
27...	35 49	263			*Correction of +10,000 km applied
28...	43 59	278	235	300	
29...	51 56	315			
30...	4 01 11	400			
31...	07 19	460			
32...	15 57	548			
33...	23 39	620			Base partially obscured by cloud but crest intact
7534...	33 57	720			

* Correction applied on account of oversetting of the occulting disk of the spectroheliograph.

VERTICAL MOTION

In the measures of the heights of the prominence of May 29 settings were made with the scale on what seemed to be the center of the body of the object, also the greatest and least heights of that body. From a number of trials the height of apparent center and mean height were quite identical, but the latter means of measurement were adopted as being somewhat more accurate. Fig. 1 is the plotted curve of the heights of the prominence as ordinates with the Greenwich Mean Times of the exposure as abscissas. The unit of height is 10,000 km and the unit of time is 10 minutes. Each cross then represents the height of the center at a given instant of G.M.T.

In the measures of the prominence of July 15 the settings were made on a narrow ropelike structure which passed centrally through it, enduring throughout the ascent period. Settings were made directly on this structure. Fig. 2 is the plotted curve of the height of the prominence in ordinate with reference to the Greenwich Mean Time in abscissa. The unit of height is 10,000 km and the unit of time is $2\frac{1}{2}$ minutes.

It is at once apparent that both curves are best represented by broken straight lines, that of May 29 having four breaks and that of July 15 having one. A mean line was drawn through each portion of the curve.

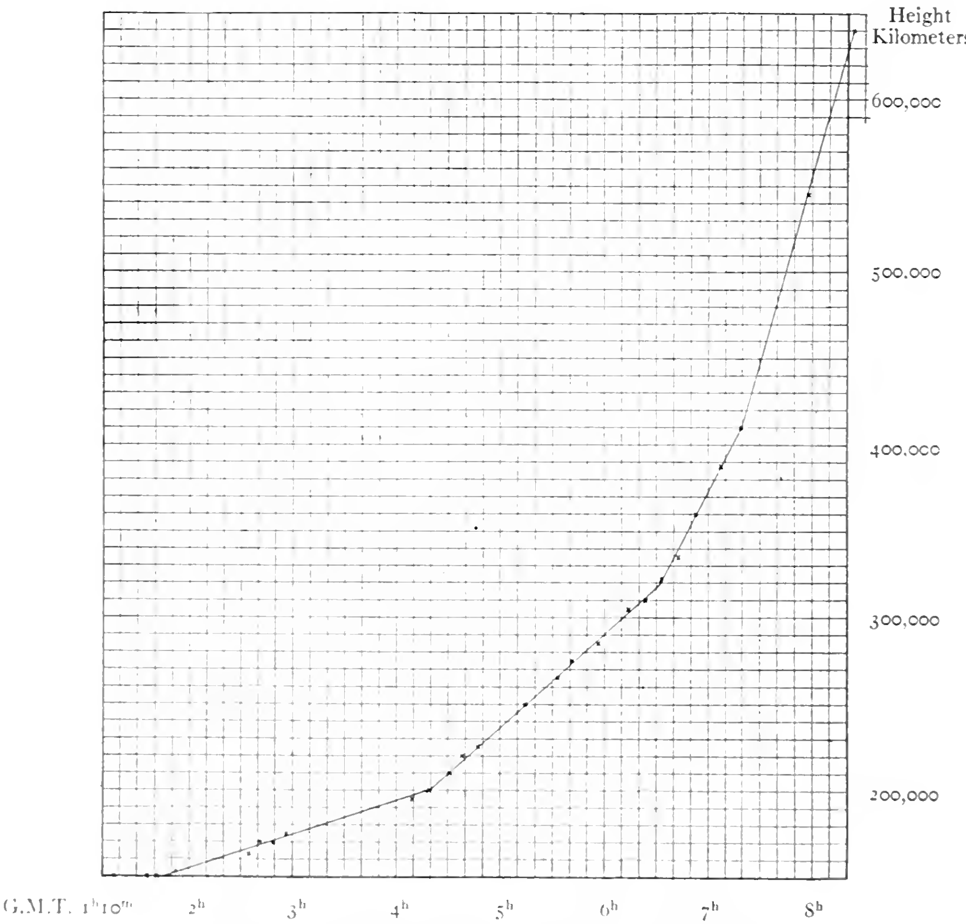


FIG. 1.—Graph of measured heights of the prominence of May 29, 1919

This fact, then, seems to be established: these prominences began their ascent with uniform motion, after a time receiving an *impulse* which increased the velocity of ascent, the motion remaining uniform, however. This process continued till the prominence disappeared.

Table III exhibits a comparison between observed data obtained from the graphs and the corresponding data computed from the gravitational theory.

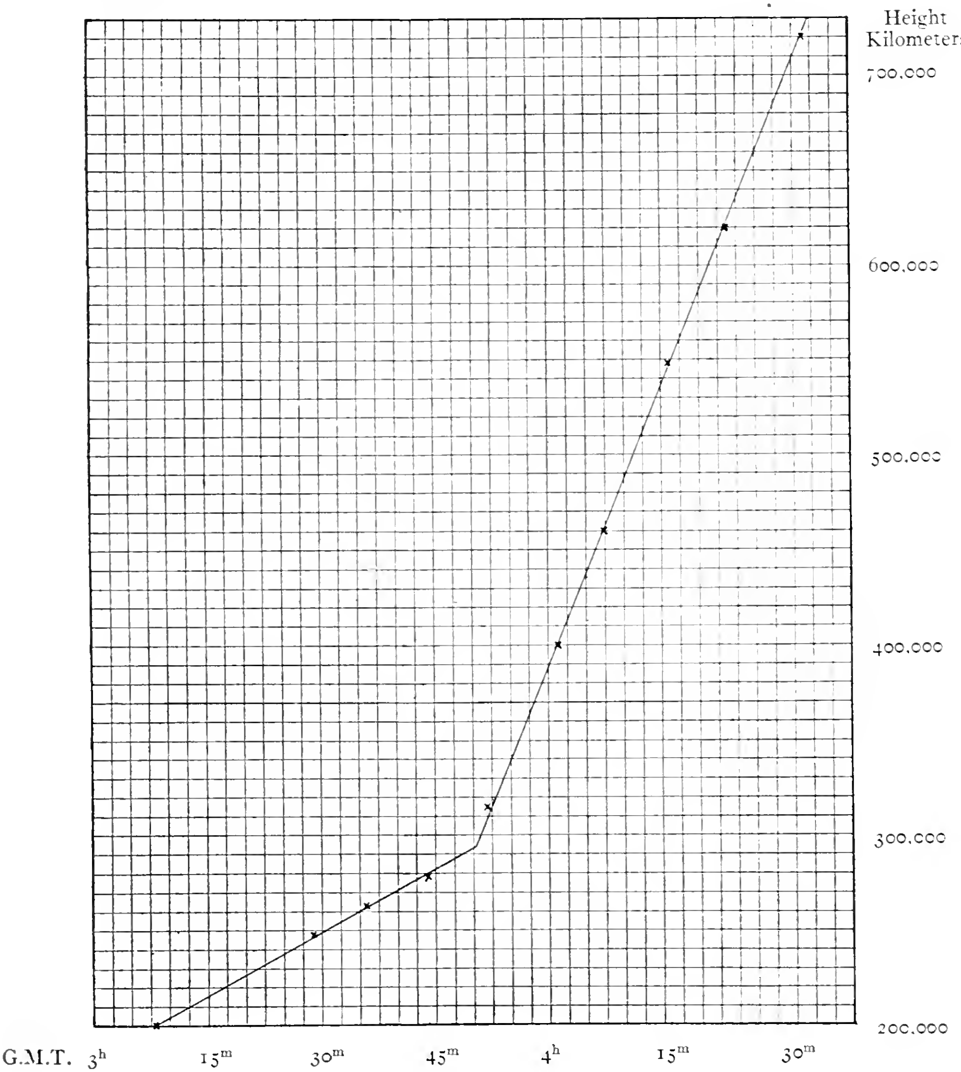


FIG. 2.—Graph of measured heights of the prominence of July 15, 1919

The first column gives the height of the prominence above the sun's surface at the instant of change of velocity, i.e., at the break in the curve. The second column records the velocity of ascent as computed from the plotted line. "ΔV" is the change in velocity at the break in the curve. "Rise at this velocity" is the distance the prominence rose while moving with the velocity given in the second column. The column "Theoretical maximum height" was computed on the basis of the supposition that the prominences

TABLE III

Height of Prominence above Sun	Velocity of Ascent	ΔV	Rise at This Velocity	Theoretical Maximum Height at This Velocity	Observed Duration (Const. V.)	Theoretical Duration (Dimin. V.)
The Prominence of May 29						
km	km/sec.	km/sec.	km	km	min.	min.
150,000	5.5	5.5	50,000	83	164	0.51
200,000	14.7	9.2	119,000	665	133	1.51
319,000	27.9	13.2	91,000	3,010	57	3.57
410,000	60.0	32.1	230,000	17,059	63	9.53
The Prominence of July 15						
200,000	37.0	?	294,000	4,211	43	3.80
294,000	163.9	126.9	426,000	111,753	47	23.56

began with the observed velocity and distance from the sun's surface and rose in the same manner and under the same conditions as a body projected vertically upward while affected only by the sun's gravitative force. "Observed duration" gives the time during which the observed velocity was maintained. "Theoretical duration" is the interval of time required for the prominence to attain the "Theoretical maximum height" and computed on the same basis as the latter. These two columns may be conveniently computed from the following formulae:

$$H = s_1 - s_0 = \frac{V_0^2 s_0^2}{2gR^2 - V_0^2 s_0^2}, \qquad (1)$$

$$T = \frac{A\frac{\pi}{2} - AB + C}{D} \qquad (2)$$

$$t=\frac{A\sin^{-1}\left[\frac{2s-s_1}{s_1}\right]-AB+C-\sqrt{s_1s-s^2}}{D}, \tag{3}$$

where $A=\frac{s_1}{2}$, $B=\sin^{-1}\left[\frac{2s_0-s_1}{s_1}\right]$, $C=\sqrt{s_1s_0-s_0^2}$, $D=R\sqrt{\frac{2g}{s_1}}$

g = acceleration of a falling body at the sun's surface (0.27 km/sec.)
H = "Theoretical maximum height attained" measured from sun's surface
R = radius of the sun (695,553 km)
s = distance at any instant from the sun's center
s_0 = distance from the sun's center at the time when the velocity V_0 begins (measured from the break in the graph of the heights of the prominence)
s_1 = "Theoretical maximum height attained" measured from the sun's center
T = time required to attain height H ("Theoretical duration")
t = time required to attain distance s from sun's center
V_0 = "Velocity" observed, measured from the plot

It is interesting at this point to compare the observed heights acquired by the prominences with those obtained from the theory mentioned. Thus in the case of the prominence of May 29 the initial velocity was 5.5 km/sec., and according to the theory it should have risen only 83 km with diminishing velocity, finally falling back to the surface of the sun, whereas it actually rose 50,000 km with uniform velocity.

It might be surmised that other eruptive prominences may obey the same law as these two, viz., that they rise with uniform motion and receive impulses at intervals which increase the velocity, but preserve the uniform character of the motion. On our series of spectroheliograph plates, now numbering 7800 of both prominence and disk, extending back to 1903, there seems to be only two eruptions from which it is likely that information of this kind may be expected. One of these was taken by Mr. Slocum, March 25, 1910 (5 plates), and the other by Mr. Lee, October 21, 1914 (6 plates). Plotting the curves of these it is found that the same principle obtains as in the two prominences discussed here, one break showing in each curve. It is hoped that other observations of these

prominences may be obtained from other sources so as to increase the number of points on the plots. This material will be published in the future.

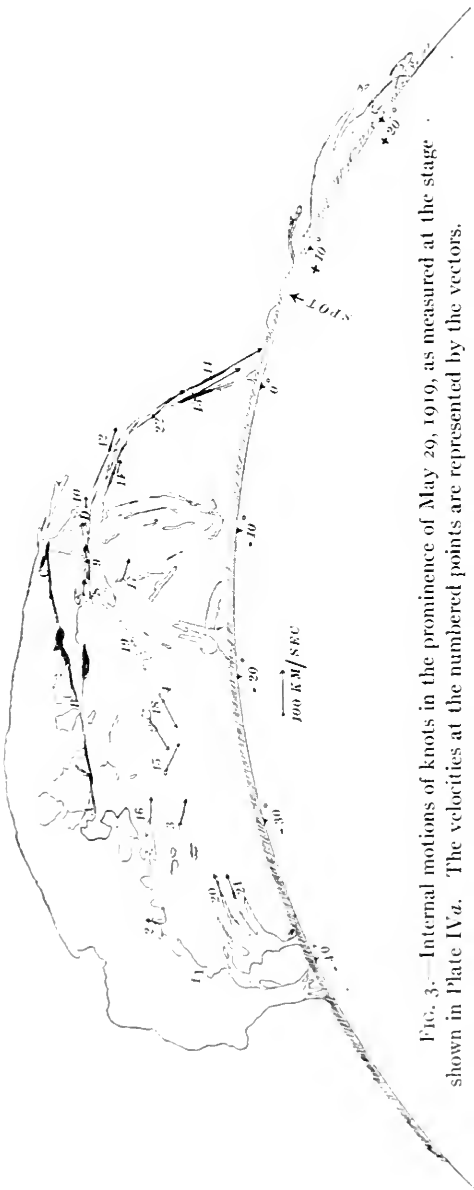


Fig. 3.—Internal motions of knots in the prominence of May 29, 1919, as measured at the stage shown in Plate IV*a*. The velocities at the numbered points are represented by the vectors.

INTERNAL MOTIONS

The appearance of the prominences readily suggests that there may be a "pouring" of matter into the spot in each case connected with it. Indeed investigation shows this to be the case, but as this has not yet been completed only a partial report can be given.

The plates are measured in pairs on the stereo-comparator with the position-micrometer attached. Two plates at the beginning of the series are placed in the comparator and oriented in position angle by the dust lines, and in X and Y by the two small prominences which remained fixed throughout the series of exposures. The shift of the knots in the prominence and the position angle of this shift are then measured with the micrometer. After the measurement the earlier plate is removed and the next later one inserted, and the measures are repeated. After the series is finished the measures are repeated with the plates in

reversed holders. Fig. 3 is a pantograph drawing of the prominence shown on Plate IV*a*, indicating the points measured and showing the vectors of motion. The values of these quantities are given in the following table:

TABLE IV

MEASURES OF KNOTS ON PLATES NOS. 7270 AND 7271

Point No.	Velocity km/sec.	Position Angle (through North-west)
1	6.5	5°0
2	22.2	19.0
3	55.5	104.9
4	88.1	245.6
8	31.2	95.9
9	36.1	87.4
10	22.3	93.1
11	28.7	114.4
12	50.8	112.2
13	122.4	153.5
14	130.6	153.5
15	65.9	118.4
16	59.6	95.8
17	44.1	109.3
18	83.2	242.4
20	49.0	72.2
21	58.5	71.1
22	91.1	141.2
AB	11.9	260.5
BC	8.0	255.8
BD	8.3	270°6

General acceleration of motion is shown along the stream of matter into the spot, varying from 7 km/sec. at points most remote to 131 km/sec. for points near the spot. The general outline of this stream is elliptical. The torn, shredded structure beneath the prominence shows high velocities (60–88 km/sec.) toward the sun's surface, as will be seen on an examination of the table.

The heavy lines *AB*, *BC*, *BD* of Fig. 3 (see also Plate IV*a*) show velocities upward of 8 to 12 km/sec. Now these two plates were taken just before the prominence began to rise as a whole; hence the underside was experiencing an upward pressure tending to compress the whole body along the lines indicated (*AB*, *BC*, *BD*, Fig. 3).

THEORY

It must be admitted that the phenomena we have before us requiring explanation are rather complex. We have, first, the uniform motion upward of the body as a whole; second, the impulses with uniform motion preserved but increased velocity; third, the accelerated motion of knots of matter into the spot in a reverse direction.

If we may suppose that all eruptive prominences obey these principles, and the differing physical characteristics of those just discussed strengthens the idea, then the following speculations may not be amiss.

Evidently uniform motion in a straight line can only exist, so far as we know, (*a*) when the force of gravitation and other centripetal forces are always neutralized by forces acting in the reverse direction, (*b*) when the body is suspended in a medium with propelling power.

It is difficult to imagine the condition (*b*) to exist without introducing fanciful suppositions, for impulses alone, acting on a body, can only produce negatively accelerated motion, not uniform. With regard to the possibility (*a*) the following considerations must be dealt with:

1. The photographs being spectrographic results, it must be borne in mind that we are dealing with a true gas under low pressure.

2. If it is supposed that, originally, gravitation was balanced by light-pressure and electrical repulsive forces, they would become unbalanced as soon as the prominence rose slightly, since gravitation acts as if from the center of the sun while the other forces act from the surface—hence the balance will not be preserved. Also (1) must be taken into consideration.

3. Electrostatic repulsion due to induced charges in the prominence. It might be questioned here, however, whether the gas (calcium vapor) is conducting to a sufficient degree.

SUMMARY

a) The following principles have been found to govern the motion of the eruptive prominences of May 29 and July 15, 1919.

1. Before the ascent begins the under side of the prominence is compressed as if a force were pressing on it over a large area.

2. The velocity is at all times uniform.

3. The velocity suddenly increases at intervals, as if the prominence were acted upon by an impulse of very short duration—possibly a matter of only several seconds.

b) Knots of matter in the prominence of May 29 were moving in an elliptic arc into the spot with velocities increasing on approach to the spot.

c) Principles (2) and (3) were found to apply to two other eruptive prominences.

Since writing the above 12 other eruptive prominences observed either visually or photographically have been found to be governed by the same principles.

d) That principles stated under (*a*) may apply generally to eruptive prominences is strengthened by the fact that the four cases cited differ in general form and characteristics other than motion.

e) It is urged that spectroheliograph observers should attempt to secure material suitable for further studies of this kind on occasions of future eruptions.

Besides those already mentioned my thanks are due to Professor Frost and other members of the staff for suggestions and assistance in other ways.

Yerkes Observatory
September 1, 1919

ON RADIATION-PRESSURE AND THE QUANTUM THEORY

A PRELIMINARY NOTE

By MEGH NAD SAHA

After the prediction by Maxwell of the existence of the pressure of radiant energy on the basis of his theory of stresses and strains in ether, other ways of arriving at the same result have been found by Bartoli (thermodynamical), Poynting (flow of momentum along a ray of light), and Larmor (electromagnetic wave-theory of light). A review of these methods shows that they are all statistical, i.e., the result holds only when the surface encountered by radiation is large compared with the wave of light and is thickly set with matter.

Schwarzschild and more recently Nicholson[1] and Klotz[2] have worked out, on the basis of the continuous theory, the value of the radiation-pressure, when the size of the obstructing mass is gradually decreased, ultimately being reduced to the scale of the wave-length of light. In this case the effect of repulsing light-pressure gradually preponderates over any gravitative force to which the particle may be subject, but at the same time it appears that there is a limit to this process of reduction. If the particle be too small, it is no longer capable of acting as a barrier to the advancing light-waves, and consequently experiences no radiation-pressure. It appears from these investigations that for particles of the molecular size (radius $=10^{-8}$ cm) the effect of light-pressure is totally evanescent.

But this conclusion from the old continuous theory is rather contradictory to the requirements of astrophysics, for in order to explain tails of comets and other astrophysical phenomena (such as solar prominences, corona) which take place on the surface of luminous heavenly bodies we have to assume the existence of certain repulsive forces[3] (levity) acting on the ultimate gaseous mole-

[1] *Monthly Notices*, **74**, 425, 1914.

[2] *Journal of the R.A.S. of Canada*, **12**, 357, 1918.

[3] See Agnes M. Clerke, *Problems of Astrophysics*, p. 51.

cules and thus reducing the gravitational attraction on them. But a still stronger ground for rejecting the conclusion is furnished by the experimental demonstration by Lebedew[1] of the existence of radiation-pressure on molecules of absorbing gases like CO_2, methane, propane, etc. It may thus be taken for granted, in spite of the failure of the continuous theory, that the molecules do really suffer a radiation-pressure, which in the aggregate conforms to Maxwell's law.

Professor R. W. Wood[2] is inclined to the opinion that the gas-molecule may be capable of stopping the radiation by resonance, and may thus experience a radiation-pressure, but precisely what is meant by stoppage of radiation by resonance is not clear. An explanation of the existence of radiation-pressure on molecules is furnished when we apply the quantum theory in the place of the old continuous theory of light. Instead of assuming that "light" is spread continuously over all points of space, let us suppose that they are localized in pulses of energy $h\nu$ (ν = frequency of light, h = Planck's universal radiation constant). Let this pulse encounter a molecule m and be absorbed by it. Then in doing so the molecule will be thrust forward with an impulsive momentum of $\frac{h\nu}{c}$ (c = velocity of light); for we may suppose the pulse to have the mass $\frac{h\nu}{c^2}$ and the momentum $\frac{h\nu}{c}$; the absorption of the pulse by the molecule may be taken as a case of inelastic impact, the whole momentum being communicated to the molecule. The velocity with which the molecule will move forward $=\frac{h\nu}{cm}$.

Let us consider the effect of the absorption of a pulse of the hydrogen light corresponding to the line $H\alpha$ by the hydrogen atom. The velocity imparted at each kick of light,

$$v=\frac{h\nu}{cm}=60 \text{ cm per second,}$$

$$\left(\text{taking } h=6.54\times 10^{-21}\right.$$

$$\left.\frac{c}{\nu}=\lambda=6.563\times 10^{-5}\text{cm}, \quad m=\frac{1}{6.062\times 10^{23}}\text{ gms}\right)$$

[1] *Annalen der Physik*, 32, 411, 1910. [2] *Physical Optics*, p. 51.

This velocity is rather a small quantity (compared to the orbital velocity of the molecules), but it should be remembered that it is really an impulsive velocity and is of the nature of an acceleration. The total velocity acquired by a hydrogen atom per second will depend upon the number of kicks of light it experiences per second, and provided this is sufficiently great the velocity acquired may rise to enormous values. But a priori we cannot say what this number will amount to without a preliminary examination of the physical conditions.

This conclusion explains Lebedew's results, which cannot be explained by the continuous theory, and at the same time offers a general explanation of the radiation-pressure. The pressure $=\frac{1}{c}\Sigma\Sigma h\nu$, where the summation extends over all the pulses absorbed in unit time, within unit area. It thus equals AI, where $I=$ intensity of light, $A=$fraction absorbed. The aggregate effect remains unchanged, but it is now supposed to be concentrated on a few active molecules, the inactive molecules remaining unaffected.

The explanation offered closely resembles Einstein's explanation of the velocity of emission of the photo-electrons. According to Einstein when a pulse of light ($h\nu$) falls upon an atom it is instantly absorbed and goes to increase the energy of the system. Consequently certain of the electrons of an atomic system acquire a velocity which is greater than the critical velocity required for retaining these electrons in their orbit. Let A be the energy required for detaching an electron from the parent atom. Then the velocity of escape is given by the law

$$\sum \tfrac{1}{2}mv^2 = h\nu - \sum A$$

The maximum velocity occurs when only one electron is emitted. Then

$$\tfrac{1}{2}mv^2 = h\nu - A$$

Actual experiments by Millikan[1] and others have established the truth of the law quantitatively. Besides, the phenomenon is instantaneous whatever be the intensity of the light. This feature is not capable of explanation by the continuous theory of absorption. N. R. Campbell[2] has found that in certain cases the con-

[1] *Physical Review*, 7, 18, 1916. [2] *Modern Electrical Theory*, p. 240.

tinuous theory requires that the atom must be illuminated for at least 15 minutes before it can acquire the energy sufficient for the emission of the electron, while actually the emission takes place in less than $\frac{1}{1000}$ of a second after illumination.

Let us now see how the number of kicks of light experienced by the hydrogen atom or molecule varies with the existing circumstances. The number will clearly depend upon the following factors: (1) the density of pulses of light in the region traversed by the molecule; (2) the time of retention by the molecule or the atom of the capacity for the absorption of light. We shall first take the second point. Hydrogen under ordinary circumstances does not absorb its characteristic radiation (represented by the Balmer lines), as has been demonstrated by the repeated failures of the experiments for obtaining the reversal of the hydrogen lines. But the experiments of Ladenburg and Loria[1] have thrown a new light on the cause of these failures; they find that hydrogen is capable of absorbing its characteristic radiation only when it is in an active state, i.e., when it is in a state of luminescence. This conclusion is also borne out by the theoretical investigations of Bohr,[2] for according to his theory a hydrogen line is emitted when the attendant electron leaps from orbit 3 to orbit 2, while in the natural state the electron is at orbit 1. We may symbolically express the idea as in Fig. 1.

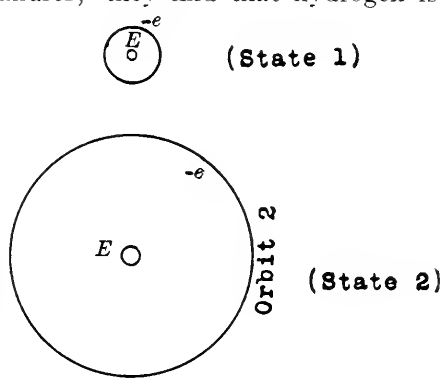


FIG. 1

State 1, natural state when inactive for the Balmer lines.

State 2, active state (when emitting the Balmer lines).

In order that an H atom may absorb a Balmer line, it must be, to start with, at state 2.

We may thus take it for granted that the H atoms which absorb the Balmer lines are not the ordinary H atoms, but an active

[1] *Verhandlungen der deutschen Physikalischen Gesellschaft*, **10**, 858, 1908.

[2] *Philosophical Magazine*, **26**, 1, 476, 857, 1913.

modification thereof, the electron being at orbit 2 instead of at orbit 1. When light corresponding to any line of the Balmer spectrum traverses a mass of hydrogen, it is only the active particles which will absorb this light and be subjected to the impulsive kicks of this light.

Taking it for granted that an active molecule suffers a discontinuous kick of light given by the formula in the process of absorption, let us see how it will behave when placed in a field of radiation. To visualize matters, we shall take an active H atom moving near the photosphere of the sun. The H atom, if active to start with, will pick out from the continuous spectrum the pulse corresponding to $H\alpha$ or $H\beta$ and with an instantaneous velocity of 60 to 31 cm per second. It is true that, as the particle emits light, it suffers an equal recoil opposite to the direction of emission. It should be borne in mind that the emission does not take place in any specified direction, but in any direction according to the law of chance, while the pulses which are absorbed come from a specified direction, viz., the center of the sun. Hence if the particle continues active for a sufficient length of time, the H atom may ultimately acquire a velocity exceeding the critical velocity of 6.12×10^7 cm per sec. (the velocity required for the escape of the particle from the gravitational influence of the sun).

The precise velocity which a particle acquires depends upon a large number of unknown factors: (1) the intensity of the field of radiation—the influence of this factor is to a certain extent known—the density of pulses varies as the intensity of light, and therefore follows the law of inverse square; (2) the persistence of the activity of the H atom, or rather, if the activity be lost, the quickness with which it is regenerated; (3) the actual proportion of active particles in any region.

Nothing is known about the second and the third factors; consequently it is not possible to work out a quantitative theory of the effect of radiation-pressure on the expulsion of the molecules. But the general considerations show *that radiation-pressure may exert an effect on the atoms and molecules which are out of all proportion to their actual sizes. It also shows that the radiation-pressure exerts a sort of sifting action on the molecules, driving the active ones*

radially outward along the direction of the beam. The cumulative effect of the pulses may be sufficiently great to endow the atoms with a large velocity—the velocity with which the tops of solar prominences are observed to shoot up.

The velocity of the red prominences are sometimes found to be as high as 6×10^7 cm per second.

The solar prominences have sometimes been explained on the assumption that they are due to the convection of hot masses of vapor from the solar photosphere, which, after reaching the atmosphere, are supposed to expand adiabatically and develop the large velocities with which the prominences are observed to shoot up. But both Pringsheim and Nicholson[1] have pointed out several insuperable difficulties in the way of the acceptance of this hypothesis, including the deduction that the maximum velocity obtainable from adiabatic expansion is less than $\frac{1}{45}$ of the velocity with which the prominences are observed to shoot forward (6×10^7 cm). Nicholson has suggested that some unknown forces of electrical origin may be the cause of these large velocities, but even granting that the electrical fields exist in the sun it is difficult to see how this can act upon the luminous hydrogen particles, which are most probably uncharged. According to the hypothesis put forward in this paper, the effect of radiation-pressure on the separate particles is altogether disproportionate to the dimensions of the particles and may cause them to be endowed with a "levity"[2] long sought for in the explanation of the prominences, the corona, and other solar phenomena, including the extension of the solar atmosphere.[3] The hypothesis presents the problem of the radiative equilibrium of the solar atmosphere in a new light.

[1] *Monthly Notices*, **74**, 425, 1914.

[2] Ch. Fabry, lecture delivered before the Astronomical Society of France, 1918 (*L'Astronomie*, **32**, 14, 1918).

[3] Attention may be called to a comprehensive paper by D. Brunt (*Monthly Notices*, **73**, 568, 1913), who has shown that neither of the three theories of the equilibrium of the solar atmosphere (isothermal, adiabatic, or radiative) can account for an atmosphere extending to the observed height of the solar atmosphere. The results of the spectroheliographic observations are distinctly unfavorable to Julius' theory of anomalous dispersion (see *Astrophysical Journal*, Papers by Hale, St. John, and others).

These ideas may be applied to the explanation of the tails of comets. The tails of comets are undoubtedly caused by some sort of repulsive action exerted by solar light, but since, on the older theory, the effect was found evanescent on particles of the molecular size, the tail was supposed to consist of some sort of cosmic dust. But the spectroscopic examination of the light from the tails shows that they consist, at least partly, of luminous gases (CO, CO_2).[1] Now the explanation is quite easy, if the considerations advanced in this paper hold. As the comet approaches the sun, more and more pulses of light from the sun traverse the nucleus and the coma. Light-pulses of suitable frequency are picked up by the gaseous particles, which thus gradually gain in velocity in a direction away from the sun. The cumulative effect of the absorbed pulses may endow the particle with a velocity sufficient for its escape from the main mass of the cometary matter and form into the tail.

It is hoped to develop these ideas further in a future communication.

University College of Science, Calcutta
March 4, 1919

[1] Bohr, *loc. cit.*

Minor Contributions and Notes

POLARIZATION OF THE NIGHT SKY

It is known, and indeed may readily be verified by anyone, that the background of the sky at night, or while the stars appear, is far from being absolutely dark. Charles Fabry[1] has recently pointed out the interest of determining whether or not this light is polarized, like the light of the daylight sky. In this way we may hope to decide whether or not the light is scattered sunlight.

I have recently obtained some preliminary results on this subject. A Savart polariscope was arranged for photography. A pair of quartz plates, each about 2.5 mm thick, and cut so that their normals were about 16° from the optic axis (true angle, corrected for refraction), were crossed and placed in the appropriate azimuth over a Nicol of about 27 mm clear circular aperture. At the other end of the Nicol was a plane convex lens (not achromatic) of 50 mm focal length, which focused the sky on a photographic plate. The field of view of the apparatus was about 20°. It was pointed toward the pole, so that the image of the pole star was in the middle of the circular field; thus the direction of observation was nearly at right angles to the sun, independently of the time. The principal plane of the Nicol passed through the sun. To maintain this adjustment it was necessary to keep the apparatus in rotation about its axis at the rate of one turn in twenty-four hours. Practically it was enough to turn it discontinuously every fifteen minutes.

A short exposure made at twilight showed Savart bands of strong contrast covering a circular field 17 mm in diameter. The distance between successive dark bands was 1.9 mm. Night exposures were made at Terling, Essex, England, on April 6 and May 10, 1918, both very clear nights. The moon was below the horizon throughout the exposures. Both photographs gave the

[1] *L'Astronomie*, **32**, 15, 1918.

same result. Taking that of May 10, the exposure was continued from one hour before (true) midnight to one hour after midnight. On development the circular field proved to be adequately exposed, and the Savart bands, though somewhat disguised by the short star-trails superposed upon them, could be seen without doubt at a glance. The contrast between the dark and bright bands was, however, far less marked than in a twilight exposure of equal average intensity. The inference is that the night sky shows some polarization, but much less than the day sky.

R. J. STRUTT (RAYLEIGH)

November 11, 1918

NOTE ON THE POLARIZATION OF THE NIGHT SKY[1]

At the suggestion of Lord Rayleigh (then Professor Strutt), attempts were begun at Mount Wilson during the past spring to detect the presence of polarization in the background of the night sky. During the preceding year he himself had made some observations in England for this purpose, and very kindly loaned to us the large Nicol prism and pair of quartz plates which he had used. When combined with a short-focus lens of large relative aperture, these form a photographic Savart polariscope well adapted to the problem. It was found, however, that the weight of the assembled apparatus and the losses of light therein were undesirably large, and a simplified arrangement was developed (after Cornu) which has proved very satisfactory.

For monochromatic light the variation of intensity across the field of a Savart polariscope takes the form of a smooth curve, i.e., the maxima and minima are separated by regions of intermediate intensity. Furthermore, when a considerable range of wave-lengths is employed, there is a flattening of the scale of contrasts, due to the change of spacing of the bands with the wave-length. This effect must tend to diminish the sensitivity of the Savart polariscope, especially when it is used photographically to detect a small percentage of polarized light in a large amount of ordinary light. In the polariscopes described below, areas of maximum and

[1] *Contributions from the Mount Wilson Observatory* No. 171.

minimum intensity are placed side by side in the field of view, unseparated by regions of intermediate intensity, and they suffer no loss of sharpness when white light is used, so that advantage is taken of the well-known apparent enhancement of contrast due to juxtaposition of areas of different blackness upon the photographic plate. Furthermore, the number of air surfaces in the optical system may be reduced to four and the total thickness of solid media may be kept down to a couple of centimeters or even less, so that the losses of light may be made very small.

The first instrument which was designed to fulfil these conditions consisted of a Nicol prism of the Glan-Thompson type, having a compound half-wave plate of mica attached to one side. The Niçol was selected on account of the uniformity of its polarization over the angular field, and the half-wave plate was made of mica strips 2×8 mm with axes alternately at $0°$ and $45°$ to the principal plane of the Nicol. The mica was of such a thickness as to introduce a relative retardation of $\lambda/2$ for the region of the spectrum to which ordinary photographic plates are most sensitive. The alternate strips of mica appear light and dark, their contrast depending upon the amount of polarized light in the incident beam, as well as upon the orientation of the Nicol with respect to the plane of polarization. This polariscope was found to be highly sensitive and readily adaptable to such problems as the one at hand.

Upon consultation with Dr. Anderson, however, he offered a suggestion as to a still simpler method of constructing a polariscope which would fulfil the conditions discussed above. He then very kindly prepared the two essential elements required, which are described below, and most of the observations have been made with this apparatus. It is a pleasure to acknowledge here my indebtedness to him.

In its simplest form this polariscope consists of a small plate of calcite 5 or 6 mm thick, having cemented to one side a glass screen containing a series of equal opaque and transparent strips. The width of strip is made equal to the separation of the two images of a point seen through the calcite plate, and the glass screen is so oriented upon the calcite as to bring the two images of each transparent strip just into contact side by side.

It is evident that if the incident light is wholly or partially polarized the strips will in general appear alternately light and dark. When the plane of polarization is at 45° to the principal directions in the calcite the illumination will be uniform, but if the angle is 0° or 90° there will be maximum contrast between the two sets of images, dependent upon the percentage of polarized light in the incident beam.

The glass screen was made by photographic reduction from a model having dimensions about 8 by 10 inches. Upon a glass plate of this size were mounted a row of pieces of rolled-metal strip, the alternate ones being afterward removed, so as to leave a grid having equal black and white spaces when viewed by transmitted light.

The calcite plate and attached screen were mounted in one end of a tube about ten inches long, at the other end of which was an aplanatic hand magnifier lens of 37 mm focal length. A simple metallic screw-cap held a photographic plate in the proper position behind the lens to receive a sharp image of the strips of the screen as seen through the calcite plate. Diaphragms were used to prevent undesirable light from reaching the plate.

This arrangement proved an exceedingly delicate detector of polarized light, but its sensitivity was still further increased by the addition of a narrow strip of mica at right angles to the strips of the glass screen. The thickness of the mica was such that it introduced a relative retardation of one-half a wave-length for blue light, and its axis was at 45° to the principal directions in the calcite. As a result the portion of the field covered by the mica has the relative intensity of the alternate strips reversed as compared to the rest of the field. This is found to be of material assistance when the difference between alternate images is very small.

The polariscope was provided with an index and a graduated circle by means of which the principal planes of the calcite plate could be oriented with respect to the position of the sun. The most satisfactory method of adjusting was to mount the polariscope upon the tube of a small telescope having an equatorial mounting. The polariscope was set for the neutral position (where sensitiveness of setting is a maximum) by daylight in a direction at 90° from the sun. A rotation of 45° then brings one of the principal planes of

the calcite through the sun, and a reading of the hour circle of the telescope and the time makes it possible to set the polariscope correctly for the subsequent exposure.

All the plates were taken in the northern sky in a direction very nearly 90° from the position of the sun. All were made within a day or two of the time of new moon and nearly all of the exposures were of two hours' duration, symmetrical about the time of true midnight. Fast plates were used and a contrast developer. Upon one of the nights the atmosphere was clear enough so that three observers agreed in their separate locations of the *Gegenschein*. It must be said, however, that for some weeks past, probably including the last two periods of observation, the transparency of the atmosphere on Mount Wilson has been below its best. Sunset colors have been brighter than usual and there has been a slight milkiness apparent in the daylight sky. Visual measures of the daylight polarization of the sky by Anderson in Pasadena have shown a decrease in the percentage of polarization during the past few weeks.

The photographs suitable for discussion number eight. There is evidence upon two or three of them of a very slight difference between adjacent strips, but considering the marked sensitiveness of the instrument, it must be said that the total amount of polarized light shown is extremely small.

Quantitative measures of the sensitiveness have not been made, but it is probably safe to say that we have not found more than 1 per cent of polarization in the background of the night sky.

Harold D. Babcock

Mount Wilson Observatory
August, 1919

THE AURORAL SPECTRUM

V. M. Slipher's result of the position of the principal auroral line as given in this *Journal* for May 1919, **49,** 273, is certainly very surprising. In *Nature* for June 28, 1883 (**28,** 209), I summarized all the observations of the auroral spectrum I was acquainted with up to that time, and while many of the rough observations of the principal line gave a greater wave-length than 5578, no single one

of those that appeared to be more accurate exceeds 5572. But the superiority of the photographic method over the visual leaves no room to doubt the accuracy of Slipher's result.

As regards the existence of the principal auroral line over the night sky at all times, without there being any definite aurora, I have frequently looked for it with my rough apparatus, and have failed to see it; but I have seen it on 13 occasions (between 1872 and 1895). On 8 occasions besides I have suspected it, or else the continuous spectrum has faded abruptly in its position. On 3 nights I also perceived the line I have called ϵ, about wave-length 5226, and which Vogel thought probably belonged to the red part of auroras. These two lines, the principal and ϵ, are the only ones in the auroral spectrum I have certainly seen when there was no actual aurora. On one of these occasions there may possibly have been what I call a faint "irregular aurora" as one was visible later in the night. I have also suspected the line ϵ on 5 other nights. I have twice suspected the line about 4688.

T. W. Backhouse

West Hendon House Observatory
Sunderland, England
September 9, 1919

EDWARD CHARLES PICKERING

From a painting by Mrs. Sarah G. Putnam, [illegible]

THE
ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME L NOVEMBER 1919 NUMBER 4

EDWARD CHARLES PICKERING, 1846–1919

By SOLON I. BAILEY

Whenever a man in any sphere of human activity accomplishes results which seem to place his name among the Immortals, the world is interested to know wherein lay his power.

Edward Charles Pickering was born on Beacon Hill, Boston, July 19, 1846. He was most fortunate in his heritage. Of an old and distinguished colonial family, he was heir to neither riches nor poverty, but to splendid opportunity which he eagerly grasped. From early boyhood to late manhood, in ill health as in good, his zeal in whatever scientific problem engaged his attention was unbounded. His education was begun in private schools. The years spent at the Boston Latin School were always regarded by him as largely wasted. He had small love of the classics and, in common with companions since more or less famous in science, he made no attempt to attain high rank in them. At the Lawrence Scientific School, however, he entered upon his work with that great enthusiasm which marked all the activities of his later life. He was graduated from this school *summa cum laude* at the age of nineteen, and was at once appointed instructor of mathematics in the same institution. Two years later he became assistant in physics at the Massachusetts Institute of Technology, and in the following year Thayer professor of physics, a position he held until called to the

directorship of the Harvard College Observatory. During these nine years forty-one scientific papers were published by him or by students under his direction, as well as two volumes of his pioneer textbook on practical physics entitled *Physical Manipulations*. He established, in connection with these volumes, the first physical laboratory in America for the use of students. The idea of such a laboratory had been suggested a few years earlier by President W. B. Rogers, of the Institute, but the details of its operation and its successful installation and management were first developed by Pickering in 1869. The great object to which his work at the Institute was directed was research. The value of the course in physical manipulation was measured to him by the success with which it taught the student to think for himself and fitted him to solve problems experimentally. Although research was his chief interest, his duties as a teacher consumed the greater part of his time. His investigations were largely concerned with light, which formed a fitting foundation for his future life-work. In later years he found that few astronomers thoroughly understood the principles involved in the various photometers which he devised for astronomical studies at the Harvard Observatory and elsewhere.

A notable contribution of a different character, however, was furnished by his early experiments with the telephone. In 1870, several years before the invention of the telephone now in common use, he devised, constructed, and tried a receiver consisting of a flexible iron diaphragm supported at the edges and replacing the armature of an electromagnet. This apparatus appears to differ in no way in principle from the receiver later employed. The possibility of protecting such a device by patent would never have been considered by him, since he held the conviction consistently throughout his life that a scientific man should place no restriction on his work which would prevent the repetition by others of any experiment of scientific interest.

Mr. Pickering also in 1876 founded and became the first president of the Appalachian Mountain Club. The great value of this club for popular purposes is undoubted, but its primary aim was scientific. Notable achievements have been accomplished in both

fields. His own chief contribution was the perfection of a light and portable micrometer level, weighing only twelve pounds, for the rapid determination of approximate positions and altitudes. To measure the positions and heights of a wide group of mountains, it was only necessary to ascend two of them and to know the horizontal positions of two and the altitude of one of the observed objects. Mr. Pickering made many thousands of observations of various points of interest in the White Mountains with this instrument. The intensity of his interests and the enthusiasm and success with which he carried out his plans made a deep impression on his associates. This is evident from the profound regard in which he is still held by the survivors among his scientific friends of those years.

Professor Pickering was chosen director of the Harvard College Observatory in 1876, at the age of thirty, and entered upon his duties February 1, 1877. The appointment of a physicist to direct an astronomical observatory caused some natural criticism from astronomers of the old school. The time was ripe, however, for the introduction of astrophysics. The beginnings had indeed been well made by Secchi, Huggins, and others, but their work constituted only a beginning which served to open the way for greater surveys and profounder details. The old astronomy of position and motion still occupied nearly all the time of the great observatories. Even the magnitudes of the stars were not fixed on any scientific basis, and the estimates of different astronomers sometimes varied by several magnitudes. In such a condition of the science the accumulation of great masses of data was necessary in various lines of investigation before any authoritative solution of most stellar problems would be possible. Of a certain great astronomer, who recently died, it was said that he put "theory first, practice in the second place." This should be exactly reversed in the case of Pickering. His plan, early arrived at, was to furnish to astronomers those facts, concerning the stars especially, which were most necessary to the progress of modern astronomy.

At first the range of his researches was sharply limited by the equipment of the observatory. There were two instruments of great power and perfection for that day, the 15-inch equatorial

refractor and the 8-inch meridian circle. The financial resources of the observatory were insufficient to keep these actively employed and to publish the results obtained. His first care was to secure additional funds sufficiently large to accomplish this, and also to extend his researches into new fields. His *Annual Report* for 1877 contained an appeal for money, and every succeeding report included direct or indirect appeals of the same nature. He early realized that to accomplish the great schemes which he had in mind he could not depend on himself alone but must command the services of many minds and hands. This involved a great increase of endowment. Little by little this was secured, until the observatory became sufficiently strong to carry on many widely distributed investigations. His own part in this increased income was large. In all, he gave to the observatory a sum more than equal to the salary which he drew as director during forty-two years.

The first and one of the greatest of his achievements was in stellar photometry. It was necessary to bring the stars and planets into an orderly sequence, which should be on some definite and "absolute" scale. He undertook to do this for all the brighter stars in the *Harvard Photometry*. For this, however, a special apparatus must be devised, and until this was ready, and indeed throughout his life, various photometric investigations were carried on by himself and others with the 15-inch telescope. A touch of romance was lent to the beginning of these labors by the measurement, in 1877, of the newly discovered tiny satellites of Mars. Their observation taxed the 15-inch telescope to the limit of its power. Other photometric investigations with the same instrument were the observation of the eclipses of Jupiter's satellites, the determination of the relative brightness of the components of double stars, and measures of the magnitudes of Algol and other variable stars and of planets and asteroids. The accuracy of many of these observations has never been surpassed for faint celestial objects.

These results were not attained with instruments already made. In attacking these problems it was necessary to invent suitable instruments and to have them constructed and attached to the

PROFESSOR E. C. PICKERING AT THE AGE OF 45 (1891)

telescope. A long series of photometers was devised by him, depending for the most part on the principle of polarization. A Nicol prism was attached to a double-image prism so that it could turn freely around its axis. The angle was read on a graduated circle. The star to be measured was compared with another star assumed to be standard. By the revolution of the Nicol the light of the two stars was equalized, and the resulting readings gave a determination of the relative light of the two objects. Accepting the law of Pogson that the ratio of one magnitude to the next should be expressed by the quantity whose logarithm is 0.4, Mr. Pickering reduced all the stars to a system which in general left the bright stars of about the same magnitudes as given in the *Almagest*. While there was no scientific necessity for such a basis and system, the wisdom of this deference to ancient usage is amply justified. Wedge photometers were also used in various forms.

While engaged in carrying on the researches with the 15-inch telescope, the meridian photometer was constructed for the measurement of all the brighter stars. The first meridian photometer had lenses of only one and a half inches' diameter. With this he measured the brightness of 4260 stars. The results proved to be of such value that a larger instrument was constructed with which the work was extended to all stars of the magnitude 7.5, and brighter, in the northern sky. A similar work carried out below the equator made the *Harvard Photometry* complete for the whole sky and established its use almost universally.

Many years ago Mr. Pickering saw the possibilities of a photographic photometry. Many experiments were tried soon after the beginning of astronomical photography. The difficulties proved to be very great, but these were to some extent eliminated and the work is now being systematically carried forward by his former associates at the Harvard Observatory. The investigation is also being pushed at other observatories and promises to add much to our knowledge of the stars. In the last few years Mr. Pickering was much interested in photo-visual magnitudes, which may be so derived with the use of filters and isochromatic plates as to agree closely with visual magnitudes.

An appropriation from the Bache Fund in 1885 made possible the beginning of photographic investigations. These were later greatly extended by the help of the Boyden Fund and the generous gifts of Mrs. Draper. The invention of the dry plate had opened the way for swift and sure development in nearly all lines of astronomical research. Something of romance was thus lost, indeed, but the gain in efficiency was tremendous. The laborious charting of a field of stars could be done in an hour, the resulting photograph showing more stars than the eye could see with a telescope of equal size. For certain purposes the plates themselves are sufficient without reduction. Professor Pickering early grasped the idea that a large collection of such photographs, made through a series of years, would have immense value in answering questions which would constantly arise. That is, he planned to collect a history of the stars during his life, and to make this record as complete as possible. This scheme was made to cover the whole sky by a station in the Southern Hemisphere. Various kinds of photographs were undertaken, especially charts and spectra of stars obtained with the objective prism. They were made with instruments of widely different powers. As extremes may be mentioned a wide-angle ½-inch Ross-Zeiss lens covering a field of about 60° square, so that the entire sky available at any one time and place could be covered in a single night, and the 24-inch Bruce doublet. With an exposure of one hour the former of these showed stars to about the ninth magnitude, the latter to the seventeenth magnitude. The extended discoveries of novae, asteroids, variable stars, and other interesting celestial objects from this collection of photographs are ample proof of its value. A series of plates of four hours' exposure with the 24-inch Bruce was proposed and a considerable number of excellent photographs were made at Arequipa from the South Pole northward. Such a series, if it could be completed for the whole sky, would contain a hundred million stars, and from it might be derived definitive lists of clusters and nebulae for the determination of the laws of their distribution, distance, and motion. The scheme would require a long time for its completion with a single telescope, and meanwhile the "selected areas" of Kapteyn and Pickering's own "standard regions" made this complete plan less necessary.

The study of stellar spectra, carried on by several observers under Mr. Pickering's direction, constitutes one of the observatory's greatest achievements. The practical completion of the *Henry Draper Catalogue*, in which his interest was intense, even in the last days of his life, forms a fitting close to his career. To estimate its importance, one needs to consider how small was our knowledge of the nature of the stars when he began to photograph them with the objective-prism, and how intimately the Harvard classification has entered into relation with nearly all lines of astronomical research. For detailed study of bright stars and for the determination of motion in the line of sight, the objective-prism does not compete well with the slit-spectrograph, but for a *Durchmusterung* of the spectra of two hundred thousand stars no other method was possible.

Aside from the classification of the spectra, the objective-prism plates yielded enough in the way of by-products to justify Mr. Pickering's enthusiasm: several novae, hundreds of new variable stars, and long lists of peculiar stars of special interest. These results occupy much space in the *Annals*. The Harvard classification, though not final, is now universally accepted as the best available system and has received international sanction. Nothing pleased Mr. Pickering more than to know that the results obtained at the observatory were those most needed by astronomers in their investigations. Certainly no better example could be found of a recognized astronomical need than the classification of spectra to be furnished in the nine volumes of the *Henry Draper Catalogue*. The publication of this work has been awaited with unusual interest by astronomers. Indeed, in many cases it has not been awaited, since the spectra of nearly forty thousand stars have been furnished by special request in advance of publication.

During many years Professor Pickering searched for efficient means by which photographs of stars made with the objective-prism might be used for the determination of motion in the line-of-sight. Several methods were proposed. One of these was to turn the prism 180° between two exposures on the same star on a single plate. Another and perhaps more hopeful method was by the use of an absorbing medium placed in the path of the rays. Of all

the substances tried, neodymium, suggested by R. W. Wood, gives the best absorption lines for the purpose. Though not yet carried out to a successful issue, the results thus far obtained indicate that even if precise individual determinations are not possible, mean values of great importance may be hoped for where large numbers of stars are under consideration.

This conception of a vast collection of photographs of the stars, destined in time to give a history of the sky, was unique. Its execution was carried out with enthusiasm and success. To some these half-examined plates, made in many cases for the purpose only of securing as complete a record as possible, appeared unnecessary and even excited ridicule. This seems absurd now that their value has been so fully demonstrated. Hardly a nova or new variable star has been discovered for many years whose history could not in some degree be traced upon these photographs. Eros furnished an early example of their value. Discovered in 1898, its history was traced backward on the Harvard plates to the interesting opposition of 1894, thus promptly furnishing material for a precise determination of its orbit. This collection in some sense symbolizes a large part of Mr. Pickering's achievements. From it have been derived all the studies of the spectrum, all the photographic photometry, much of the work on variable stars, and discoveries in many lines. It still exists, its possibilities by no means exhausted, its value in many ways increasing as the years go by. Of course, photographic plates are not immortal, and in time their films may decay, but meanwhile they may assist in solving many problems.

One fascinating possibility still needing study on these plates is that of spectral parallaxes. The direct determination of stellar parallax is perhaps the most difficult problem in astronomy and has been impossible for the more distant stars. By the work of Adams and others it now appears that the absolute magnitude of a star can be determined from the varying intensities of the lines of certain spectral types, the relation of which to the apparent magnitude gives the distance.

Within this great collection of stellar photographs, therefore, by means of such problems as these, there still remains for Professor

Pickering the possibility of an immortality of scientific labor, more unique and worthy than ordinary fame.

It is possible that Mr. Pickering's best work was in photometric and spectroscopic lines, but he was active in many other fields. The study of variable stars has been a marked feature of the observatory under his administration. When he began his work, about 200 variable stars were known. At the time of his death 3435 variables had been discovered at the Harvard Observatory. He published in 1880 a classification of variable stars which is the accepted notation at the present time. He soon began to encourage their observation on a scale hitherto unknown. This was possible not only through the increasing resources of the observatory but also by the assistance of amateurs. This is a field wherein the amateur can make systematic observations of real value. When the American Association of Variable Star Observers was formed, he gave the members the assistance which they needed. The spirit in which this aid was given and received is well shown in the regard and affection in which he was held by the members of this Association. At their meeting in 1918 they presented him with a beautiful gift, when their president made the following reference to him: "He has assisted us in everything that we have undertaken and has carefully watched our progress along every step of the way, and the manner of his so doing has been that of the Big Brother."

The astronomy of position has not been neglected at any time in the history of the Harvard Observatory. Two zones of the *Astronomische Gesellschaft*, those from $+49°55'$ to $+55°10'$, and from $-9°50'$ to $-14°10'$, have been observed with the 8-inch meridian circle. This work has occupied the time of one professor and several assistants during the last half-century, has cost approximately $200,000, and fills a dozen volumes of the *Annals*.

When Mr. Pickering came to the observatory, only a dozen volumes of the *Annals* had been published or were ready for printing. At the time of his death nearly a hundred of these quarto volumes had been issued or were practically ready for the printer. Many of these, indeed, were chiefly the work of others, and supervised and edited by him. On the other hand, an enormous amount was his own. His interest also in the work of others

seemed as intense as in his own. His desire was to see results. If he was fond of quantity, the painstaking care with which he examined and re-examined all that he did or supervised is evidence that quantity was not sought at the expense of quality. Loyalty to his predecessors in office was one of his marked characteristics. He devoted much time and badly needed financial resources during the early years of his directorship toward completing and publishing their unfinished work.

One of the most cherished objects of Pickering's life was to secure an international fund for the benefit of astronomers of all nations. He wished to be able to assist the special man in carrying out his ideas. Such a fund would form the most fitting memorial to his name. Similar in scope was his plan for an international southern telescope, which would be devoted to the needs of astronomers everywhere.

Mr. Pickering loved to discuss but refused to dispute. He believed in the "inanity of rivalry, the pettiness of jealousy, and the joyfulness of association for the good of mankind." Over his youth and early manhood had hung that curse of New England, tuberculosis, but through all his life he worked with that tremendous enthusiasm without which, it is said, nothing great was ever accomplished. Under the liberal faith of the modern Bostonian he had a puritan conscience, which impelled him to the accomplishment of certain duties, perhaps at times against the advice of his judgment.

He caught the true spirit of the age, but was ever a little in advance of it. He had the rare gift of knowing his own powers and of making the most of himself. He did not waste time in trying the impossible. What he attempted he performed. In the surveys of the sky which he carried out he was another Herschel. Theoretical reasoning not based on well-established data had little attraction for him. He recognized that the best service he could render to astronomy was the accumulation of facts. To this end he massed all the forces he could command and instituted great pieces of research, sometimes a vast routine, that in the end a sufficient basis should be furnished for a solution of stellar problems. Not a mathematician, he was yet a master of mathematics.

His practical nature led him to adopt graphical, in place of analytical, methods, whenever nothing was thereby lost in accuracy.

Mr. Pickering loved appreciation but was not swerved from his course by its presence or absence. His persistence in what he believed was right was only equaled by the readiness with which he accepted new ideas. Until the very last of his life he had an alert, unprejudiced mind, and was ready to reject the plans of a lifetime if convinced of their error. He was prompt to give advice whenever it was asked; perhaps, in some cases, where it was not desired. Always eager for friendly criticism himself, he could not believe that anyone would wish to go on in a doubtful course when it was possible to find a better one.

He was a natural leader, but he was an indefatigable worker as well. He worked for the love of it, carrying on observations for several hours each clear night, in addition to his arduous duties as director. Of the two million observations of light concerned in the visual *Harvard Photometry*, more than half were made by him.

Mr. Pickering's patriotism was intense. His desire to assist his country in time of war was shown by several valuable suggestions to the government. He heartily condemned all practices which he regarded as contrary to civilized warfare, but retained throughout a high regard for old astronomical friends among the enemy nations.

As remarkable as were Mr. Pickering's scientific accomplishments, equally rare were his personal qualifications. For men and women he had an equal charm. His grace of manner and conversation was the constant wonder of all who knew him intimately. Over all, old and young, wise and witty or ignorant and stupid, who seemed to have any claim upon him, he threw the glamor of his personality. To astronomers especially he was ready with unlimited service. Many astronomers remember him as an ideal host. The entertainment of the American Astronomical Society in 1918 gave him special pleasure.

The following quotations from men of high scientific rank show the appreciation of his attitude: "A great, kind, unselfish man has gone." "The loss to astronomy and to science generally is inestimable, but all his friends will mourn him for his lovable personal

qualities." "His wonderful energy and enthusiasm, his alertness, his unvarying courtesy, his wide vision and generous heart, make his passing a keen personal loss even to those of us who knew him only slightly. For a number of years I have thought of him as the Dean of American Science." The memory of association with him is cherished by those who knew him intimately as one of life's choicest gifts.

Professor Pickering received nearly all the honors which the world has to bestow on a scientific man. These he valued highly as an expression of the appreciation in which his work was held.

Unknown probably to most, Mr. Pickering had strong poetic and religious elements in his nature. On his tomb he asked to have engraved the one word, "Thanatopsis." To him this word of doubtful coinage meant perhaps that view of death which after all might be a new view of life. At any rate, to his eager and always open mind, the words of Bryant appealed with special power:

> So live, that when thy summons comes to join
> The innumerable caravan, which moves
> To that mysterious realm, where each shall take
> His chamber in the silent halls of death,
> Thou go, not like the quarry-slave at night,
> Scourged to his dungeon, but, sustained and soothed
> By an unfaltering trust, approach thy grave,
> Like one who wraps the drapery of his couch
> About him, and lies down to pleasant dreams.

Harvard College Observatory
August 1919

PLATE IX

a—

Fig. 1

b—

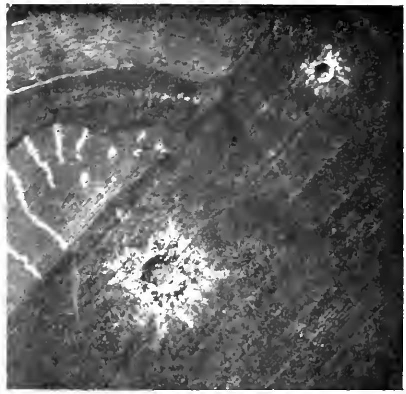


Fig. 3

Fig. 4

Fig. 2

d

Fig. 5

Fig. 1. Reproduced from W. H. Pickering's *The Moon*

Fig. 2. From a Photograph of the Moon at 18 days, Yerkes Observatory

a. Copernicus *c.* Plato

b. Archimedes *d.* Theophilus and Cyrillus

SOME LARGE-SCALE EXPERIMENTS IMITATING THE CRATERS OF THE MOON

By HERBERT E. IVES

The origin of the characteristic crater-like features of the moon's surface has been the subject of frequent discussion, and a solution satisfying all who have studied the problem is far from being reached. The explanation readiest to hand, that the rings, pits, and peaks are the result of volcanic action, does not appear to be adequate when closely studied. While superficially similar in appearance to terrestrial volcanoes, the lunar "craters" exhibit significant differences of structure from these. The crater floors are lower than the surrounding country instead of higher, as they are in most terrestrial volcanoes, the central peak is often missing, and the amount of material piled up in the ring mountain is less than would be deposited there by the volcanoes we know. It has, however, been pointed out by W. H. Pickering that the volcanoes of Hawaii are quite similar to the lunar configurations.

Opposed to the volcanic theory is the meteoric or impact hypothesis. This assumes that the lunar craters are the result of the impact of meteors. Objections that have been raised to this theory are the almost uniformly circular shape of the craters, which offer difficulty on the ground that many meteors would strike at a glancing angle, the elevated central peak, and the enormous number of the impacts represented, while the earth has apparently been immune. Reference will be made more in detail to these and other objections in the discussion of the subject-matter of this paper, which supports the meteoric as opposed to the volcanic hypothesis. Excellent reviews of the theories of the cause of the moon's surface structure will be found in W. H. Pickering's *The Moon* and in Moulton's *Introduction to Astronomy*.

Reference to Plate IX (Figs. 1 and 2) will bring to mind the characteristic features of the moon's surface. Copernicus, the large crater in the upper part of Fig. 1, exhibits the circular ring, the level

floor, and the central small elevations or peaks, all characteristic details of the typical lunar crater. The radiating streaks or "rays" are visible in the photograph, though not shown as well as under some other conditions of lighting, and are never so extensive as the rays from Tycho, which extend over a large portion of the Southern Hemisphere. To the left and below Copernicus are smaller characteristic craters, both with and without the central peak. To the left are the large ringed plains Archimedes and Plato, which have no central peaks at all, and whose floors are practically flat. All gradations between the extremes represented by the small craters with sharp central cones and the ringed plains are to be found. Figure 2 is inserted chiefly to show one of the most striking instances of overlapping craters, Theophilus and Cyrillus, a phenomenon which must be readily covered by any suggested explanation of the lunar configurations.

Various attempts have been made in the past to duplicate the craters of the moon by laboratory experiment. Some of these are reproduced in W. H. Pickering's *The Moon*. Reference may be made to that work for detailed explanation and discussion of these experiments, which show "impact" craters, made by shooting clay pellets at a clay surface, and other pictures of effects obtained in cooling slag and paraffin, and therefore belong more properly to the "volcanic" theories. A point of considerable importance, shown in his first picture (his Plate A), and also found in experiments made by shooting lead bullets at a lead surface, is the occurrence of the central elevation or peak, formed apparently by a species of rebound. This answers one of the earlier objections which appeared rather difficult to meet on the meteoric theory.

All these laboratory experiments are on a very small scale, compared with the ten, twenty, or even fifty miles which represent the diameters of many lunar craters. The present paper describes some experiments on a scale which, while still small compared with the size of the feature it is desired to duplicate, is nevertheless enormously greater than the laboratory scale of those mentioned above. The experiments were not made with any reference to lunar theories but were incidental to the development of munitions

PLATE X

Photograph from Airplane of Craters made by Bombs Dropped from Airplanes at Langley Field

of war. They are therefore a scientific by-product of the Great War. Specifically the craters which are illustrated and discussed below are bomb craters made by the explosion of experimental bombs, dropped from airplanes at Langley Field, Virginia. A photograph of the bombing target and a number of craters is shown in Plate X. The resemblance to the pitted surface of the moon will strike the most casual observer.

With special reference to the scale of the experiments, we mention that twenty men with joined hands just span one of the craters. This particular crater was caused by the explosion of several hundred pounds of T.N.T. Also in connection with the scale on which the experiments were performed belongs a description of the observing platform. Obviously craters of this size would not be easily photographed from any position on the ground; the laboratory bench used was in fact an airplane, and Figs. 3, 4, 5, and Plate X are aerial photographs.

Attention may be called in some detail to the points of resemblance between these bomb craters and the lunar configurations. Figure 3 shows two types of crater. The smaller one, in the upper right-hand corner, exhibits all the features of a lunar crater of medium size. It has the circular surrounding wall, the central peak, and a few short radiating streaks. The larger crater resembles more nearly the ringed plains. Its floor is flat, due to the seeping in of water. This of course would not happen on the moon, with its present waterless condition, but there is no objection to the suggestion that the larger ringed plains of the moon date from a period when water, carrying sediment, could have leveled off the floor of the craters in just this way.

Figure 4 shows, in addition to a large and a small crater of types already discussed, a very striking example, in the small crater at the top, of the production of a central peak. This peak is indeed much more pronounced than any on the moon, probably due to the earth in which the bomb fell being damp and softer than the materials of the moon. It is at any rate a most conclusive demonstration of the ability of a body (of the proper sort) striking a surface to produce an elevation.

Figure 5 shows, in the largest crater, a formation greatly resembling Copernicus, in its central peaks, circular wall, and radiating streaks. The resemblance, it may be remarked, was still more striking a few weeks before these pictures were taken, before the collection of water in the cavity. To the left in the same figure will be seen a pair of overlapping craters, similar to Theophilus and Cyrillus, easily explainable on an impact theory, but harder as a result of volcanic action. The smaller pits near "Copernicus" are practically identical in appearance to those near the real Copernicus.

These few words of description are sufficient, since the photographs largely speak for themselves. It is believed to be evident that they show very striking similarity between the craters produced by the explosion of bombs and the craters of the moon. What then is the significance of this similarity of appearance?

It may at first thought seem far-fetched to liken meteors to explosive bombs, which is the most direct application that can be made of these experimental results. But on further study this interpretation, far from being an obstacle to considering the experiments as pertinent, rather adds to their significance. It may first of all be pointed out that meteors striking the earth's atmosphere not only flash into incandescence, but do frequently burst with terrifying reports, spreading their fragments over a considerable territory. Most light on this point is furnished by some simple calculations, as follows: The velocity of meteors striking the earth's atmosphere varies between 16 and 64 km per second. Let us take a meteor traveling with the lower of these speeds, and assume that it strikes, not the earth's atmosphere, through which its velocity is slowly dissipated, but the surface of the moon, at which it would arrive with its full velocity. Calling the mass of the meteor m, its velocity v, its specific heat s, and the mechanical equivalent of heat J, we have the following equation for the temperature to which the meteor will be raised, assuming all the generated heat to remain in the meteor itself:

$$\tfrac{1}{2}\, mv^2 = ms(T-T_0)J,$$

where T_0 is its original temperature and T is the temperature to which it is raised. Putting in the above velocity, the value for J (41.8×10^6), assuming 0.2 for s, and zero for T_0, this equation gives for T the figure 150,000 degrees Centigrade! Even if we assume that nine-tenths of this heat is given up to the surroundings, we still have in the 15,000° C. a temperature amply sufficient to gasefy any known material, that is, *to produce an explosion*.

Thus our calculation leads to the conclusion that a meteor striking the moon, with even the lowest velocity at which these are observed, would become a very efficient bomb, and should therefore produce the kind of crater we can imitate on the earth only by filling our slowly moving military aerial bombs with explosive material. And not only does this explanation take care of the general appearance of the craters, but it affords an answer to the perplexing question presented by the almost uniformly circular shape of the lunar craters; for it is clear that the shape of the cavity has no reference to the angle at which the bomb strikes, but takes its form from the symmetrical explosive forces. Moreover, the available energy is so great that even if the meteor strikes at very great angles to the vertical the result will be an explosion. It has, however, been shown by G. K. Gilbert that the vast majority of meteors would strike at angles within 30° of the vertical.

Some of the objections which have been raised to the meteoric theory may be touched on in conclusion, considered more especially in reference to the suggestions of this paper. One is that the heat generated by the impact of the meteor would be so great as to melt the crater walls. Obviously this criticism errs only in not going far enough into the matter and finding that the generation of heat is so much beyond that necessary for the melting of rock as to put an entirely new face on the problem, leading, as we have seen, to the conception of the meteor as an explosive bomb. A second objection is that the earth should show similar effects of bombardment. We may note in passing that the Cañon Diablo, the most perfect imitation we have of a lunar crater, bears numerous evidences of having been caused by an explosion of other than subterranean origin. But the

most complete answer to this criticism is found by noting, first, that the earth is surrounded by an atmosphere which in previous ages must have been much denser than now and so would dissipate the energy of falling meteors, as indeed we see it doing now; and second, that the earth's surface has been undergoing the processes of upheaval and weathering for perhaps countless ages since the collision with the giant meteor swarms which permanently marked the dead and atmosphereless lunar surface.

Philadelphia
June 1919

MEASUREMENTS ON THE NEAR INFRA-RED ABSORPTION OF SOME DIATOMIC GASES

By ELMER S. IMES

The importance of the study of the near infra-red absorption bands of gases is being more clearly realized the farther this study is carried. This importance arises from two main considerations. The first is the information which the absorption in this region gives with regard to the structure and mechanics of the molecule. This includes, of course, the inferences as to atomic structure, which are not only possible, but become necessary if the molecular facts are to be explained. The second is that there is found here a new application and test for the quantum theory in that it is extended to the originally excluded region of the rotational energy of molecules.

SUMMARY OF THEORY AND PREVIOUS WORK

It was Drude[1] who first announced the theory that the infra-red absorption and emission bands of the majority of substances have their origin in the vibrations of electrically charged atoms and molecules, and not in the oscillations of the electrons within the atoms. The two widely separated absorption regions usually appearing in the infra-red spectra of gases were naturally assigned, the one in the far infra-red to molecular rotation and that in the near infra-red to atomic vibrations within the molecule. N. Bjerrum[2] pointed out, however, that in all probability the shorter-wave absorption was due to a combination of the two frequencies, that of rotation and that of vibration. This observation was based on Lord Rayleigh's[3] combination principle: viz., that an oscillator which emits and absorbs at a frequency ν_0 due to its oscillations alone would, when rotating about an axis perpendicular to its line of vibration with a frequency ν_r, emit and

[1] *Annalen der Physik* (4), **14**, 677, 1904.

[2] *Nernst Festschrift*, p. 90, 1912.

[3] *Philosophical Magazine* (5), **24**, 410, 1892.

absorb at the new frequencies $\nu_o+\nu_r$ and $\nu_o-\nu_r$. The assumption of a Maxwellian distribution of rotational velocities would require that the near infra-red band consist of two broad absorption areas having maxima at $\nu_o\pm\bar{\nu}_r$, where $\bar{\nu}_r$ is the most probable rotational frequency, and a sharp maximum for ν_o due to molecules whose rotation frequency was zero or in a plane perpendicular to the line of sight. W. Burmeister's[1] work in this region showed the two broad areas for most of the gases investigated, but gave no sign of the sharp line corresponding to ν_o, from which it was concluded that either the dispersion used was not great enough to show it, or, at least in the case of diatomic molecules, there might be no absorption for ν_o, i.e., no molecules having no rotation.

H. Rubens and H. von Wartenberg[2] had found in the far infra-red the bands for some of the gases investigated by Burmeister, and the values for $\bar{\nu}_r$ computed from Burmeister's doublet maxima agreed well with their directly obtained values.

A complication arose, however, in the discovery that these near infra-red absorption bands did not always present, even with the low dispersion available, continuous areas with a single maximum each for $\nu_o+\nu_r$ and $\nu_o-\nu_r$. Rubens'[3] work on the water-vapor band at 6 μ and even F. Paschen's[4] much earlier work on the same band showed many separate maxima in these areas. Finally Eva von Bahr[5] in her work on water-vapor and hydrochloric acid showed such marked discontinuity in these bands that an extension[6] of the theory of their origin, abandoning the assumption of a Maxwellian distribution of rotational velocities and introducing the quantum theory, was made necessary. As a matter of fact W. Nernst[7] had previous to this time arrived at the conclusion that the quantum theory must be applied to molecular rotation. This

[1] *Verhandlungen der deutschen physikalischen Gesellschaft*, **15**, 589, 1913.

[2] *Ibid.*, **13**, 706, 1911.

[3] *Sitzungsberichte Preussische Akademie*, p. 513, 1913.

[4] *Wiedemanns Annalen*, **52**, 215, 1894.

[5] *Verhandlungen der deutschen physikalischen Gesellschaft*, **15**, 710, 731, 1150, 1913.

[6] N. Bjerrum, *loc. cit.*; E. von Bahr, *loc. cit.*; A. Eucken, *Verhandlungen der deutschen physikalischen Gesellschaft*, **15**, 1159, 1913.

[7] *Zeitschrift für Elektrochemie*, **17**, 265, 1911.

conclusion was based on two observations: first, that molecular rotation causes radiation in the case of charged molecules; and, second, that even for infinitely thick layers of gas no shorter wave-lengths are emitted. Work was done on the specific heat of gases at low temperatures with a view to finding this quantum effect, and various explanations of the results arrived at were attempted. Perhaps the best of these was that of P. Ehrenfest[1] who proposed the equation

$$\tfrac{1}{2}I(2\pi\nu_r)^2 = n\frac{h\nu_r}{2}\ ,$$

(where I is the moment of inertia of the molecule, ν_r the rotation frequency, n a whole number, and h Planck's constant), as representing the energy of rotation of a diatomic molecule and as a starting-point in the desired explanation. This equation differs from that proposed by Bjerrum[2] by the factor 2 in the denominator of the second member of the equation, since Ehrenfest concluded that the rotation quantum of energy is $h\nu_r/2$ instead of $h\nu_r$ as assumed by Bjerrum. E. C. Kemble[3] gives a derivation, based on the classical statistical mechanics, which leads to the equation

$$I = \frac{RT}{4\pi^2\nu_r^2 N}$$

as giving the moment of inertia of a diatomic molecule in terms of the rotation frequency, $\bar{\nu}_r$, obtained from the Bjerrum doublet. He shows that this checks with Ehrenfest's assumption rather than with that of Bjerrum.

It is unfortunate that only the order of magnitude of numerical results computed from such formulae is possible of verification. This, however, in no sense minimizes the importance or the desirability of further and more exact work on the infra-red absorption of diatomic gases. There can be little hope of interpreting properly the results already obtained in the cases of water-vapor and CO_2, to say nothing of more complex molecules yet to be studied, until by

[1] *Verhandlungen der deutschen physikalischen Gesellschaft*, **15**, 451, 1913.

[2] *Loc. cit.*

[3] *Physical Review* (2), **8**, 689, 1916.

systematic study of these simpler diatomic molecules the material for generalization is gathered.

In the hope of adding to the amount of such material available for theoretical work the writer undertook the study of the absorption of HCl in the near infra-red, with greater dispersion than had previously been available. Burmeister's[1] original curve for this gas showed only the doublet consisting of broad areas having maxima at $3.4\ \mu$ and $3.55\ \mu$. Von Bahr[2] succeeded in resolving these into twelve separate maxima, of which five were on the long-wave side of the center, thus making only five pairs available for measurement. Finally J. B. Brinsmade and E. C. Kemble[3] have published, since this work was begun, a curve for HCl showing eight maxima on the long-wave side of the center of the band at $3.46\ \mu$, as well as a partially resolved curve for the "harmonic" at $1.76\ \mu$.

It has been possible in the present work to extend both of these, there being twelve pairs of maxima given for the band at $3.46\ \mu$ and the band at $1.76\ \mu$ being resolved over eight pairs of maxima. In addition to these curves for HCl the writer has also obtained curves for the HBr band at $3.91\ \mu$ and the HF band at $2.52\ \mu$. For the former of these Burmeister has published a doublet having maxima at $3.84\ \mu$ and $4.01\ \mu$. For the latter the writer has found no published work.

APPARATUS

The apparatus designed and used by W. W. Sleator[4] in his work on water-vapor was admirably adapted to this present work and was fortunately available.

The galvanometer. The galvanometer is of the Paschen type, a modification of the Thomson four-coil astatic instrument, and was built in the department's shop. The resistance of this galvanometer as used in the present work is approximately 2 ohms and

[1] *Loc. cit.*

[2] *Philosophical Magazine* (6), **28**, 71, 1914; *Verhandlungen der deutschen physikalischen Gesellschaft*, **15**, 1150, 1913.

[3] *Proceedings of National Academy of Sciences*, **3**, 420, 1917.

[4] *Astrophysical Journal*, **48**, 125, 1918.

with the scale 200 cm distant and the period adjusted to 6 seconds the sensitivity is about 2.2×10^{-10} amp. per mm deflection.

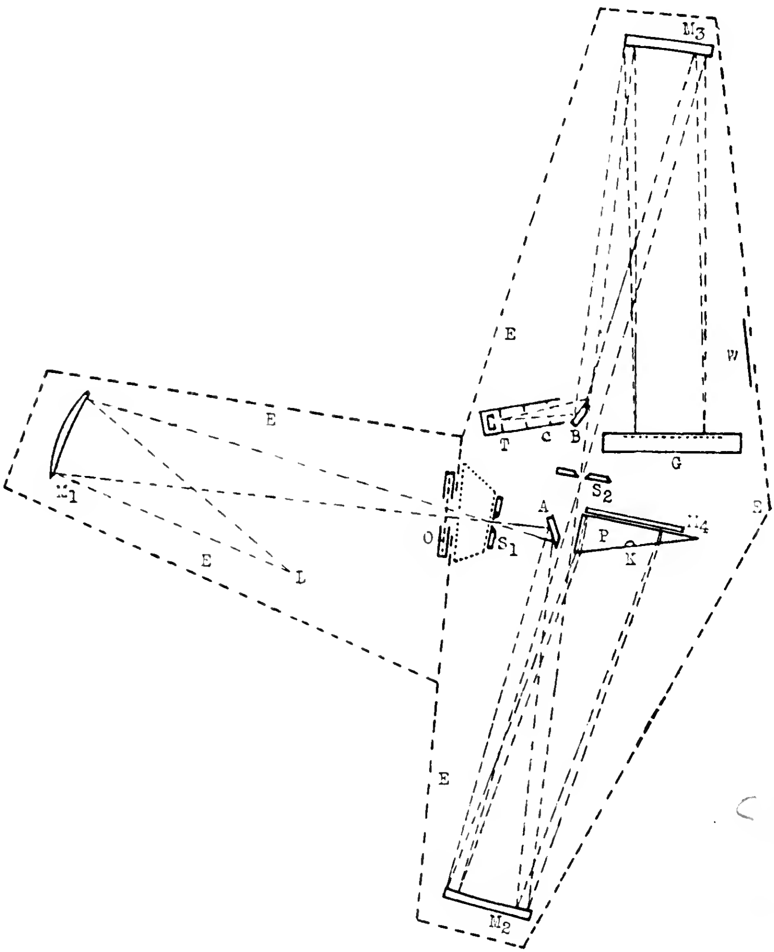


FIG. 1.—The spectrometer

L, Nernst glower; S_1, S_2, slits; M_1, 10 cm mirror, f = 20 cm; P, salt prism; M_2, M_3, 10 cm mirrors, f = 50 cm; M_4, A, B, plane mirrors; G, grating; C, case for T, the thermopile; W, window in box E; O, shutter. The path of the light is $LM_1S_1AM_2PM_4PM_2S_2M_3GM_3BT$. A spectrum appears at S_2. P and M_4 rotate together about K, so that any region of the spectrum may be isolated for the grating, and the overlapping of spectra is avoided. (From paper by W. W. Sleator, *Astrophysical Journal*, **48**, 127, 1918.)

The spectrometer.—Figure 1 shows the arrangement of the spectrometer, which consists really of two spectrometers, both of the mirror type. The first is a prism spectrometer, $S_1AM_2PM_4M_2$, which presents a portion of its spectrum to the slit, S_2, of the grating spectrometer, $S_2M_3GM_3BT$. The prism is of rock salt and has a refracting angle of about 18°. Its face is about 12×14 cm. Three gratings were used in the course of the work—a brass grating by Hilger, a 7500-line grating, and a 20,000-line grating, both on speculum metal by Anderson, of Johns Hopkins University. With the brass grating the spectrometer constant is 211,476 A, while with the 7500-line grating it is 67,693 A and with the 20,000-line grating it is 25,375 A.

The double-spectrometer method has proved highly successful in bringing the desired high dispersion of the grating to bear on the infra-red problem. Obviously the grating must not be called upon to analyze a spectral range containing wave-lengths which are integral multiples of each other, if the results are to be interpreted. This is especially true where, as in the present case, photographs are impossible. In the double-spectrometer method a very limited portion of the prism spectrum is thrown upon the slit of the grating spectrometer. If this slit is narrow enough to allow no multiple wave-lengths to pass, the problem of overlapping is solved.

The theoretical value of the resolving power of the grating spectrometer is given in the equation

$$d\lambda/\lambda = 1/Nn$$

(where $d\lambda$ is the wave-length separation of two lines which may just be seen as separate lines, N the number of grating lines used, and n the order of the spectrum observed).

For the 7500-line grating at the center of the HCl band at $3.46\,\mu$, the beam covering 7.5 cm of grating surface and the observations being taken in the first-order spectrum, this equation gives

$$d\lambda = \frac{3.46}{22500 \times 1} = 0.00016\,\mu = 1.6\text{ A}.$$

But this is based upon the assumption of infinitely narrow slits, which is not at all the case in this work, both slits of the grating spectrometer being 0.5 mm wide. A better idea of the resolution obtained may be gained from the following consideration. The width of the thermopile slit corresponds to 1.7 minutes' angular displacement of the grating. This is $d\theta$ in the equation

$$d\lambda = k \cos \theta d\theta$$

derived from the spectrometer equation $\lambda = k \sin \theta$. Substituting this value of $d\theta$ and that of $\cos \theta$ for $\lambda = 3.46\ \mu$, it is found that

$$d\lambda = 29.1\ \text{A}.$$

In other words the thermopile slit includes 29 A of the spectrum formed by this grating at 3.46 μ. Kemble worked with approximately 70 A and von Bahr with 100 A for their best results.

The thermopile.—The "eyepiece" of the spectrometer consists of a thermopile and the galvanometer. The thermopile is a linear bismuth-silver group made by Coblentz. It has ten junctions in its center line and a resistance of about 2 ohms. It is mounted behind a 0.5 mm slit at the focus of the mirror M_3, the beam from M_3 being reflected by the plane mirror B at about 90°, so that the thermopile will not obstruct the beam from the slit S_2 to M_3. The mounting is so designed as to keep the thermal junctions at the same temperature, except when radiation is absorbed by the row exposed to the slit.

The source.—A Nernst glower was used as the source of energy. The work was started with the Nernst lamp in practically its original form and driven by D.C. from a storage battery for the sake of steadiness. Toward the end of the work, however, the original mounting of the filament had given out and it was impossible to obtain new parts for its repair. Accordingly the expedient of simply mounting the filament in a suitable alundum cement mold and using a bank of tungsten lamps as ballast was resorted to. In order to avoid a troublesome amount of polarization of the filaments A.C. was used at this stage. As all of the observations were taken between 1:00 A.M. and 5:00 A.M., when the mechanical and magnetic disturbances to which the galvanometer responds

so decidedly were at a minimum, the fluctuations of voltage on the A.C. lines were practically negligible. Furthermore, current was drawn almost directly from a transformer on the 2300-volt lines, having no other load, and the glower circuit was so arranged that the fall of potential across the filament was only a fraction of the total fall in the circuit, thus making steadiness quite assured.

The absorption chamber.—The absorption chamber was of brass, 15 cm long and about 8 cm in diameter. For the greater part of the work thin mica plates were used as windows. They were cemented over the ends of the chamber, and another pair of plates, cut from the same sheets, was so mounted as to be in the beam when the chamber was out. The thickness of these plates was of the order of 0.03 mm. For work in the region of $2.5\ \mu$ and at shorter wave-lengths certain specimens of glass plates were available as windows.

Although the total length of the air path of the beam of light is more than 5 meters, the greater part of it is inside the box that contains the spectrometer, and is dried by vessels of calcium chloride, while the length of the absorption chamber is quite 20 per cent of that of the undried air path outside the box. Accordingly, for the part of the work done in regions of strong water-vapor absorption—notably at $2.6\ \mu$—it was thought best to provide a compensating chamber similar in dimensions to the absorption chamber and carefully filled with dried air that had also been freed from CO_2 by passing through KOH solution.

METHODS AND RESULTS

Hydrochloric acid.—The first work was done with HCl in the region of $3.5\ \mu$. The gas was generated by dropping H_2SO_4 on $CaCl_2$ and dried by being passed through concentrated H_2SO_4. Both chemicals were the "analyzed" product of the Baker and Adamson Company. A slow stream of the gas was kept passing through the absorption chamber and disposed of by absorption in water. Gum rubber tubing was used, since the ordinary vulcanized tubing was attacked by the HCl giving rise to enough H_2S to be distinctly perceptible by its odor. The unequivocal nature of the curves obtained is taken as sufficient evidence of the

purity of the HCl thus generated, so far as the presence of any substances having overlapping absorption is concerned.

For each point of the curve—one minute of arc apart—six to eight readings of the galvanometer deflection were taken through the absorbing gas and a like number through only the compensating plates, the deflections being obtained by opening or closing the shutter in front of the slit S_1. These readings were not taken consecutively, but alternately in pairs—two with the chamber "in," then two with it "out," and so on. The percentage of absorption was computed from the two averages, the direct ratio between the "in" and "out" averages being translated into a per cent transmission, and the difference between this and 100 per cent being the per cent absorption plotted against the grating setting for that point of the curve. This method has obvious advantages over a method used by some observers who have gone over the entire band first with no absorbing medium (sometimes vacuum) in the chamber, and then with the chamber filled with the substance under observation. Any failure to reproduce a spectrometer setting must result in a corresponding error in the value so obtained. This error would be especially large in the case of a substance having, for example, the very sharp absorption maxima shown by HBr (Fig. 6).

The extreme sensitivity of the galvanometer used in this work to the slightest mechanical or magnetic disturbance is a source of regret. Many times it was not possible to obtain consistent deflections even during the favorable hours, between midnight and dawn, chosen for observation. No claim for extreme accuracy is made for the percentages obtained, but it is not likely that there is any uncertainty with regard to the location of the maxima, which have all been repeated—some as many as four or five times and never with a greater disagreement than two or three angstroms. The location of these maxima, and not their magnitude, has been the problem.

Figures 2 and 3 give the curves obtained for the HCl band at $3.46\ \mu$ with two different gratings. Table II gives the measurements and computations made from the better curve. Figure 5 gives the curve obtained for the so-called harmonic of HCl at

1.76 μ with the 20,000-line grating and Table III gives the measurements and computations from this curve.

These curves as well as those for HBr and HF (Figs. 6 and 7) have been idealized to the extent of omitting small irregularities

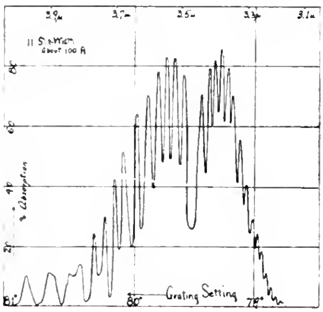


FIG. 2.—The HCl band at 3.46 μ, mapped with brass grating.

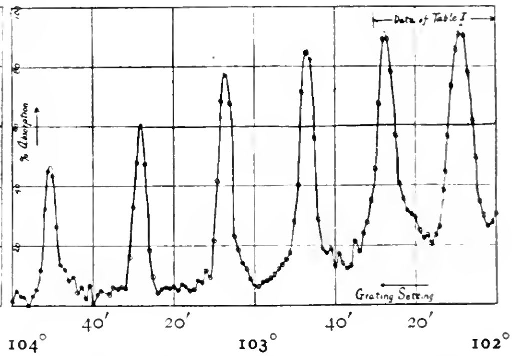


FIG. 4.—A portion of the HCl band at 3.46 μ, plotted from a single set of data.

The curves given in Figs. 2, 3, 5, 6, and 7 are plotted from averages of such sets of data and have omitted the slight irregularities appearing in the minima above, since they are not significant.

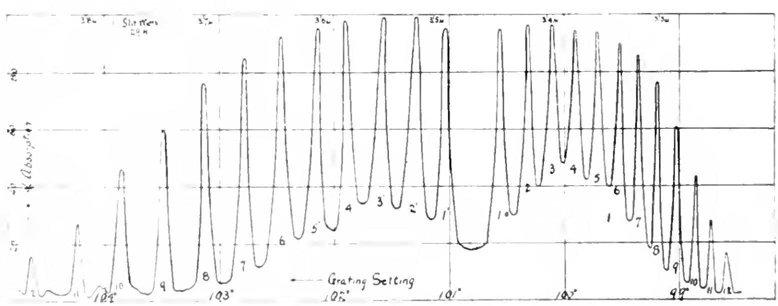


FIG. 3.—The HCl band at 3.46 μ, mapped with 7500-line grating. HCl at atmospheric pressure.

and averaging all the percentages obtained for each point. Table I gives specimen data and Fig. 4 shows the actual curve plotted from this data for a small portion of the HCl band at 3.46 μ.

Hydrobromic acid. HBr was obtained in a satisfactorily pure state by the direct union of hydrogen and bromine. The hydrogen

was obtained by the electrolysis of NaOH solution and was carefully dried. Chemically pure bromine was washed in a KOH solution and twice distilled at as low a temperature as possible. The hydrogen was then bubbled through the bromine and the mixture passed through a combustion tube in which there was a platinum coil heated to a bright red by an electric current. The product was delivered through a long vertical glass tube surrounded by a mixture of snow and calcium chloride to condense any uncombined bromine as well as any remaining traces of water-vapor. Burmeister[1] speaks of drying HBr by passing it over P_2O_5. This process would give rise to volatile compounds of phosphorus and bromine, and would no doubt account for some of the difficulty he had in obtaining satisfactorily pure HBr.

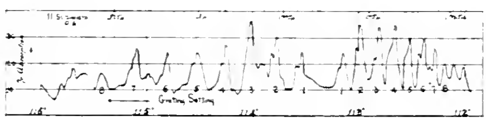


FIG. 5.—The HCl band at 1.76 μ, mapped with 20,000-line grating. HCl at atmospheric pressure.

Because of the rather elaborate nature of the apparatus required for the generation of HBr no attempt was made to keep a stream of the gas passing through the absorption chamber. It was thought sufficient to start each new series of observations with a freshly generated supply, the chamber having been washed out by drawing dry air through it by means of an aspirator for half an hour before it was filled with the gas. It was difficult to determine when the pressure of the gas in the chamber was that of the atmosphere; i.e., when all the air was displaced. Furthermore, it was aimed to generate the HBr so that an excess of hydrogen would be present and no simple means of determining constancy in the amount of this excess was at hand. The result is that the relative intensities of the various absorption maxima in the curve obtained for HBr cannot be taken as significant unless they were obtained in the same series of observations. This is not important, as

[1] *Verhandlungen der deutschen physikalischen Gesellschaft*, **15**, 596, 1913.

TABLE I

TYPICAL DATA

HCl Band, 7500-line grating, June 18, 1917, 12:45 A.M.
Glower current = 0.75 amp.
Galvanometer period = 7 sec.
Grating temperature = 20°.5 C. Grating zero = 70°0′0″

Circle	In	Average	Out	Average	Percentage Trans.	Percentage Abs.
102°0′	7.5-7.4-7.8-7.7-7.6-7.5	7.6	11.5-11-10.5-10.8-10.9-11	10.95	69.4	30.6
1′	7.4-7.8-7.7-8.0-8.0-7.6	7.75	10.5-10.7-10.8-10.7-10.9-10.7	10.7	72.3	27.7
2′	8.0-7.8-8.0-7.9-8.0-7.8	7.9	10.5-10.7-11-10.5-10.8-10.7	10.7	73.7	26.3
3′	7.0-7.4-7.6-7.5-7.4-7.4	7.4	10.6-10.4-11-10.6-10.5-10.6	10.6	69.8	30.2
4′	7.0-6.5-6.9-6.9-7.0-7.0	6.9	10.5-10.8-10.5-10.6-10.4-10.5	10.55	65.4	34.6
5′	5.0-5.5-5.0-5.5-5.3-5.5	5.3	10.3-10.2-10.5-10.6-10.3-10.5	10.4	50.9	49.1
6′	4.0-4.0-4.2-4.0-4.1-4.0	4.05	10.5-11-10.2-10.9-10.6-10.9	10.65	38.0	62.0
7′	2.5-2.4-2.0-2.4-2.3-2.3	2.3	10.4-10.5-10.4-10.4-10.5-10.5	10.45	22.1	77.9
8′	About 1.0	1.0(?)	10.6-10.5-10.7-10.7-10.6-10.5	10.6	9.4	90.6(?)
9′	About 1.0	1.0(?)	10.3-10.3-10.8-10.2-10.3-10.5	10.4	9.6	90.4(?)
10′	1.5-1.5-1.4-1.4-1.5-1.4	1.45	10-11-9.6-10.5-10.2-10.1	10.2	14.2	85.8
11′	2.8-2.5-2.8-2.7-3.0-2.7	2.75	10.3-10.7-10.4-10.5-10.6-10.4	10.5	26.2	73.8
12′	4.8-4.5-4.7-4.6-4.6-4.7	4.65	10.8-11.2-10.2-10.8-10.7-10.8	10.75	43.2	56.8
13′	7.0-6.5-6.0-6.5-6.0-7.0	6.5	10.5-10.5-11-10.5-10.7-10.5	10.6	61.3	38.7
14′	7.0-7.5-7.0-7.5-7.0-7.5	7.25	10.0-11-10.5-10.4-10.5-10.5	10.5	69.0	31.0
15′	7.8-7.9-7.5-8.5-8.0-8.0	7.95	10.2-10.8-10-10.7-10.3-10.5	10.4	76.4	23.6
16′*	11.5-11.2-11.7-11.4-11.5-11.4	11.45	14.2-15-15-14-14.5-14.6	14.55	79.2	20.8
17′	11-11-11-10.7-11-10.8	10.9	14-14.5-14-14.2-14.1-14.3	14.2	76.7	23.3
18′	11-10.5-11.5-11-11	11.0	14-14.2-14-14.7-14-14.3	14.2	77.5	22.5
19′	11-10.5-11-11-11-10.9	10.9	14.7-14-15-14.2-14-15	14.5	75.2	24.8
20′	10.5-10.5-10.5-10.2-10.3-10.5	10.4	14.6-14.8-14.7-14.9-14.7-14.8	14.75	70.5	29.5
21′	10-10-10-10.5-10-10	10.1	14.5-14.5-14.6-14.6-14.5-14.6	14.55	69.4	30.6
22′	10-10-10.2-10-9.8-10	10.0	14-14.9-14.6-15-14.5-14.7	14.6	68.4	31.6
23′	9.5-9.5-9-9.5-9.5-9.5	9.4	14.9-14.2-15-14.3-14.4-14.8	14.6	64.3	35.7
24′	8.5-8.0-9.0-8.5-8.0-9.0	8.5	14-15-14.5-14-14.2-14.7	14.4	59.0	41.0
25′	7.0-5.0-6.0-6.0-5.5-6.5	6.0	13.8-14-14-14.2-14.1-13.9	14.0	42.8	57.2
26′	3.0-3.0-2.8-3.0-2.9-3.0	2.95	13.8-13.8-13.7-14-13.7-13.8	13.8	21.4	78.6
27′	1.5-1.5-1.5-1.5-1.5-1.5	1.5	14-14.5-14.3-14-14.5-14.5	14.3	10.5	89.5
28′	1.5-1.3-1.5-1.7-1.5-1.5	1.5	14-14-14.5-13.5-14-14	14.0	10.7	89.3
29′	4.5-4.8-4.5-4.8-4.6-4.7	4.65	14-14.5-14.5-14-14.5-14	14.25	32.6	67.4
30′	7.5-8.0-7.5-7.0-7.5-7.5	7.5	13.5-13.8-14-13.2-13.7-13.8	13.75	54.4	45.6

* At this point the prism was reset so as to give a greater amount of energy of the particular wave-length under study.

TABLE II

HCl Band at 3.4 μ (from curve of Fig. 3)

n	λ (in μ)	ν_n	$\Delta\nu_n$	ν_{on}	ν_{on} Computed	ν_{rn}	ν_{rn}/n
12......	3.23868	3087.68					
			12.85				
11......	3.25224	3074.83					
			13.27				
10......	3.26631	3061.56					
			14.47				
9......	3.28182	3047.09					
			15.89				
8......	3.29903	3031.20					
			14.92				
7......	3.31534	3016.28					
			16.30				
6......	3.33336	2999.98					
			17.62				
5......	3.35305	2982.36					
			17.38				
4......	3.37270	2964.98					
			18.62				
3......	3.39402	2946.36					
			18.35				
2......	3.41529	2928.01					
			20.24				
1......	3.43907	2907.77					
			41.00				
1'......	3.48897	2866.17		2886.97	2886.73	20.80	20.80
			21.75				
2'......	3.51565	2844.42		2886.22	2885.83	41.70	20.89
			22.48				
3'......	3.54366	2821.94		2884.15	2884.33	62.21	20.73
			22.49				
4'......	3.57214	2799.45		2882.21	2882.23	82.76	20.69
			23.04				
5'......	3.60178	2776.41		2879.38	2879.52	102.97	20.59
			23.85				
6'......	3.63298	2752.56		2876.27	2876.22	123.71	20.62
			23.67				
7'......	3.66450	2728.89		2872.58	2872.32	143.69	20.53
			25.30				
8'......	3.69878	2703.59		2867.39	2867.81	163.80	20.48
			25.03				
9'......	3.73335	2678.56		2862.83	2862.71	184.26	20.47
			25.91				
10'......	3.76982	2652.65		2857.11	2857.03	204.45	20.45
			26.15				
11'......	3.80735	2626.50		2850.69	2850.67	224.16	20.38
			26.62				
12'......	3.84633	2599.88		2843.78	2843.78	243.90	20.33

NOTE.—In this and succeeding tables and figures n refers to the number of the absorption maximum counting from the center of the band. The subscripts o and r refer to vibration and rotation, as in the "Discussion of Results." Wave-numbers are given throughout instead of actual frequencies. ν_n stands for $\nu_n/3\times10^{10}$.

remarked previously, since the interest has been only in the location of these maxima, and variations in pressure do not displace them laterally.

TABLE III

HCl Band at 1.76 μ (from curve of Fig. 5)

n	λ (in μ)	ν_n	$\Delta\nu_n$	ν_{on}	ν_{on} Computed	ν_{rn}	ν_{rn}/n
8.......	1.72711	5790.00	10.85				
7.......	1.73036	5779.15	12.58				
6.......	1.73413	5766.57	14.28				
5.......	1.73844	5752.29	14.18				
4.......	1.74274	5738.11	15.55				
3.......	1.74747	5722.56	16.58				
2.......	1.75255	5705.98	17.58				
1.......	1.75797	5688.40	42.03				
1′.......	1.77015	5646.37	23.17	5667.38	5666.38	21.01	21.01
2′.......	1.77835	5623.20	23.16	5664.59	5664.61	41.39	20.70
3′.......	1.78570	5600.04	23.70	5661.30	5661.67	61.26	20.42
4′.......	1.79329	5576.34	25.80	5657.22	5657.55	80.88	20.22
5′.......	1.80163	5550.54	25.58	5651.41	5652.25	100.87	20.17
6′.......	1.80997	5524.96	26.64	5645.77	5645.78	120.80	20.13
7′.......	1.81874	5498.32	29.71	5638.74	5638.12	140.42	20.06
8′.......	1.82862	5468.61		5629.31	5629.29	160.70	20.09

The 7500-line grating was used and the long-wave limit was determined by the strong atmospheric absorption beyond 4.2 μ, which cut down the galvanometer deflections to so small a figure as to magnify unduly any observational errors in computing the differential effect sought. It was possible, however, to obtain nine good maxima on the long-wave side of the center of the band. The curve is given in Fig. 6, and the table of values read and computed appears as Table IV.

A search was made for the "harmonic" of HBr at $2\ \mu$ found by Brinsmade and Kemble.[1] Nothing was found by any one of the three gratings, the conclusion being that the length of the

TABLE IV

HBr Band at $3.9\ \mu$ (from curve of Fig. 6)

n	λ (in μ)	ν_n	$\Delta\nu_n$	ν_{on}	ν_{on} Computed	ν_{rn}	ν_{rn}/n
9	3.72089	2687.53					
			11.82				
8	3.73133	2675.71					
			12.96				
7	3.75550	2662.75					
			13.46				
6	3.77460	2649.29					
			13.96				
5	3.79460	2635.33					
			13.46				
4	3.81407	2621.87					
			15.17				
3	3.83628	2606.70					
			15.37				
2	3.85902	2591.33					
			15.64				
1	3.88245	2575.69					
			33.59				
1′	3.93376	2542.10		2558.89	2558.93	16.79	16.79
			16.65				
2′	3.95969	2525.45		2558.39	2558.23	32.94	16.47
			18.05				
3′	3.98819	2507.40		2557.05	2557.08	49.65	16.55
			18.00				
4′	4.01704	2489.40		2555.63	2555.46	66.23	16.56
			18.49				
5′	4.04709	2470.91		2553.12	2553.38	82.21	16.44
			18.69				
6′	4.07793	2452.22		2550.76	2550.84	98.54	16.42
			19.33				
7′	4.11034	2432.89		2547.82	2547.74	114.93	16.42
			19.40				
8′	4.14338	2413.49		2544.60	2544.37	131.11	16.39
			20.19				
9′	4.17833	2393.30		2540.41	2540.44	147.11	16.35

column of absorbing gas was not great enough to give strong absorption in this region, nor was it possible in the time available to do the remodeling of the apparatus that a sensibly longer chamber would require.

[1] *Loc. cit.*

Hydrofluoric acid.—For the purposes of this study it was thought best to investigate a third gas of the halogen acid group. Of the two remaining, HF was thought the more likely to prove of theoretical value, and, aside from the difficulty of handling, easier to obtain in a pure state.

The inside of the absorption chamber was flowed with ceresin, as also was that of what tubing it was absolutely necessary to use. The gas was generated in a small iron retort by the action of concentrated H_2SO_4 on NaF. No attempt was made to dry it, but the absorption chamber was washed out with dry air before being filled with the HF, and the absence of any fuming as the gas entered the chamber was taken as evidence of the absence of

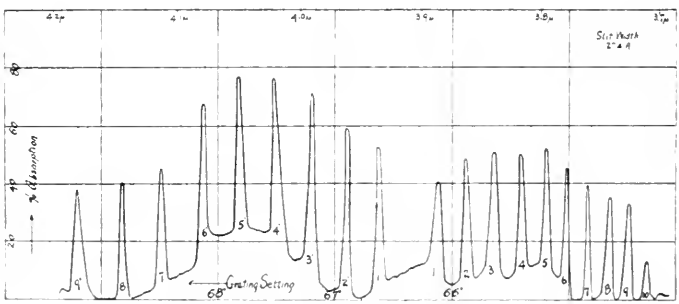


FIG. 6.—The HBr band at 3.9 μ, mapped with 7500-line grating

sensible amounts of moisture. Because of the gradual destruction of the surface of the plates used on the ends of the absorption chamber the percentages of apparent absorption ran gradually up until, usually at the end of from thirty minutes to an hour, it was necessary to change the plates. The expedient of spreading a thin layer of oil, grease, or even wax on the inner surfaces of the plates was tried, but greatly cut down their transparency in the region under study. This was doubly undesirable because on the long-wave side of the center of the band there is the very strong water-vapor band at 2.6 μ, which also cuts down the energy in spots so as to make the observational errors unduly large. It was here that a compensating chamber filled with dried air was thought of great importance. Also a new absorption chamber

was built, so designed that the plates could be clamped in place rather than cemented, thus saving much time in the necessarily frequent changes. Both mica and glass plates were used, those of mica proving the more satisfactory.

It is to be noted that the product of the action of HF on silica, present in both glass and mica, is a gas, SiF_4. In order to make certain that the absorption observed was not due to this second gas, observations were made at the beginning of each run with fresh plates, on one or more of the maxima previously obtained. If these had been absent or very much lowered the conclusion

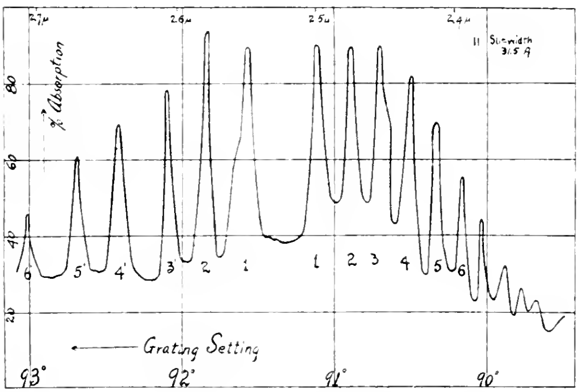


FIG. 7.—The HF band at $2.52\ \mu$, mapped with 7500-line grating

would have been that they were due to SiF_4 which was then present in very small quantity. No such absence or lowering was observed.

The long-wave limit of this work was determined by the strong atmospheric (water-vapor) absorption beyond $2.6\ \mu$, but it is to be noted that the number of maxima on the short-wave side of the center showing strong absorption is small as compared with that observed in the case of HCl.

The 7500-line grating was used in this work, except for the preliminary exploration done with the brass grating. The curve obtained for HF is given in Fig. 7, the measurements appearing in Table V.

A preliminary search was made for the possible "harmonic," to be expected in the region of $1.25\ \mu$, but if it exists it was effectively masked by the rising general absorption due to the action of the gas on the windows of the absorption chamber.

TABLE V

HF Band $2.5\ \mu$ (from curve of Fig. 7)

n	λ (in μ)	ν_n	$\Delta\nu_n$	ν_{on}	ν_{on} Computed	ν_{rn}	ν_{rn}/n
7.......	2.37911	4203.25					
			28.56				
6.......	2.39539	4174.69					
			31.84				
5.......	2.41380	4142.85					
			32.35				
4.......	2.43280	4110.50					
			34.36				
3.......	2.45330	4076.14					
			36.24				
2.......	2.47531	4039.90					
			37.88				
1.......	2.49874	4002.02					
			80.17				
1′.......	2.54982	3921.85		3961.93	3961.71	40.09	40.09
			42.75				
2′.......	2.57791	3879.10		3959.50	3959.43	80.40	40.20
			44.41				
3′.......	2.60778	3834.69		3955.41	3955.63	120.72	40.24
			44.63				
4′.......	2.63848	3790.06		3950.28	3950.33	160.22	40.05
			46.06				
5′.......	2.67094	3744.00		3943.43	3943.49	199.42	39.88
			47.93				
6′.......	2.70557	3696.07		3935.38	3935.13	239.31	39.88

DISCUSSION OF RESULTS

Reference has been made to the variance between the assumptions of Bjerrum and of Ehrenfest with regard to the size of the quantum of rotational energy. From the newer point of view, which states more generally and precisely the postulate of stationary states, the Ehrenfest[1] equation may be justified. To do this use is made of A. Sommerfeld's[2] extension of the Planck phase-integral

[1] Proposed also by A. Eucken. See *Verhandlungen der deutschen physikalischen Gesellschaft*, **15**, 1159, 1913.

[2] *Annalen der Physik* (4), **51**, 1, 1916.

in which Sommerfeld splits up the general integral for f-degrees of freedom,

$$\int_{i=1}^{i=f} \Pi(dq_i dp_i) = h^f$$

(the q's are the position co-ordinates and the p's the corresponding momentum co-ordinates), into f separate integrals,

$$\int dq_i dp_i = h,$$

one equation for each degree of freedom. For a rotating diatomic molecule this quantum equation becomes

$$\int_o^{2\pi} I\omega_n d\theta = nh,$$

which must be regarded as a fundamental assumption.

In the case under consideration—that of rotation with constant angular velocity—this integral evaluates quite simply into

$$2\pi I\omega_n = nh,$$

which, written in terms of kinetic energy and rotation frequency, becomes the Ehrenfest equation,

$$\tfrac{1}{2}I(2\pi\nu_r)^2 = n\frac{h\nu_r}{2}.$$

From this equation there is obtained directly

$$\nu_r = n\frac{h}{4\pi^2 I}, \tag{1}$$

giving rotation frequency in terms of the moment of inertia. This equation states that when the moment of inertia is constant, ν_r must have fixed values which are integral multiples of $h/4\pi^2 I$.

If, now, there is not the Maxwellian distribution of rotational velocities, but this series of frequencies differing by $h/4\pi^2 I$, it is evident that, still applying the combination principle, there would be expected in the near infra-red a series of pairs of absorption maxima corresponding to the frequencies $\nu_o \pm \nu_{rn}$ grouped symmetrically about ν_o, where ν_o is, as before, the frequency of vibration of the atoms within the molecule and ν_{rn} the rotation frequency corresponding to a given integral value of n.

Denoting the frequencies corresponding to a given pair of maxima by ν_n and $\nu_{n'}$ (the prime subscript referring to the long-wave side of the center of the band), ν_{rn} may be computed from the equations

$$\nu_n = \nu_o + \nu_{rn},$$

$$\nu_{n'} = \nu_o - \nu_{rn}, \tag{2}$$

whence

$$\nu_{rn} = \frac{\nu_n - \nu_{n'}}{2}. \tag{3}$$

The moment of inertia of the molecule will be, from equation (1),

$$I = n\frac{h}{4\pi^2\nu_{rn}}. \tag{4}$$

From the moment of inertia thus determined it is possible to compute the length of the molecule. Assuming the masses, m_1 and m_2, of the atoms concentrated at their nuclei which are a distance l apart,

$$I = \frac{m_1 m_2}{m_1 + m_2} l^2,$$

whence

$$l = \left(I\frac{m_1 + m_2}{m_1 m_2}\right)^{\frac{1}{2}}. \tag{5}$$

For $n=1$, i.e., for molecules having one quantum of rotational energy, the following experimental values are obtained by means of equations (4) and (5):

TABLE VI

Molecule	$I\times10^{40}$	$l\times10^8$	$I\times10^{40}$ (Kinetic Theory)
HF	1.37	.94	....
HCl	2.64	1.28	2.45
HBr	3.27	1.42	3.35

Von Bahr[1] gives values of I and l for the HCl molecule computed from her experimental data and the Bjerrum hypothesis, differing from the values in the foregoing table by the factor 2 in the case of I and $1/\sqrt{2}$ in the case of l within very close agreement.

[1] *Philosophical Magazine* (6), 28, 82, 1914.

Some check is desirable on any such values as these. In the present case the only one available is that furnished by the kinetic theory for the moment of inertia of a diatomic molecule in terms of its most probable rotational frequency. Accepting the equation previously given,

$$I = \frac{RT}{4\pi^2 \nu_r^2 N}, \tag{6}$$

and using the values obtained by Burmeister for the doublet maxima as corresponding to the most probable frequency through the equations $\nu_m = \nu_o \pm \bar{\nu}_r$, values may be obtained for the moments of inertia of HCl and HBr. (The present work, it is to be observed, furnishes no data for the determination of the most probable frequency.) Taking $R = 8.26 \times 10^7$, $N = 6.12 \times 10^{23}$, and $T = 292°$, equation (6) gives:

for HCl, $I = 2.45 \times 10^{-40}$,
for HBr, $I = 3.35 \times 10^{-40}$.

It may, indeed, be questioned whether this is a real check, but at any rate the agreement is significant.

If one turns to the curve of Fig. 3 and the accompanying table, Table II, it is at once apparent that the predicted symmetry of the maxima, i.e., the arithmetic progression expected in the frequencies of the maxima, does not exist. Instead, there is a gradual increase in frequency difference between two adjacent maxima as one goes farther into the infra-red.

Starting again with equations (2), the expression

$$\nu_o = \frac{\nu_n + \nu_{n'}}{2} \tag{7}$$

is obtained as giving the frequency of vibration of the molecule. A value of ν_o computed from each pair of maxima appears in column 5 of the table. There are as many centers, then, as pairs of maxima, and the asymmetry of the band is the consequence of the shifting of these centers farther into the infra-red as the rotation velocity increases. In other words, it appears that

the vibration frequency of the atoms in the molecule is dependent on the rotation frequency of the molecule.

Denoting the series of vibration frequencies by ν_{on}, where n in the subscript refers to the number of the pair of maxima from which the value of ν_o is computed, and plotting ν_{on} against n, the curve of Fig. 8 is obtained. Fitting it to the equation $\nu_{on} = A - Bn^2$ by the method of least squares, the agreement between observed

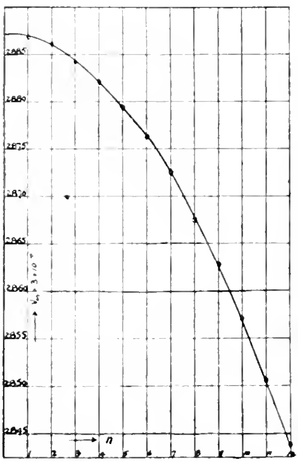


FIG. 8

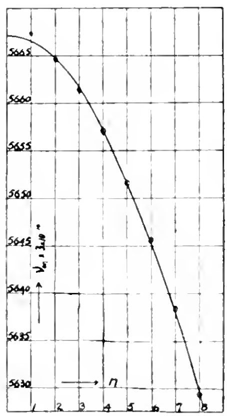


FIG. 9

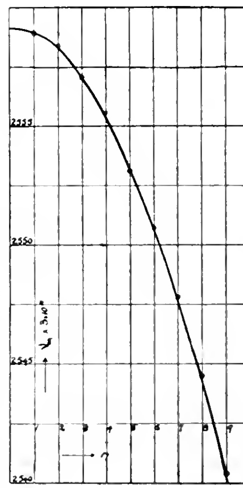


FIG. 10

FIG. 8. $\nu_{on} - n$ curve for the HCl band at $3.46\ \mu$

FIG. 9. $\nu_{on} - n$ curve for the HCl band at $1.76\ \mu$

FIG. 10. $\nu_{on} - n$ curve for the HBr band at $3.9\ \mu$

and computed values shown in columns 5 and 6 of Table II is found. The equation of this curve is

$$\nu_{on}/3 \times 10^{10} = 2887.03 - 0.30n^2.$$

For ν_{oo}, the extrapolated center of the band, the value is of course A of the equation above, and the corresponding wave-length is $3.4637\ \mu$, as against $3.475\ \mu$ obtained from the doublet maxima.[1]

Figures 9, 10, and 11 give respectively the parabolas of centers of the HCl band at $1.76\ \mu$, the HBr band at $3.91\ \mu$, and the HF band at $2.52\ \mu$. From these curves Table VII is obtained:

[1] See Brinsmade and Kemble, *loc. cit.*

The last column of each of Tables II to V gives the experimental values obtained for ν_{rn}/n and confirms a previous observation of von Bahr that this value is not constant but decreases with increasing values of the rotation frequency. The variation is small, but as it appears in each of the four curves it cannot be regarded as accidental. Eucken suggests that this divergence from the constancy predicted by the Bjerrum formula is not due to a falsity of the formula but to an increase of the moment of inertia with the velocity of rotation. That such an increase does take place may be seen from the following values of the moment of inertia of the HCl molecule computed from the values of ν_{rn} given in Table II.

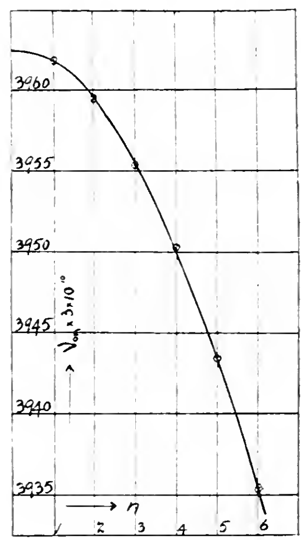


FIG. 11. $\nu_{on}-n$ curve for the HF band at $2.52\,\mu$.

The relation between I_n and n is practically linear as shown by the equation obtained from the observed values above by the method of least squares,

$$I_n=(2.63+0.0055n+0.000007n^2)\times 10^{-40}.$$

The values computed from this equation (neglecting the term in n^2) are given parallel with the observed values in Table VIII.

The "harmonic."—Kemble[1] has advanced the theory that if the amplitude of vibration of the molecule be that required by even a single quantum the infra-red absorption bands discussed so far might be expected to be accompanied by faint harmonics. The center of the first harmonic according to this theory should be at one-half the wave-length of the center of the fundamental and the spacing of the doublet maxima should be one-fourth of that of the maxima of the fundamental doublet (in wave-length). A band approximately answering to these requirements had been observed by Burmeister in the case of carbon monoxide, and this

[1] *Physical Review* (2), 8, 701, 1916.

was resolved into a doublet by Brinsmade and Kemble. They also obtained doublets for HCl and HBr approximately agreeing with the prediction. The disagreement was in the location of the centers, which were all farther in the infra-red than demanded by the theory.

Figure 5 gives the curve obtained for the HCl harmonic in the present work. The extrapolated value of the frequency of the center of this band is $5666.97 \times 3 \times 10^{10}$, while that of the center

TABLE VII

Band	$\nu_{on}/3\times 10^{10}$	Extrapolated Center	
		$\nu_{oo}/3\times 10^{10}$	λ_{oo} (in μ)
HF	$3962.47-.76n^2$	3962.47	2.5237
HCl (3.46 μ)	$2887.03-.30n^2$	2887.03	3.4637
HCl (1.76 μ)	$5666.97-.59n^2$	5666.97	1.7646
HBr	$2559.16-.23n^2$	2559.16	3.9075

TABLE VIII

I_n	Observed	Computed
I_1	2.64×10^{-40}	2.636×10^{-40}
I_2	2.67×10^{-40}	2.641×10^{-40}
I_3	2.65×10^{-40}	2.647×10^{-40}
I_4	2.65×10^{-40}	2.652×10^{-40}
I_5	2.66×10^{-40}	2.658×10^{-40}
I_6	2.66×10^{-40}	2.663×10^{-40}
I_7	2.67×10^{-40}	2.668×10^{-40}
I_8	2.68×10^{-40}	2.674×10^{-40}
I_9	2.68×10^{-40}	2.680×10^{-40}
I_{10}	2.68×10^{-40}	2.685×10^{-40}
I_{11}	2.69×10^{-40}	2.690×10^{-40}
I_{12}	2.70×10^{-40}	2.696×10^{-40}

of the fundamental is $2887.03 \times 3 \times 10^{10}$, the ratio being 1.963:1 instead of the expected 2:1. Or, to compare the bands as Brinsmade and Kemble compare them, the center of the harmonic is found 0.033 μ farther in the infra-red than the predicted position. This displacement is greater than the width of all eight maxima on the short-wave side of the band and greater than that observed by Brinsmade and Kemble. It disposes at once of their explanation that the disagreement observed in their work might be due to a slight error in the dispersion curve on which

their wave-length measurements were based, since there is no uncertainty as to dispersion in the present observations, made with a grating.

It is interesting to note that the frequency differences between adjacent maxima at the centers of the two bands (fundamental and harmonic) are sensibly the same, as shown in column 4 of Tables II and III. Also the moment of inertia of the molecule computed by means of equation (4) is the same for fundamental and harmonic.

It is to be regretted that only this one harmonic was obtained, and it is to be hoped that others may be carefully measured in the near future. The apparent tendency of some of the maxima to resolve into doublets in the case of the HCl harmonic may be due to errors of observation, but it seems significant that the small secondary maxima are all on the long-wave side of the principal maxima which they accompany. It is, of course, possible that still higher dispersion applied to the problem may show even the present curves to be composite.

Accuracy.—The question of the accuracy attained in such a problem as this is of necessity quite involved. An adequate estimate, however, of the accuracy of the determinations of the positions of the absorption maxima may be obtained by reference to columns 5 and 6 of Tables II to V inclusive. Column 5 of each table gives the frequency of the centers of the various pairs of maxima of the accompanying curves from wave-length readings on the curves. Column 6 of each table gives the values for the same centers computed by the method of least squares. In the case of Table II, the greatest variation of the observed value from the computed value is in the case for $n=8$, where $d\nu=0.42\times3\times10^{10}$. This corresponds to a value of $d\nu$ given by the equation

$$d\lambda=\frac{c}{\nu^2}d\nu.$$

Substituting the values $\nu=2872.6\times3\times10^{10}$ and $d\nu=0.42\times3\times10^{10}$, it is found that

$$d\lambda=5\times10^{-8}\text{ cm}=5\text{ Å}.$$

This is the extreme case. The average agreement is within ±1.9 Å.

SUMMARY

The object of the present work has been to obtain more extended and more accurate data with regard to the near infra-red absorption bands of certain diatomic gases than have been hitherto available. To this end the closely related HF, HCl, and HBr bands have been mapped, using greater dispersion than has been used before in this particular problem.

Curves are presented showing in greater detail the HCl bands at 1.76 μ and 3.46 μ and giving for HBr, instead of the simple doublet hitherto known, a curve resolved into its quantum lines. In addition, a similar curve is presented for HF which has not been studied before.

Several peculiarities in these curves are pointed out for the first time and one or two uncertainties in previous work are settled by material presented here.

From experimental results the lengths and moments of inertia of these three molecules have been computed.

It is hoped that in the material presented there will be found some of real value in the work which is yet to be done in reconciling theory and fact in the extremely interesting field of molecular mechanics. It is a very real pleasure to acknowledge the writer's indebtedness to Professor Randall, who suggested the problem, for his continued interest and encouragement during the progress of the work.

Physical Laboratory
University of Michigan
April 30, 1918

ON PROGRESSIVE CHANGES OF THE WAVE-LENGTHS OF LINES IN STELLAR SPECTRA WITH CHANGE OF TYPE

By SEBASTIAN ALBRECHT

In the *Astrophysical Journal* for March 1918 (**47**, 137) J. Voûte published the results of an investigation which was undertaken in order to ascertain to what extent stellar wave-lengths determined with the McClean telescope of the Royal Observatory, Cape of Good Hope, show relative changes with type of the kind previously announced by the present writer.[1] Voûte selected four stars for this purpose, namely: α Canis Majoris, type A0; α Canis Minoris,[2] type F5; α_2 Centauri, type G0; and α Boötis, type K0. The number of plates measured were respectively 10, 5, 8, and 10. Voûte concludes that his wave-lengths agree in general fairly well with mine, the most pronounced progressions being generally confirmed by his observations.

F. E. Baxandall, discussing the two sets of wave-lengths in the July number (**48**, 59), apparently concludes that the agreement is not so good as might be desired, and that the subject deserves further attention.

As apparently both Voûte and Baxandall are unaware of a somewhat more complete and slightly revised list of wave-lengths, though still entirely preliminary, published by the writer in *Boletin No. 1* of the Córdoba Observatory, it seemed worth while to make a more detailed comparison of my wave-lengths and those of Voûte. I shall confine the comparison to published data.[3]

[1] The expression "progressive changes" has been used by the writer in the sense of progressing continuously with the sequence of types, i.e., without any discontinuities, but also without any restrictions whatever as to the direction in which the wave-length may vary. Thus changes in direction of variation are not excluded and in fact have been observed for numerous lines.

[2] Harvard gives F5 as the type for this star; Voûte calls it F.

[3] I have accumulated a considerable amount of additional data, for which the completion of the reductions and the publication have been unavoidably delayed.

It may be appropriate to recall briefly that there are strong indications that the physical conditions in the stars as we pass from type A to type Mb vary roughly in the same direction as from sun to sun-spots. The solar spectrum very closely resembles the spectrum of a G-type star. Adams has shown that a striking similarity exists in the intensities of sun-spot lines and of the corresponding lines in the spectrum of Arcturus, a K-type star. This refers to the intensities of the lines and of the components of lines. It is a well-known fact that in the series of stellar spectra lines change progressively in intensity. Some lines which are strong in the so-called early types become weaker and often disappear entirely as we pass to the later types. In fact, our systems of stellar classification depend upon the presence or absence and changing intensities of the spectrum lines. Thus changes which are progressive with change of type are normally to be expected, and changes which are apparently abrupt or discontinuous would, to a considerable degree, contain the presumption of abnormality and would lead one to look for additional effects superimposed upon the changes dependent on spectral type. Thus recent work by Mount Wilson observers indicates a dependence of line-intensity, for certain lines, upon the absolute luminosity of the star and possibly also upon distance. Other factors are involved which help to complicate matters. For example, the best available evidence seems to indicate that great differences in absolute luminosity are accompanied by great differences in stellar density and therefore in size. Accompanying differences in size, it is a natural step to infer also considerable differences in the depths of the reversing layers and in the pressures and other factors, which in turn will give rise to spectral differences in the integrated stellar light secondary to the main spectral characteristics. Some of the changes of line-intensity which are thus introduced are likely to involve line displacements of the order of magnitude considered in this note, especially when blended with other components. Therefore in extensive studies of changes of wave-length with type such effects will have to be taken into account, and, besides separating the stars according to type, they must be separated also according to other factors, such as absolute luminosity. The limited comparison made below is not

likely to be appreciably changed by partly neglecting these distinctions.

In Table I, which is largely self-explanatory, are given the lines for which both Voûte and I have published wave-lengths. The weights were assigned more or less arbitrarily according to suitability for determining the difference (Albrecht − Voûte), and were increased by 50, 75, or 100 per cent, according as both observers have measures respectively for the same two, three, or four types. My wave-lengths are taken from curves drawn to represent best the wave-lengths observed for each type (*loc. cit.*); those of Voûte are from his published list (*loc. cit.*), corrected to my system by

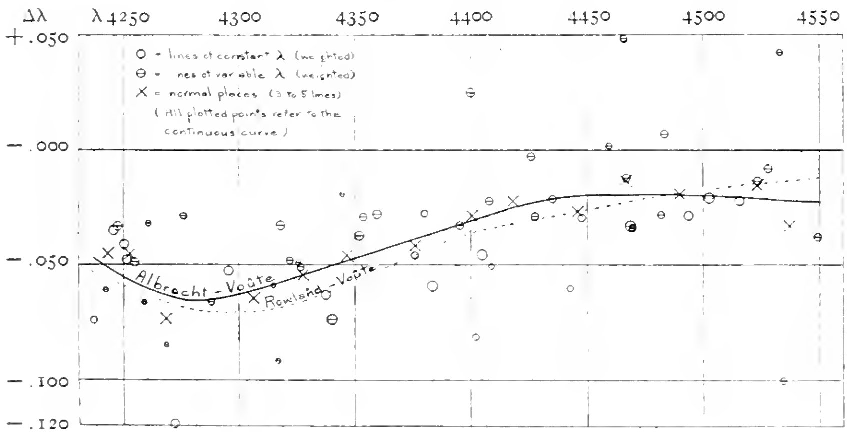


FIG. 1

means of the corrections Δλ shown in column 3. These corrections, which were applied purely to facilitate the intercomparison, do not differ greatly from those required to reduce Voûte systematically to Rowland, as is seen from the figure. The small difference between the two curves represents mostly a small systematic difference, for this particular group of lines, between myself and Rowland. The reasons for the large systematic deviation of Voûte from Rowland cannot be gone into without having available the complete details of the reductions. Moreover, we are here concerned primarily with the changes in the wave-lengths depending on stellar type rather than with the systematic differences between the two observers.

TABLE I

λ	Weight	Albrecht Voûte	Corr. Applied (Δλ)	Wave-Lengths (By A and V) A0	F5	G0	K0	Constant	No. of Obs.	Rowland λ in Sun	Blend According Intensities	Inten. in Sun	Inten. in Sun-Spots (Adams)	Enhanced (Lockyer) El.	Spark	Arc	Remarks
4236.0	1	A	. . . .	. . . .	. . . .	. . . .	. .	A5 to Mb .079	26	.112		Fe 8	n.c.	. . . .	. . . .	. . . .	Principally Fe. The companions are relatively too weak to blend satisfactorily.
		V	−.05	.10	. . . .	.14	.07	. . . .	. . . .	.279		1					
										.420		Ni 1					
4242.5	½	A	. . . .	.500	.530	. . . .	. . . .	. . . .	11	.322		0	. . . .	. . . .	. . . .	. . . .	In types A and F principally the spark lines of Mn and Cr.
		V	−.05	.52	. . . .	.61	. . . .	. . . .	. . . .	.443		0	0	Mn Cr	4 6–7	0 0	In types G and K not measured on account of the numerous components. In M the spark lines probably absent.
										.535		2					
										.615		2					
										.766	.703	2					
										.897		Fe 2					
4245.4	1¾	A	. . . .	. . . .	. . . .	. . . .	. . . .	F to Mb .438	36	.243		ON	. . . .	Bis	3	0	A good line for radial-velocity work with three prisms.
		V	−.05	. . . .	.41	.43	.43	. . . .	. . . .	.422	.455	Fe 4	8				
										.520		2					
4246.9	1¾	A	. . . .	. . . .	.093	.980	.973	. . . .	40	.729		000	. . . .	Mo	1	0	The shift toward the violet, though small, seems definite. Cause not apparent.
		V	−.05	. . . .	.96	.96	.98	. . . .	. . . .	.996		Sc 5	3				
4250.2	1¾	A	. . . .	. . . .	. . . .	. . . .	. . . .	A5 to Mb .296	44	.287s		Fe 8	7	. . . .	. . . .	. . . .	A good line for radial-velocity work with three prisms.
		V	−.06	. . . .	.26	.27	.30										
4250.9	1¾	A	. . . .	. . . .	. . . .	. . . .	. . . .	A5 to Mb .955	43	.863		0	. . . .	Mo	6	2	Lockyer gives for Mo λ .90 or .85. The stellar line is good for radial-velocity work with three prisms.
		V	−.06	. . . .	.93	.94	.96	. . . .	. . . .	.945 s		Fe 8	n.c.				
										.071 s		1					
4254.5	1¾	A	. . . .	. . . .	.504	.502	498	. . . .	39	.505s		Cr 8	10	. . . .	. . . .	. . . .	In sun-spots "widened, prob. winged." In K types the line is broad and fuzzy toward the violet.
		V	−.06	. . . .	.48	.50	.49										
4258.4	¾	A	. . . .	. . . .	. . . .	. . . .	.494	. . . .	. . . .	.201		Zr 0	. . . .	Zr	2–3	1	A very broad blend. In types K to Mb practically no change in λ.
		V	−.06	. . . .	. . . .	. . . .	.50	. . . .	. . . .	.219		IN	0–1				
										.477		Fe 2	4				
										.639		ON					
										.774		Fe 2					
4260.6	¾	A	. . . .	. . . .	.643	.658	.712	. . . .	18	.151	.50	Fe 2	. . . .	. . . .	. . . .	. . . .	Fe λ .640 is "apparently narrowed" in sun-spots. It seems likely that Voûte's λ in G is a v. broad blend, including λ .15 and λ .28. The observed change in wave-length with type needs strengthening by additional measures.
		V	−.06	.61	.67	.51	.75	. . . .	. . . .	.282		Fe 3d?					
										.494		ON					
										.640 s		Fe 10	8				
										.768		ON					
										.888		1					
										.991		Ti 0					

4267.9...	½	A	+.003		.964	.986	.045		15	.900 .985 .138 .266		I Fe 3 Zr 0 1N					My published wave-lengths for this line are in error by −.091 A. Apparently Fe is weakened, and either Zr or line to red much strengthened.
		V	−.06				.07										
4271.3...	1¾	A						F to Mb .322	22	.212 .325 .538		0N Fe 6 0	5				My observed wave-lengths, with no. of obs., are: F, .317₅; F5 and F8, .310₅; G, .352₂; G5, .321₁; K, .304₄; K2 and K5, .310₃; Ma and Mb, .310₂.
		V	−.06	.31	.28	.38	.49										
4274.9...	1½	A			.962	.955	.950		39	.746 .958	.911	Ti 2 Cr 7d?	12				In sun-spots "much widened to violet." Shift to violet not as great as expected.
		V	−.06		.92	.00	.95										
4288.1...	1	A			.122	.143	.166		38	.038 .140 .310	.134	Ti 2 Ni 1 Ti, Fe 1	2	Ru	1	0	Lockyer gives λ for Ru as .15 or .20. Probably all of the components vary in intensity. Shift toward red is quite definite.
		V	−.06		.11	.14	.20										
4294.2...	1¾	A						A5 to Mb 277	30	.077 .204 .301	.273	000 Ti 2 Fe 5	8	Ru Ti	3 7	0 3	A weakening of the enh. Ti component might be expected in the late types, but no corresponding shift to the red is shown in either set of measures.
		V	−.06	.27	.27	.26	.28										
4314.3...	⅓	A			.331	.330	.371		30	.248 .381 .470	.321	Sc 3 1 Ti 1	2				The changes in direction of variation of λ in types require verification by additional measures.
		V	−.06				.37										
4315.1...	⅔	A			.186	.176	.151		49	.673 .964 .138 .262	.178	0N Ti 1 Ti 3 Fe 4	2–3 2 5	Ti	8	1	The shift to violet is quite definite and is caused by the relative greater strengthening of λ .964 than of λ .262.
		V	−.06	.21	.28	.18	.15										
4318.8...	1¾	A			.850	.864	.882		78	.8178		Ca, Mn? 4	6				"Winged" in sun-spots. Hasselberg gives a Ti component (Kilby), spark 3, arc 5. Average variation of λ with type is small.
		V	−.06		.79	.86	.87										
4321.0...	1½	A			.002	.975	.920		34	.907 .119	.992	Sc 3 2	3 1	Ti	3	1	Shift to violet expected as the red component is a spark line and is weakened in sun-spots.
		V	−.06		.97	.96	.93										
4325.1...	⅔	A			.186	.183	.225		35	.977 .152 .306 .516	.183	0 Sc 4 Ti, Cr 1 Ni, Zr 1	6				The usual strengthening of Ti and Cr in the later types accounts for the general shift to the red. The cause for my secondary alternating shifts, if real, is not so apparent. Possibly Voûte's λ in F5 is Sc alone?
		V	−.06		.11	.21	.24										

TABLE I—*Continued*

λ	Weight	Albrecht Voûte	Corr. Applied (Δλ)	Wave-Lengths (By A and V) Ao	F5	Go	Ko	Constant	No. of Obs.	Rowland λ in Sun	Blend According Intensities	Inten. in Sun	Inten. in Sun-Spots (Adams)	Enhanced (Lockyer) El.	Spark	Arc	Remarks
4325.9	1¼	A V	.... −.06	.. .94	.950 .93	.084 .06	.975 .00	....	39	.777 .9398 .119 .213	.941	1 Fe 8 1 0	 7	....	......	....	The Fe line is "apparently narrowed in spots" and is much sharpened at the limb. My wave-lengths for the M type need strengthening.
4337.2	1¾	A V	 −.05			 .24	 .24	F to Mb .227	36	.216 .414		Fe 5 0	6	....	......	....	"Winged" in sun-spots. Red edge frequently fuzzy in stars, probably on account of the faint companion.
4340.6	2	A V	.... −.05	.648 .66	.650 .73	.658 .68	.638 .63	....	77	.192 .207 .634 .7 or .8		000 Cr 0 Hγ 20N	.... 0–1 8	K Bis	2 5	0 0	Hγ is "weakened and narrowed on red edge mainly" in sun-spots. The wave-length drops off very rapidly from K2 (.628) to Mb (.560), Hγ being faint in Mb and blended into a broad and fuzzy line with Cr λ .297. The wave-lengths of Voûte and Albrecht are in fair agreement, except in F5.
4344.5	½	A V	 −.05		.550	.576	.621 .59	....	50	.451 .670	.597	Ti −2 Cr 4	n.c. 6	Ti	3	1	The observed shift to the red is in accord with the indicated strengthening of Cr in sun-spots. Also, the Ti line is weakened at the limb.
4351.9	1¾	A V	...... −.05	.981 .99	.019 .01	.019 .98	.008	....	80	.711 .871 .930 .083 .235	.006	Fe 2 0000 Cr, Fe 5 Mg 5 Nd? 0N	 11	.... Fe	 5	 0	For blend in sun-spots, "strengthening very doubtful." The Mg line is very slightly weakened in sun-spots, and is weakened and moderately sharpened at the limb. Albrecht's observed stellar wave-lengths may find an explanation in a moderate weakening of λ .930 in F and G due to a great weakening of the Fe spark line and a strengthening of Cr, and a slightly preponderating weakening of λ .083 in K and M due to a slight weakening of Mg and a considerable strengthening of Cr.

4352.9 ..	$1\frac{1}{2}$	A	....	....	.036	.917	.975	....	53	.718 .908s .044		ON Fe 4 V 0	5 7	....	......	.....	The blend (Fe, V) is "widened to red" in sun-spots. Probably the vanadium component is principally responsible for the observed shift to the red in stars. The wave-length in Mb may indicate an appreciable weakening of Fe?
		V	−.05	....	....	.01	.07	....	.....								
4359.8...	$1\frac{3}{4}$	A	....	....	.803	.784	.798	....	64	.654 .784s .907		Ni 0 Cr 3 Zr 0	5	Zinc Zr	3 4	0 1–2	The curve representing the wave-lengths is weak from A5 to F5, but strong and well determined from G to Mb. Cause for the observed shift is not apparent.
		V	−.04	....	.80	.76	.79	....	.....								
4376.1...	$1\frac{1}{2}$	A	....	....	....	....	....	F to Mb .104	56	.107s		Fe 6	8	....	......	.....	"Probably winged" in sun-spots. Consistently small range within each type. Very good for radial-velocity work with three prisms.
		V	−.04	....	....	.08	.14										
4379.3...	1	A	....	....	....	....	....	G to Mb .383	43	.396		V 4	7	....	......	.....	"Very broad" in sun-spots.
		V	−.04	....	....	....	.37										
4383.7...	2	A	....	....	....	....	....	A to Mb 721	70	.720s		Fe 15	n.c.	....	......	.....	In the M types strong, broad, and usually fuzzy, but not shifted.
		V	−.04	.72	.72	.74	.78										
4395.2...	$1\frac{1}{3}$	A	....	.203	.244	.255	.276	....	104	.201 .413	.286	Ti 3 V, Zr 2	2 3	Ti	9	5	The definite shift toward the red is due to the strengthening of the component to the red and a weakening of the component to the violet.
		V	−.03	.21	.22	.27	.29	....	.....								
4399.9...	$1\frac{3}{4}$	A	....	.948	.931	.917	.885	....	66	.770 .935		Ni 0 Ti, Cr 3	2	Ti	7	3	In the A type the line is practically the Ti spark line alone; in F and later types it is a blend with Cr and possibly also Ni. Three measures in α_2 Centauri give λ .89.
		V	−.03	.95	.87	.81	....	....	.....								
4401.6...	1	A	....	....	....	....	....	F to Ma .613	44	.456 .613 .709	.589	Fe 2 Fe 1 Ni 2	....	....	......	.....	Although from F to Ma my wave-length is apparently constant and in fair agreement with the blend according to Rowland's intensities in the sun, in Mb I obtain λ .71 for a "faint, broad, and very fuzzy" line.
		V	−.03	....	....	.67	.66	....	.....								
4404.9...	2	A	....	....	....	....	....	A to Ma .932	63	.927s		Fe 10	9	....	......	.....	Rowland gives very faint companions on each side, which apparently do not become effective in stellar spectra.
		V	−.03	.94	.94	.97	.94										
4407.8...	$1\frac{1}{2}$	A	....	....	.857	.853	.843	....	54	.810 .871s	.851	V 2 Fe 4	8	....	......	.....	The stellar line is principally Fe in F type and V in Mb.
		V	−.03	....	....	.81	.87	....	.....								

TABLE I—*Continued*

λ	Weight	Albrecht Voûte	Corr. Applied (Δλ)	Wave-Lengths (By A and V) Ao	F5	Go	Ko	Constant	No. of Obs.	Rowland λ in Sun	Blend According Intensities	Inten. in Sun	Inten. in Sun-Spots (Adams)	Enhanced (Lockyer) El.	Spark	Arc	Remarks
4108.5	$\frac{3}{4}$	A						A8 to Mb .579	43	.364		V 2	3−4	....	......	....	My observed stellar wave-lengths show no shift with type, nor is much shift to be expected where the components on each side vary together in intensity.
		V	−.03			62	.58	....	....	.582	.549	Fe 3	7				
										.683		V 2					
4425.6..	$1\frac{1}{2}$	A			615	.600	.872	......	29	.608s		Ca 4	5−6	....	......	....	"Winged" in sun-spots. Variation of λ in F and G weakly determined. In K and M the measures represent a blending with λ 26.201, forming a very broad double line, resolved with moderate slit widths.
		V	−.02		60	64											
										.201		Ti ON d?					
4427.4..	$1\frac{1}{4}$	A			440	443	.432	......	78	.260		Ti 2	3	....	......	....	My wave-lengths show a small but definite shift, indicating that the strengthening of the weaker component is more effective than the strengthening of the stronger.
		V	−.02		.45	44	.46	....	....	.482	.420	Fe 5	7				
4435.1.	$1\frac{1}{4}$	A			160	186	.230	......	62	.129s		Ca 5	7	....	......	....	Shift to red in stars is quite definite, showing that the strengthening of the weak component is more effective than the strengthening of the strong component.
		V	−.02		15	.20	.23	....	....	.321	.184	Fe 2	3				
4442.5	1	A					..	A5 to Mb .529	66	.510		Fe 6	7				
		V	−.02			...	.57										
4447.9	$1\frac{1}{2}$	A				...	...	A5 to Mb .915	74	.892s		Fe 6	7	....	......	....	"Winged" in sun-spots. The stellar line may possibly be shifted slightly toward the violet in the M type.
		V	−.02			92	.93										
4459.2...	1	A	......	.226	302	.317	.327	....	103	.199		Ni 2	7	....	......	....	Cr is "widened" in sun-spots. Shift of blend is small in types F5 to M. Blend shifted back slightly toward the violet in Mb?
		V	−.02	....	.30	.27	.31	....	.....	.301	.304	Fe 3					
										.525		Cr 1	1−2				
4464.6...	1	A	......	.609	.749	.811	.885	....	21	.617		Ti? 2	n.c.	Ti	3	1	In the A type the observed λ is that of Ti alone. The large shift toward the red in the K types seems to imply also a weakening of the Ti component?
		V	−.02	....	.69	.75	.80	....	.....	.844	.772	Mn 2	3−4				
										.938		Fe 1					
4466.7...	$1\frac{1}{4}$	A	......	....	.743	.740	.779	....	63	.548		Ni 0	......	....	......	....	Fe "widened and hazy" in sun-spots.
		V	−.02	....	.73	.74	.78	....	.....	.727		Fe 5	7	K	1	0	The stellar wave-lengths show the widening to be principally toward the red.

4468.6...	2	A V	.... −.02	.662 .69	.651 .66	.660 .66	.695 .71	....	117	.663		Ti– 5	4	Ti	9	4	Variation of λ in stellar spectra is quite definite. It may be a blend?
4469.4...	1	A V	.... −.02	....	.496 .51	.530 .54	.593 .61	....	44	.316 .441 .545 .731 .873		Ti 1 0000 Fe 4 Co o d? V 00	0–1 4 0	......	......	......	Fe is "narrow, probably winged," in sun-spots. Variation with type seems quite definite. In the F type λ .316 and in the M type one or more of the components to the red probably become effective with λ .545.
4481.3...	1¼	A V	.... −.02	.381 .43	.395 .41	.400 .36	.410	....	160	.298 .438 .515		0 Ti 1 0	000 2	Mg	10	0	A slight increase in λ with type seems indicated. Shall discuss this line in detail at another time.
4482.3...	1⅓	A V	.... −.02	....	.393 .36	.394 .35	.393 .39	....	57	.338 .438	.376	Fe 5 Fe 3	11	......	......	......	My stellar wave-lengths are practically constant from F to K but diminish rapidly from K2 to Mb.
4494.7...	1¾	A V	.... −.02		74	70	.73	A5 to Mb .714	30	.548 .656 .738s .903		00 00 Fe 6 000	.. 8	Zr	2–3	0	Lockyer gives a spark line of Zr at λ .55 or λ .78. In stellar spectra this line is fuzzy on one or both edges, and therefore not especially good for radial-velocity work.
4501.4...	2	A V	.... −.02	.444 49	.437 .45	.435 .37	.430 .44	....	75	.264 .455s .622 .7 or .8		Cr, Mn 0 Ti– 5 00N	.. 3	Ti ..	8 4–5	4 0	The indicated small progressive variation with type is still uncertain. Caesium spark line.
4515.5...	1⅓	A V	.... −.02	.61	.48	.. 47	..	A to Mb .517	40	.342 .508		0 3	1	Fe	4	trace	
4522.7...	1½	A V	.... −.02	795 .82	.847 .82	861 .80	.830 .87	....	48	.691 .802 .974	.871	Fe? 0 3 Ti 2	... 1 4	Fe	6	2	The observed shift to the red is to be expected, due to weakening of the spark line and strengthening of the Ti component. Cause for drop in λ in K type (if real) is not apparent.
4528.7...	1½	A V	.... −.02	741	.794 .79	.810 .79	.825 ...	....	43	.647 .798 .929 .990		0N Fe 8 0 0	n.c.	V	6	1	Vanadium spark line at λ .70 or λ .73. The change in wave-length is most rapid in types A to G. In types later than G the vanadium spark line has practically ceased to be effective. This is in accord with the indicated no change in sun-spots.
4533.2...	1	A V	.... −.02	....	.288 .20	.301 .27	.336 .27	....	34	.133 .219 .419		1 0 Ti 4	5–6	......	......	......	The observed shift toward the red is accounted for by the expected strengthening of the Ti spot line. In M the violet components seem to be relatively too faint to be in the least effective.
4534.1...	1	A V	.... −.02	.195 .23	.190	.186 .31	.175	....	48	.139 .340	.168	Ti-Co 6 1	5	Ti	5	2	Although a small shift toward the violet seems indicated, many more measures are required to establish it.
4549.6...	1½	A V	.... −.02	.685 .65	.736 .81	.753 .75	.783 .82	....	88	.642 .808 .990	.776	Fe 2 Ti-Co 6d? 0	0 4	Fe Ti	7 8	1 4	The shift toward the red is due to the more rapid weakening of the violet component, which seems to have practically disappeared in Mb.

Table II has been prepared to bring out the fact that the differences (Albrecht − Voûte) run practically the same for the lines of constant wave-length (Albrecht's) as for the lines for which the wave-length changes with spectral type. The slight difference in

TABLE II

(Albrecht − Voûte), in Units of 0.01 A

Constant Lines						Variable Lines					
λ	A0	F5	G0	K0	Range	λ	A0	F5	G0	K0	Range
4236.0	−2	......	−6	+1	7	4245.2	−1	......	......	......	......
4245.4	......	+3	+1	+1	2	4246.9	......	+3	+3	−1	4
4250.2	......	+4	+3	0	4	4254.5	......	+2	0	+1	2
4250.9	......	+2	+1	−1	3	4260.6	......	−3	+15	−4	19
4258.4	......	......	......	−1	......	4267.9	......	......	......	−2	......
4271.3	(+1)	+4	−6	−17	21	4274.9	......	+4	+6	0	6
4294.2	(+1)	+1	+2	0	2	4288.1	......	+1	0	−3	4
4337.2	......	......	−1	−1	0	4314.3	......	......	......	0	......
4376.1	......	......	+2	−4	6	4315.1	......	−9	0	0	9
4379.3	......	......	......	+1	......	4318.8	......	+6	0	+1	6
4383.7	0	0	−2	−6	6	4321.0	......	+3	+2	−1	4
4401.6	......	......	−6	−5	1	4325.1	......	+8	−3	−2	11
4404.9	−1	−1	−4	−1	3	4325.9	......	+3	+2	−2	5
4408.5	......	......	−4	0	4	4340.6	−1	−7	−2	+1	8
4442.5	......	......	......	−4	......	4344.5	......	......	......	+3	......
4447.9	......	......	0	−1	1	4351.9	−1	+1	+4	......	5
4469.4	......	−1	−1	−2	1	4352.9	......	......	+4	0	4
4494.7	......	−3	+1	−2	4	4359.8	......	0	+2	+1	2
4515.5	−9	+4	+5	......	14	4395.2	−1	+2	−2	−1	4
						4399.9	0	+6	+11	......	11
						4407.8	......	......	+4	−3	7
						4425.6	......	+2	+2	......	0
						4427.4	......	0	0	−3	3
						4435.1	......	+1	−1	0	2
						4459.2	......	0	+5	+2	5
						4464.6	......	+6	+6	+8	2
						4466.7	......	+1	+1	0	1
						4468.6	−3	−1	0	−2	3
						4481.3	−5	−2	+4	......	9
						4482.3	......	+3	+4	0	4
						4501.4	−5	−1	+6	−1	11
						4522.7	−2	+3	+6	−4	10
						4528.7	......	0	+2	......	2
						4533.2	......	+9	+3	+7	6
						4534.1	−4	......	−12	......	8
						4540.6	+4	−7	0	−4	11
Means (without sign)	2.3	2.3	2.5	2.7	5.0		2.5	3.2	3.5	2.0	5.9
Means (with sign)	−3.0	+1.3	−0.9	−2.3	......		−1.7	+1.2	+2.2	−0.3	......

favor of the constant lines may be attributed to the fact that these lines are on the average more nearly "pure," i.e., single components, than the lines of variable wave-length. Sixty-three per cent of the constant lines and 33 per cent of the variable lines are apparently pure. Also the average weight assigned according to suitability for determining the difference (Albrecht − Voûte) is 0.88 for the

constant lines and 0.74 for the variable lines. Table II also shows up the moderate systematic differences for the individual types. A few exceptionally large values appear in the table, some of which, like that for 4260.6, are probably due to the inclusion by one observer of components excluded by the other. Such differences are to be expected, as accurate descriptions of the lines have not yet been published. The large difference of 0.17 A for the pure line 4271.3, due to the large deviation of Voûte's wave-length from that of Rowland seems inexplicable. The deviation seems too large to be attributed to personality in the settings due to the close proximity of the strong line 4271.9.

The comparison of the two sets of stellar wave-lengths can best be followed by referring to the corresponding data in Tables I and II. In Table I the columns giving the components in the solar spectrum (Rowland), the intensities in sun-spots, and the behavior in spark and in arc spectra aid in indicating the degree to which the wave-lengths observed in stellar spectra correspond to the wave-lengths which might by inference be expected. For the writer's wave-lengths this correspondence is quite close; with very few exceptions—see also remarks in Table I—both for the G type and, in direction and approximate amount, for the other types. While Voûte's wave-lengths also correspond in a general way they show more frequent and larger deviations.

On the whole, I believe that Voûte's wave-lengths do confirm my variations with type. In considering this question we should bear in mind, especially as Voûte's wave-lengths are given to only two decimal places, that a considerable range of accidental variation in the differences (Albrecht−Voûte) must be anticipated, so that only a comparatively few lines—those with a large range of variation with type—are competent individually to give fairly definite evidence. Among these may perhaps be included 4288.1, 4321.0, 4395.2, 4435.1, 4464.6, 4469.4, and 4549.6. The line 4468.6. though especially good for variation with type, has only a moderate range of variation from A to K.

An inspection of my tables will show that most of Baxandall's criticisms do not strictly apply. In paragraph 2 Baxandall interprets my use of the word "progressive" in a sense different from

that clearly implied in my tables of wave-lengths and in my curves showing the variations for individual lines. This also accounts for Baxandall's third paragraph in so far as it refers to complex blends involving more than two components. For the three complex blends 4288.1, 4315.1, and 4352.0, which were cited, the agreement between Voûte and Albrecht is satisfactory, except in F5 for 4315, and the direction of variation is in harmony with the evidence furnished by sun-spot and by arc and spark data.

With the great multiplicity of fine lines, especially in the G and later types, we cannot be certain that lines which are apparently pure are not in reality affected by superimposed unknown components. Bearing this possibility in mind we may provisionally regard as pure such lines as 4468.6. For the pure lines we cannot at present predict the direction and approximate amount of variation of the wave-length with type, as can be done for the great majority of the blended lines, except as an unsymmetrical change of width in sun-spots may give an indication. For these "pure" lines the agreement between the two observers is also satisfactory for the material at hand. The observed differences for 4246.9 are too small to be regarded as discordances. The large discordance in the G type for 4260.6 seems clearly due to the inclusion by Voûte of rather widely distant companions to the violet. For 4274.96 somewhat large accidental variations may be involved in the moderate apparent discordance. For $H\gamma$ the agreement is good for types A, G, and K (F5 is discordant), and the direction of variation is that indicated by sun-spot data. As measured on the three-prism plates line 4399.9 is not pure, and this is probably true also of 4469.5. For 4468.6 the results are accordant. Although the wave-lengths are accordant for 4254.5 and 4359.8, with such small range not much can be expected in the way of confirming my variations with type.

SUMMARY

The stellar wave-lengths which Voûte determined recently were compared with those published by the writer in *Boletin No. 1* of the Córdoba Observatory, for the purpose of noting whether the former showed the same variations with spectral type as the latter.

Systematic corrections were applied to Voûte's wave-lengths purely to facilitate the comparison. The details for the comparison are contained in Table I. Bearing in mind that an appreciable range of accidental difference is to be expected for the data compared, Voûte's wave-lengths may be regarded as confirming my variations of wave-length with stellar type.

DUDLEY OBSERVATORY
ALBANY, N.Y.
December 1918

A PROPERTY OF THE PHOTOGRAPHIC PLATE ANALOGOUS TO THE PURKINJE EFFECT

SECOND NOTE

By J. A. PARKHURST

In the former note printed in the April number of this *Journal* (**49,** 202) the effect of star-color on the contrast of the photographic plate was investigated for the case of the ordinary plate, the results apparently confirming Abney's statement that the gradation of the plate increases as the wave-lengths of the light employed depart from that to which the plate is most sensitive. The amount of this increase for Seed 27 plates was found to be so slight that it is usually masked by the accidental errors.

The appearance of F. C. Jordan's important paper on "The Color-Changes of Certain Variable Stars of Short Period"[1] calls for this second note, since the question of the gradation of the plates is vital to his conclusions and is not explicitly treated in his paper. However, in Table II of his paper are given grating measurements of Pleiades stars on Seed 27 plates, which will yield evidence on this question when the stars are arranged according to color and magnitude. This evidence is shown in Table II*a* and II*b*, in which column 1 gives the designation of the stars in Bessel's notation, column 2 the B.D. number in zone +23°, column 3 the photo-visual magnitude and color-index, taken from Miss Parsons' tables in this *Journal* (**47,** 42, 1918), columns 4, 5, and 6 the difference between the square roots of the diameters (normal minus grating) and the mean value, for the three divisions of Jordan's table. Columns 7 and 8 give the mean values of these differences, grouped in Table II*a* according to magnitude, and in II*b* according to color-index.

The first table shows no appreciable systematic error depending on magnitude, if we except the two naked-eye stars Bessel *f* and *h*. These two images are 38′ from the optical axis (a distance greater

[1] *Astrophysical Journal*, **50,** 174, 1919.

than is customary to use for photometric purposes) and somewhat elongated, enough to explain the excessive values of Δ. The remaining small progression in the values of Δ, from −0.03

TABLE I*a*

EFFECT OF COLOR ON SCALE—CRAMER TRICHROMATIC PLATES, FILTER β 10
PLATE UV 504

Star	Spectrum	ΔS 8^m-1^m	Means: Spectrum	Means: ΔS	$\frac{\Delta(\Delta S)}{\Delta S}$
17	A0	9.65			
+25°2498	A0	8.42			
21	A0	8.74			
16	A2	8.76			
12	A2	9.46			
13	A2	9.33	A3	8.62	
+26°2326	A4	7.75			
+26°2329	A4	6.61			
+26°2343	A4	7.51			
14	A5	8.81			2 per cent
+26°2345	A6	8.28			
18	Fp	8.09			
+24°2455	G	9.48			
			G	8.78	
+27°2114	G0	8.97			
15	K5	8.60			

TABLE I*b*

PLATE UV 1153

Star	Spectrum	ΔS $g-f$	Means: Spectrum	Means: ΔS	$\frac{\Delta(\Delta S)}{\Delta S}$
17	A0	6.70			
21	A0	6.31			
12	A2	7.65			
13	A2	6.41	A2	6.86	
16	A2	6.64			
+26°2329	A4	7.4			
14	A5	6.93			5 per cent
18	Fp	7.18			
+27°2114	G0	7.76			
			G2	7.20	
7	G5	6.6			
15	K5	7.30			

to +0.01, is 2.6 per cent of the quantity measured and might indicate a slight error in the square-root formula used by Jordan.

The second table (II*b*), in which the stars are grouped according to color, also shows a slight difference in the gradation of the plate between the group of sixteen stars having a mean color-index of +0.01 (therefore white stars) and the group of four stars having a mean color-index of +0.33. The difference in gradation between

TABLE II*a*

EFFECT OF COLOR ON SCALE—SEED 27 PLATES, NO FILTER
ARRANGED IN ORDER OF MAGNITUDE

Designation		Mag. *P-V*	Δ_1	Δ_2	Δ_3	Means	
Bessel	B.D					Mag.	Δ
f......	557	3.73	$+0^{M}.12$			4.44	$+0^{M}.18$
h......	558	5.15	+0.24				
p......	536	6.24	+0.03	$-0^{M}.14$			
s......	556	6.56	−0.04	−0.11			
32......	561	6.66	+0.07	+0.09		6.68	−0.03
24......	540	6.93	−0.02	+0.06			
29......	553	6.99	−0.05	−0.04			
19......	537	7.04	+0.01	−0.05			
17......	535	7.07	−0.08	−0.07			
4......	512	7.18			$-0^{M}.21$		
22......	538	7.19	−0.17	−0.05		7.39	−0.02
31......	560	7.54	+0.26	+0.03			
23......	530	7.81	−0.04	+0.25			
7......	517	7.87			−0.02		
33......	562	8.01	+0.11	−0.06			
1......	510	8.06			+0.02		
18......	534	8.27	−0.21	+0.09		8.30	+0.01
27......	549	8.31	+0.12	+0.02			
13......	528	8.46	−0.09	+0.03			
15......	531	8.68	+0.04				

these two groups, 0.04 = 2.6 per cent of the range measured, is in the direction opposite to that shown by the Coma stars in the former paper, but, being within the accidental errors of the plate, cannot be considered of much significance. The conclusion therefore seems justified that for ordinary plates the gradation is not appreciably affected by star-color.

The effect on the photo-visual scale remains to be considered, for the color-sensitive plate used behind a yellow filter is only

affected by the rays of wave-lengths near the yellow, whereas the plates themselves are fully twice as sensitive to the blue rays. A priori reasoning from the behavior of the ordinary plate would seem useless in this case; therefore the plates themselves must yield the evidence. Jordan states near the foot of Table II that "trichromatic plates and filter were also tested with the grating," but he gives no details. Presumably these were also plates of the Pleiades.

TABLE II*b*

ARRANGED IN ORDER OF COLOR-INDEX

Designation		Color-Index	Δ_1	Δ_2	Δ_3	Means	
Bessel	B.D.					Color-Index	Δ
h	558	$-0^{\mathrm{M}}.19$	+0.24				
29	553	−0.17	−0.05	$-0^{\mathrm{M}}.04$			
f	557	−0.13	+0.12				
32	561	−0.12	+0.07	+0.09			
31	560	0.00	+0.26	+0.03			
23	539	+0.01	−0.04	+0.25			
22	538	+0.02	−0.17	−0.05			
4	512	+0.04			$-0^{\mathrm{M}}.21$	$+0^{\mathrm{M}}.01$	$+0^{\mathrm{M}}.01$
33	562	+0.04	+0.11	−0.06			
13	528	+0.05	−0.09	+0.03			
19	537	+0.08	+0.01	−0.05			
7	517	+0.08			−0.02		
p	536	+0.09	+0.03	−0.14			
1	510	+0.09			+0.02		
24	540	+0.11	−0.02	+0.06			
18	534	+0.16	−0.21	+0.09			
27	549	+0.21	+0.12	+0.02			
15	531	+0.37	+0.04				
s	556	+0.30	−0.04	−0.11		+0.33	−0.03
17	535	+0.43	−0.08	−0.07			

with a small range of color, as is seen in Table II*b*, and therefore not so well adapted for the purpose as plates of the Coma group. However, two plates of the Coma group taken with the Zeiss ultra-violet camera are available. These were Cramer trichromatic plates taken behind filter β 10, the counterpart of filter β 7 used by Jordan on the 2-foot reflector, so that the results will be strictly comparable with those found by Jordan. The data obtained from measures of these two plates are given in Tables I*a*

and Ib, where column 1 gives the Bayer or B.D. number of the star, column 2 the spectral class found from objective-prism plate OP 38, column 3 the difference in scale-reading between two exposures. In case of plate UV 504 the exposures were of different times, eight and one minutes respectively; on UV 1153 the free image (f) was compared with the central image (g) of an exposure through grating R 6. The measures in column 3 were made by comparing the focal images with an artificial scale formed by a series of images of a polar star taken with a constant time-ratio, $\sqrt{2}$. The means of spectral class and ΔS are given in columns 4 and 5 and the percentage of increase in column 6. As in the previous tables, these differences are taken in the sense that they increase with the gradation (contrast) of the plate; therefore with the trichromatic plate and filter, these two plates show a slightly greater contrast ($3\frac{1}{2}$ per cent) for the colored stars of spectral class G than for the white stars. This may be interpreted as following the same rule as the ordinary plate, the contrast being greater for the yellow light to which the combination of plate and filter is most sensitive, and less for the white light. In this case also the amount of the increase is of the same order as the accidental errors of the plate.

SUMMARY

1. The effect of star-color on the gradation (contrast) of the photographic plate is small, being of the same order as the accidental errors.

2. The direction of the change in gradation is found to agree with Abney's law that it should increase as the wave-length of the light employed departs from that to which the plate is most sensitive.

3. This is found to be true for the ordinary (Seed 27) plate, giving photographic magnitudes; and also for the combination of Cramer trichromatic plate and yellow filter, giving photo-visual magnitudes.

Yerkes Observatory
July 1919

ON THE DIFFUSING ACTION OF THE SUN'S GASES AS THE CAUSE OF THE APPARENTLY SHARP SOLAR BOUNDARY

By C. G. Abbot

I have recently received a pamphlet entitled *Etudes sur le Rayonnement Solaire* (*Extrait des Archives Neerlandaises des Sciences Exactes et Naturelles* (III A), **5**, 1, 116, 131, 1918). This pamphlet contains papers by J. Spijkerboer, W. H. Julius, and B. J. Van der Plaats. In the first of these papers Mr. Spijkerboer deals with my views as to the diffusing effect of the gases of the sun as the cause of the sharp boundary of it. It seems to me he has not carefully read my statements and so has overlooked some considerations which go to support my conclusions. As the matter is of great interest, I venture to explain my views somewhat further. Referring to page 109 he says:

Abbot aussi s'est occupé de l'évaluation de la profondeur des couches diffusantes du soleil, mais il arrive à une valeur qui me paraît trop petite.

Il dit que, d'après des mesures effectuées à Mount Wilson, il se perd par diffusion dans l'atmosphère terrestre 5 pour cent du rayonnement solaire pour la lumière jaune. A son avis une couche contenant 75 fois plus de gaz doit donc produire la perte par diffusion de 99 pour cent au moins du rayonnement. Or, cela est inexact.

A la page 50 (Tableau XVII) de notre travail il est dit qu'une couche pour laquelle H = 1/10 laisse passer, dans une direction perpendiculaire, 95 pour cent du rayonnement incident. Mais une couche pour laquelle H = 8, c. à d. 80 fois plus grand, laisse encore passer environ 17 pour cent du rayonnement incident. Pour trouver une couche qui ne donne que 1 pour cent du rayonnement et dans laquelle il se perdrait 99 pour cent par diffusion, H devrait être très grand.

La formule de Schuster

$$R_0 = \frac{2}{2+H}S$$

(voir p. 7 et se rappeler que $H = st$), qui peut être admise comme grossière approximation, nous donne aussi une idée suffisamment exacte de cette question.

Si nous admettons maintenant que la masse gazeuse doit être 10 fois plus grande que ne l'a calculé Abbot, nous ne pouvons pas dire, eu égard aux variations de pression et de température, que nous devons aussi aller à une profondeur dix fois plus grande, mais dans tous les cas il doit encore venir de la lumière de profondeurs beaucoup plus grandes que ne l'indique Abbot.

He has not noticed that my statement dealt only with the direct beam. I had stated: "If, as computed by Schuster, the quantity of gas in the vertical column of atmosphere above Mount Wilson is sufficient to scatter from the direct beam of yellow sunlight six per cent of its light, a column containing seventy-five times as much will suffice to scatter ninety-nine per cent."[1]

My result follows directly from the extinction formula of Bouguer and Lambert, whose accuracy is capable of experimental demonstration. It will be convenient to refer in this connection to a paper entitled "New Evidences of the Intensity of Solar Radiation Outside the Atmosphere," *Smithsonian Misc. Coll.*, **65**, No. 4, by Messrs. Abbot, Fowle, and Aldrich. Referring to page 7 of that publication, Mr. Fowle showed that the observed atmospheric transmission coefficients above Mount Wilson differ by very small amounts from those which would be found for dry air. Employing the values observed on clear days, with trifling corrections for humidity, Mr. Fowle has been able to compute, according to Lord Rayleigh's theory of diffusion, the number of molecules per cubic centimeter in air at standard temperature and pressure. He finds the value 2.70×10^{19}, agreeing almost identically with Millikan's value 2.705×10^{19}, which is obtained by wholly dissimilar laboratory methods.

This indicates that we may regard the results obtained on the transmission of light through the atmosphere above Mount Wilson on the clearest days as fairly representative of those which would be obtained in a gas free from dust, as we must suppose the gas of the sun to be.

Referring now to page 26 of our publication, we see that on September 20, 1914, Bouguer's formula for the atmospheric transmission represented all the observed intensities found by spectro-bolometric analysis within an error of generally much less than 1 per cent. In other words, the direct transmission of a dust-free gas is equal to a constant which we call the transmission coefficient raised to a power proportional to the number of molecules of the gas traversed. This is confirmed by experiment through a range of path including twenty times as many molecules as are found in

[1] *The Sun*, Appleton, 1912, p. 244.

the vertical layer of atmosphere above Mount Wilson, and no doubt it holds for indefinitely thicker layers where monochromatic radiation is concerned.

As the 75th power of 94 per cent is somewhat less than 1 per cent, it seems that my statement quoted from *The Sun* is correct, and we may justly conclude that less than 1 per cent will remain in the *direct* beam of yellow light which traverses a path in the solar envelope which contains 75 times as many molecules as does our atmosphere vertically above Mount Wilson.

This statement is entirely harmonious with Mr. Spijkerboer's, for while the *direct* beam is thus practically cut off in traversing such a layer, yet that does not prevent the light scattered to and fro by molecules from reaching the boundary of the sun by a tortuous course from layers situated much deeper than that which is just indicated.

I would draw attention, however, in this connection to the fact that the formula of Schuster and statements of Spijkerboer are based upon the assumption of zero absorption for the gas which forms the path of the rays in question. As shown by Plate XVIII of my book *The Sun*, Dr. Gale has found that the effect of pressure is to broaden greatly the lines of emission of the spark spectrum of titanium, so that under a pressure of seventeen atmospheres in carbon dioxide gas the narrow lines are broadened out to be a continuous spectrum of quite considerable intensity. This being so, it must follow that gases under such pressures must be not only radiating but absorbing over a continuous spectrum. The pressures which exist in the outer solar envelopes are not well known, and perhaps cannot be determined accurately, but yet in view of the enormous gravitation of the sun and the large mass of gas which would be required to contain 75 times as many molecules as exist above Mount Wilson, one would be justified in supposing that at least toward the bottom of such a layer the pressure must be very considerable, and amounts no doubt to a great many atmospheres. Hence we may suppose that the absorption of such a gas all through the spectrum is by no means negligible. This is the more probable when we reflect that a great number of chemical elements, many of which have very numerous spectrum lines, are mixed together, so

that there is scarcely a wave-length of the solar spectrum not close to some line of powerful absorption by some of these elements. Hence the tendency is to reduce our estimate of the depth from which the rays may come both in the direct beam and tortuously by scattering. I am inclined to think, in view of this consideration, that possibly my estimate is after all not too small, even if it is taken for the limiting depth from which yellow rays can come to the surface by *any* path whatever.

However this may be, the point I wish to make and to emphasize is that a *direct* beam of yellow light traveling by a geometrical path as in a homogeneous medium (by which I mean a smooth curve and not a zigzag line of many elements) will be practically cut off in the sun within a distance which contains roughly 75 times[1] as many molecules of gas as our atmosphere above Mount Wilson, owing to the molecular scattering such as produces the prevailing blue of the sky.

As for radiation of other colors, the results vary rapidly with wave-length, as shown in the following table. This gives the coefficients of vertical atmospheric transmission above Mount Wilson for dry air, as determined by Fowle[2] from our spectrobolometric observations, and the corresponding exponents giving the proportionally increased number of molecules, compared to those of our atmosphere, required for a scattering of 99 per cent from the *direct* beam.

Wave-length in μ	0.350	0.397	0.452	0.503	0.598	0.686	0.812	0.987
Coefficient......	.632	.752	.840	.885	.913	959	.980	.987
Exponent.....	10	16	26	38	50	110	228	350

Both the result expressed in this table and the relative paucity of strong absorption lines toward longer wave-lengths tend to show the condition accepted by Spijkerboer, and dwelt upon in my book *The Sun*, and by Schwarzschild as quoted by Spijkerboer, namely, that the longer wave-lengths arise at lower levels in the

[1] Or better, 50 times, as stated in the next paragraph.

[2] See Table 6 of our publication above cited. I prefer these data to those used in the statements quoted from *The Sun*.

sun than the shorter ones. At the same time it is to be noted that the immense pressures which must soon be reached tend to increase rapidly both the absorption and the scattering, so that actual increase of linear depth with wave-length is by no means so rapid as would otherwise appear.

Let us now turn to the larger question, namely, the cause of the apparently sharp boundary of the solar disk. Mr. Spijkerboer says:[1]

> Les trois explications suivantes du bord solaire méritent notre attention.
>
> A. Qu'on se rappelle qu'une hauteur de 700 km sur le soleil correspond à 1″ d'arc. Si le pouvoir rayonnant diminue, au bord, d'une façon continue, il est vrai, mais si rapide qu'à une hauteur de 700 km au dessus d'une couche émettant de la lumière blanche il se trouve déjà une couche gazeuse ne donnant plus un spectre continu, on doit alors observer un bord net (Schwarzschild).
>
> B. La limite nette du soleil s'explique par la réfraction régulière des rayons, due au gradient de densité dans le sens radial (Schmidt).
>
> C. La courbure des rayons par des gradients de densité irréguliers est la cause de la netteté de la limite (Julius).
>
> Nous excluons immédiatement une quatrième explication de la netteté du bord du soleil, celle que donne Abbot et qui est uniquement basée sur la diffusion. Abbot, en effet, commet une erreur dans ses considérations: il ne fait attention qu'à la longueur des chemins a l'intérieur de la masse gazeuse diffusante et ne tient aucun compte des conditions de rayonnement dans lesquelles se trouvent les points des chemins parcourus, chose qui, dans une saine compréhension du problème de la diffusion, aurait dû etre considérée en premier lieu.

With regard to the summary and adverse manner with which Mr. Spijkerboer deals with my view of the cause of the sharp boundary of the sun, it seems to me he has singularly misunderstood my position. At the very outset I assumed exactly the same rapid decline of temperature and density to account for the rapid decline of luminosity toward the solar boundary as does Schwarzschild and also Emden. Not only scattering but also the effect of temperature is dwelt upon by me with particular emphasis. My view was stated in the following words:

> It will be assumed: *a*) The sun, excepting perhaps in sun-spots, is wholly gaseous or vaporous. Except in sun-spots the photosphere is too hot to contain solids or liquids. *b*) The density of the gases rapidly diminishes, and their temperature rapidly falls from within outward across the apparent boundary of the sun.

[1] *Op. cit.*, p. 98.

It seems probable that gaseous scattering alone prevents us from seeing toward the center of the sun, when looking directly at the middle of the solar disk, to more than 5,000 miles below the reversing layer.

At the limb of the sun, the direct line of sight to a position at the same distance radially below the reversing layer would traverse fully 60,000 miles of gas. Accordingly, to obtain our column containing the requisite quantity of gas for practical extinction of yellow light, at the limb we should penetrate a layer which, measured along the radius, would be very much thinner than that required at the center of the disk. For, even to a radial depth of only 500 miles, the direct line of sight is almost 20,000 miles.

These considerations seem to point to a reasonable explanation of the sharp boundary of the sun. For at the edge of the disk, owing to the oblique line of sight, gaseous scattering will probably extinguish almost all yellow light starting from more than 500 miles below the chromosphere, while an even less thickness suffices for blue or violet light. It is plain that an indistinctness of outline corresponding to a layer of this depth would not be readily recognized on the solar image, since it corresponds to only about one second of arc. Furthermore, the direct line of sight takes in not only the nearer, but the farther solar hemisphere as well. A still thinner stratum than 500 miles would, therefore, suffice to contribute all the light that can be contributed to the beam directly along the line of sight. We therefore conclude that within a small part of a second of arc below the reversing layer the sun would appear as a solid body, even though entirely gaseous.

. . . .

Importance of Temperature.—It will be noted that in the solar hypothesis we are recommending the temperature plays a most prominent part. . . . . The darkening toward the limb is regarded primarily as a temperature effect, secondarily due to scattering. Owing to scattering, the effective radiating layer must necessarily be nearer the surface, and hence cooler, at the limb than at the center of the disk. We say it must be nearer the surface: For, travelling obliquely, a ray must become extinguished by scattering in the gas at the limb, before it reaches the same radial depth that it does if travelling radially at the center. The darkening at the limb would naturally be greater for violet than for red rays, firstly, because with all incandescent bodies a fall of temperature causes more decrease of radiation for short rays than for long; and, secondly, because molecular scattering is greater for violet rays than for red, and hence at the sun's edge the effective radiating layer for the violet will be more near the surface than will that for the red.[1]

See also the following:[2]

Under solar conditions, contrasted with terrestrial ones, scattering takes place on rays arising from every direction instead of from one direction alone.

[1] *The Sun*, pp. 237 and 245.

[2] *Astrophysical Journal*, 44, 39, 1916.

Accordingly the beam which seems to come from the center of the sun's disk, as (to use a homely illustration) the handle of an umbrella comes from its center, really comes largely by scattering from all sides, just as the strength of the umbrella handle arises from its ribs.

For brevity I have omitted the data which led me to the numerical values given. These the reader may consult in my book if he pleases. In the latter part of this quotation I speak repeatedly of "the direct beam." As I have said above, I do not at all deny that rays may come to the eye through an indirect zigzag course from points situated within the sun at greater distances than those indicated, but, as I have also said, the most certain existence of absorption in the solar gases tends to limit us even in this possibility, so that perhaps even the farthest depth of the medium from which the yellow rays come to us by any course whatever may not greatly exceed the depth which I have here assigned.

What seems clear, however, is this, that whatever objects might lie as a background within or behind our sun, their rays could not pass through the solar gases so as to yield any optical image of the supposed background when the path of these rays traverses a great depth below its exterior. For there would be found in the path abundantly sufficient molecules practically to extinguish all *direct* rays from the farther source. The sun up to within a few miles of its edge would then, on account of scattering alone, behave like a solid body as far as concerns interposing a perfect obstacle to the passage of the direct beams of light required to make optical images.

On account of the great diameter and spherical form of the sun the relative number of molecules found in a direct line of sight increases with extreme rapidity as the line of sight passes inward from the outer boundary of the sun. The absolute increase of the number of molecules depends of course upon the density of the gases at the region in question. The brilliance of the sun in this neighborhood depends both on the density of the gases (which influences the intensity of the emitted and scattered light) and on the temperature of the gases on which depends the intensity of the primary radiation. These are the considerations which led me to introduce the second assumption quoted above from my book *The Sun*, namely: "*b*) The density of the gases rapidly diminishes and

their temperature rapidly falls from within outward across the apparent boundary of the sun." This stipulation appears to have been overlooked by Mr. Spijkerboer in his statement to the effect that I had not considered the conditions of radiation at the limb of the sun.

When we consider the complexity of the conditions: (1) that the depth at which the rays we see arises depends on their wave-length as influencing both scattering and absorption; (2) that their intensity depends on the temperature and absorption of the gases through which they pass, as well as on the scattering by these gases, and (3) that the absorption and scattering depend upon the density, which itself is a function of the temperature of the gases, and (4) that the density depends upon radiation-pressure as well as on gravitation, it seems clear that it is almost futile to investigate theoretically the distribution of brightness over the sun's disk, as has been done by Mr. Spijkerboer and others, without first investigating by the aid of thermodynamics and otherwise the probable distribution of density and temperature toward the limb of the sun.

There is one condition which simplifies the matter, for it makes it unnecessary to investigate the density of the outermost regions of the solar gases. This is as follows:

We see the sun on a bright field of sky-light produced by scattering of the solar rays within our atmosphere. Even though the solar gases should continue with gradually diminishing density out to a vast distance beyond the apparent solar boundary, these appendages could not be seen wherever their combined scattered and emitted light is inferior to the light of the sky, except during total eclipses. The farthest visible limit to the sun must therefore be set where its combined diffused and radiated light equals in brightness the brightness of the sky. Hence we see a bigger sun during a total eclipse than ordinarily. This consideration removes from the problem the idea of very far-reaching, gradually fading extensions of the sun's brightness such as the proposition of a purely gaseous sun brings up to some minds. According to spectrobolometric measurements made at Washington,[1] May 14, 1907, the brightness of the sky outside the sun's limb was as follows. The

[1] *Annals Smithsonian Astrophysical Observatory*, **2**, 22, 1908.

brightness at the sun's center in the wave-lengths given in each case is taken as unity.

Distance in solar radii from sun's center	1.005	1.010	1.015	1.020	1.025	1.030	1.035	1.040
Brightness: Wave-length 0.46 μ	.033	.015	.007	.004	.002	.001	.000	.000
Brightness: Wave-length 1.03 μ	.058	.024	.013	.010	.007	.005	.004	.003

It is not claimed that these results are of great accuracy. They were obtained at a station where there is considerable humidity and dust which varies from moment to moment, so that the two series of measurements may not be strictly comparable. Besides this, optical and instrumental sources of error tend to make the brightness observed somewhat too large. However, they indicate that the sky's light close to the sun's disk is probably very much brighter than the inner solar corona at corresponding points, as it was observed bolometrically at Flint Island January 3, 1908, as follows.[1] The brightness at the sun's center is taken as unity.

Distance in solar radii from sun's center	1.10	1.25	1.75
Brightness	0.0000013	0.0000004	0.0000000

Unfortunately the two series do not quite overlap, but it is well known from photography of the solar corona and chromosphere at total eclipses that the growth of brightness toward the sun's limb would leave the inner coronal brightness, within the range of positions above stated, far short of the sky brightness as observed at Washington.

Having cut off from consideration the region beyond 1.01 apparent solar diameters, our problem is to explain why the sun brightens, with sudden rapidity, certainly more than tenfold, and probably more than one hundred fold, from about 1.01 to 0.99 solar radius.

The gases of the outer layers of the sun are light-giving because at high temperature. Suppose, however, for the moment that this were not the case, and that like the terrestrial atmosphere they were

[1] *Ibid.*, 3, 5, 1913.

too cool to radiate visible rays. How bright would the sun's envelope then appear? Without attempting any accurate reply to this question, we may at least partially fix our ideas regarding it by considering the observed brightness of the terrestrial atmosphere under the clearest conditions. As already stated, observations indicate that, at clearest, our atmosphere may be regarded as a purely gaseous scattering medium.

Referring to Table 50 (*Annals Smithsonian Astrophysical Observatory*, 3, 145, 1913), neglecting the very bright region of sky close to the sun, the average intrinsic brightness of the whole hemisphere of the sky, measured for all wave-lengths combined in terms of that of the sun as unity and observed from Mount Whitney, California (altitude 4400 meters), on an exceptionally clear day was 0.0000015. The corresponding figure for Mount Wilson (altitude 1700 meters) on a day not so clear[1] was 0.0000035. Taking into account that (1) the average intensity of the rays shining on the air was somewhat reduced by preliminary scattering in the higher atmosphere; (2) the mass of air producing scattering above Mount Whitney is less than that above Mount Wilson; (3) the average mass of air involved (on account of obliquity) is to be regarded as perhaps twice as great as that corresponding to zenith sun; (4) according to Rayleigh's theory scattering is as effective backward as forward; we may on the whole set the average intensity of radiation of scattering corresponding to a layer of gas equal to that of the vertical thickness of our atmosphere above Mount Wilson, for hemispherical illumination, at 0.000001 times the intensity of the incident beam.

We view the sun as 0.00001 of a hemisphere. The gases close to the sun would receive its rays over nearly a full hemisphere. Accordingly we conclude that on the basis of the preceding figures a layer of solar gas containing as many molecules as the terrestrial atmosphere vertically above Mount Wilson would appear one-tenth as bright as does the sun owing to the scattering alone.

The brightness of the scattered ray increases with the thickness of the scattering gas, but not so rapidly. Thus from the original observations at Mount Wilson September 22, 1913, which yielded

[1] *Astronomical Journal*, 28, 133, 1914.

one of the results quoted above, I find that for a solar altitude of 46° the sky brightness in the sun's azimuth was as follows:

Altitude	2°47′	7°18′	15°56′	26°5′	36°46′	57°15′
Air mass	15.5	7.4	3.6	2.3	1.7	1.2
Observed brightness $\frac{\text{sky}}{\text{sun}} \times 10^8$	860	790	660	630	880	650
$\frac{1}{2}\{1+\cos^2(\phi-46°)\}$ (See Rayleigh's theory)	0.76	0.80	0.82	0.94	0.98	0.98
Corrected brightness ratio (Line 3 ÷ line 4)	1130	990	800	670	900	660

In the last line of the table I take account of the fact that the scattering in any direction is governed by the angle which the scattered ray makes with the incident light. By this allowance I take away the influence of the angular distance of the sun from the point of observation, so that in the final line we see the variation of scattering as influenced merely by the variation of air mass indicated in line 2. As Rayleigh[1] states that the expression in line 4 is not applicable close to the direction of the incident beam, it is probable that the two last values are too high on that account.[2] The march of the values shows that the intensity of the scattered beam increases fairly slowly compared to the increase of number of scattering molecules. I do not know whether this would be so along directions nearly in line with the incident beam.

The numerical results given show that a layer of gas containing as many molecules as our atmosphere does vertically above Mount Wilson would probably scatter to the earth light of one-tenth the intensity of the sun, and that a great increase of the number of molecules in the path of the ray observed might strengthen the scattered light at least several fold. Thus neglecting radiation altogether (though in my view it is not a factor to be neglected) the scattering alone is sufficient to give illumination comparable to that which is found at the solar boundary, assuming there a very moderate thickness of solar gases at densities comparable

[1] *Phil. Mag.*, **41**, 1871.

[2] Professor R. W. Wood has lately shown me unpublished experimental results which indicate that the great brightness near the direct solar beam is due to motes of dust which produce intense diffraction of the ordinary sort.

with those in our atmosphere. The question that now remains is, How does the density of the solar gases increase as we go toward the center of the sun?

Referring to the work of R. Emden,[1] he states that the laws of thermodynamics, like the Newtonian gravitational laws, have no creative capacity. Just as little as on the ground of the gravitational laws our planetary system may be built up, even if one knows the masses of the central body and the eight planets, just so little can the relations of temperature and pressure on the sun be determined merely by the aid of thermodynamics, even though we have a correct knowledge of the values of the radius and the mass. For it is shown by the theory that for a given radius and given mass there may exist innumerable different solar organizations, each of which would remain in gravitational equilibrium. Thus it is quite impossible, according to Emden, to fix the distribution of density and temperature by merely theoretical considerations based upon the known mass and radius of the sun. His study of the matter does not at all limit him in the consideration of the probable rate of change of density and the temperature near the solar boundary. In his opinion[2] there is a layer some hundreds of kilometers thick near the boundary of the sun for which the density and pressure diminish so very rapidly that at a great distance from the sun, such as that of the earth, the 725 kilometers which correspond to an angle of 1″ of arc may diminish in brightness so rapidly as to give the illusory though apparent impression of a sharp boundary of the sun.

This condition, which Emden finds theoretically possible and which he regards as probable, is all that is required to meet my views of the sun's sharp boundary. I have shown that a layer of gas containing a moderate number of molecules is sufficient to cut off the direct beam coming from the regions which lie beyond. The same layer of gas is competent to scatter to the eye rays of nearly the same order of intensity as are found at the border of the sun's disk. It appears from the work of Emden that it is quite within the bounds of theoretical possibility that the increase of density

[1] *Gaskugeln: Anwendungen der mechanischen Wärmetheorie*, p. 404, 1907.

[2] Page 308.

as one goes inward along the solar radius may be sudden and rapid enough to meet the requirements of this view.

SUMMARY

Owing to molecular scattering and true absorption by its gases, radiation cannot travel more than perhaps 7000 kilometers in those outer layers of the sun which lie immediately beneath the chromosphere. Outside of the chromosphere the gaseous density is so small that neither by radiation nor scattering is the brightness comparable with that of our atmosphere near the sun. Parts below a depth of 7000 kilometers furnish us no rays at all but merely keep outer parts hot. Hence, so far as it is a source of radiation to the earth, the sun is like a hollow shell. This shell is thicker for longer waves than for shorter ones. It is also thicker at the center of the solar disk than at the limb. For, owing to obliquity of the line of sight, the rays traversing 7000 kilometers lie near the surface at the limb, and are there probably confined within a layer less than $1''$ of arc, or 725 kilometers thick. Hence the solar disk appears sharp, for an indefiniteness of $1''$ of arc is not distinguishable from effects of bad seeing. Terrestrial examples of gaseous scattering show that the rapid increase of brightness at the solar limb is well accounted for if only the increase of density there is as rapid as was supposed by Emden and Schwarzschild. Readers are referred to the preceding article for support of these statements.

Smithsonian Astrophysical Observatory
Washington, D.C.
September 1919

Minor Contributions and Notes

THE GENERAL ILLUMINATION OF THE SKY

In his recent interesting article, published in the May number of this *Journal* (**49**, 266), Mr. V. M. Slipher has shown by irrefutable photographic observations that the green ray of the aurora is permanently present in the light of the sky. In consequence. Mr. Slipher questions whether the light of the aurora does not contribute an appreciable part in the total light of the heavens, which has been measured visually and photographically by different observers.

As to my own photographic measurements,[1] it seems to be certain that the ray at λ 5578 could not have had an appreciable effect on the result, for the following reasons: (1) The plates employed had an extremely slight sensibility to the radiation λ 5578. If that ray had been able to affect the plates appreciably, the visual observation would have given a brightness much more intense than that which was observed. (2) The photographic measures give very constant results, both from one day to another and in different years. Our measures were made in 1909. New determinations were made in 1917 by Mr. H. Bourget,[2] who employed the same method, and his results are almost identical with mine.

As to the visual method, Mr. Slipher lays stress on the discordance between my photographic determinations and the visual measures of Mr. Yntema;[3] but, per contra, the measures of Newcomb and those of Burns are in very good agreement with the photographic measures and completely exclude the possibility that any considerable proportion of the light is due to the so-called

[1] *Astrophysical Journal*, **31**, 304, 1910.

[2] *Comptes Rendus*, **166**, 943, 1918.

[3] For bibliography see Mr. Slipher's article, this *Journal*, **49**, 266, 1919.

green radiation. I am not in a position to give an explanation of the anomalous result found by Yntema. I will only remark that a variation in brightness of from 1 to 4 appears improbable; a brightness four times greater than normal would make the Milky Way almost invisible.

Visual measures on sources of illumination more faint than those of the sky[1] should be very difficult, while the photographic measures are easy and certain. If the visual measures are regarded as useful, it would be better to make them by the photo-visual method, with the use of an orthochromatic plate and a filter.

There are several problems which remain to be solved on the subject of the general luminosity of the sky. In particular there are no reliable observations available which permit us to say whether or not there is a variation, either photographic or visual, related to the solar periodicity. Also, may we not approve of Mr. Slipher's proposition for co-operation in studying this important question? Valuable results probably can be reached with instruments that are inexpensive and with a moderate amount of work. I venture to suggest the following scheme for co-operation in such researches.

1. A small number of regions of the sky, perhaps a dozen, having been selected, the intrinsic brightness should be measured photographically at several observatories (three or four, for example) at fixed dates and fixed hours. We should thus see whether the place, the date in the year, the time of day, or the solar activity had any effect on the luminosity of the sky. It would be well to choose regions of the sky among those which had been studied in respect to the density of the stars.[2]

A single observer in each one of the observatories, devoting one evening a week to these researches, would probably be all that was necessary. The plates could be measured with the microphotometer in a single laboratory.

2. Certain photo-visual measures could be utilized; they would yield the color-index of the general light of the sky.

[1] This brightness is almost that of a perfectly white surface *illuminated* by a candle placed at 70 meters.

[2] By my photographic method it would be necessary to make the measurement on a circle at least 1° in diameter.

3. It would be useful to make observations of the green radiation, not only in a qualitative way, but also in measuring its intrinsic brightness. Mr. Slipher's observations show that the "green" radiation is present even when the aurora is not visible. There is no proof that this diurnal illumination follows the variation of the solar activity, or even that it may be due to the same cause as the aurora (cathode rays emitted by the sun).

For the study of the first two questions, the methods are completely established and may be employed almost without experimentation; but for the third question certain preliminary researches would be necessary. A necessary condition for the fruitful study of the luminosity of the sky is that the situations should be far from large cities. Unfortunately most of the European observatories do not at all fulfil this condition.[1]

CHARLES FABRY

FACULTÉ DES SCIENCES, MARSEILLE
September 18, 1910

[1] The observations by Mr. H. Bourget, as well as mine, were made in the country during a vacation.

ERRATA

Vol. **48**, July 1918, "On Some Phenomena Observed in the Foucault Test," by S. Banerji:

Page 55, sixth equation, *for* $\frac{t}{T}$ *read* $\frac{t}{\tau}$, and similarly in the next line.

Page 56, fifth line, *for K read k;* also in the fourteenth line.

Page 57, the fourth term of expression (II) should read,

$$\frac{c_1}{2C_1k}\left[\sin k(A_1+B_1s+C_1s^2-\phi\xi_1)\cdot s\right]_{-s}^{s}$$

Vol. **48**, December 1918, "Change in Brightness, Spectrum, and Temperature of Nova Aquilae No. 3," by M. Maggini:
Page 309, in the fifteenth and sixteenth lines, *for* = *read* −.

Vol. **48**, December 1918, "The Period of V Tucanae," by B. H. Dawson:
Page 312, fourth column of Table II, last line, *for* +40* *read* −40*.
Page 315, fourth line from foot, *for* $T-21415^{d}427$ *read* $T-2421415^{d}427$.

Vol. **48**, December 1918, "The Radial Velocities of 119 Stars," by Joseph Lunt:
Page 263, footnote to Table I, *for* 229 *read* 299.
Page 265, second remark, *for e read a*.
Page 266, Table III, heading, *for* Cape (3) *read* Cape (2). Table III, fourth star, *for* † *read* ††. γ Aquilae, remarks apply to star 304 not 311.
Page 267, last line of table, *for* +0.09 *read* +0.08.
Page 270, Table IV, last column, for star No. 194 place a ¶ sign.
Page 271, first footnote, *for* plates *read* parts.
Page 273, Table V, last two lines, *for* †† *read* †.
Page 276, Table VI, *for* Leoporis *read* Leporis. For star No. 100, *for* * *read* †. Footnote, *for* Hα *read* Hγ.

Vol. **50**, September 1919, "Review of Recent Work on the Series Spectra of Helium and Hydrogen," by F. A. Saunders:

Page 155, in the second column headed "Formula," *for* $\frac{1}{2^2}, \frac{1}{3^2}, \frac{1}{4^2}$ *read* respectively $\frac{1}{1^2}, \frac{1}{2^2}, \frac{1}{3^2}$.

GENERAL INDEX
TO THE *ASTROPHYSICAL JOURNAL*, VOLS. XXVI TO L

(July 1907 to December 1919)

The *General Index* by authors and by subjects to Vols. I–XXV of the *Astrophysical Journal* (January 1895–June 1907), compiled by Professor Storrs B. Barrett, of the Yerkes Observatory, was issued in 1908. It made a volume of 133 pages, of the same size and style as the *Astrophysical Journal* itself.

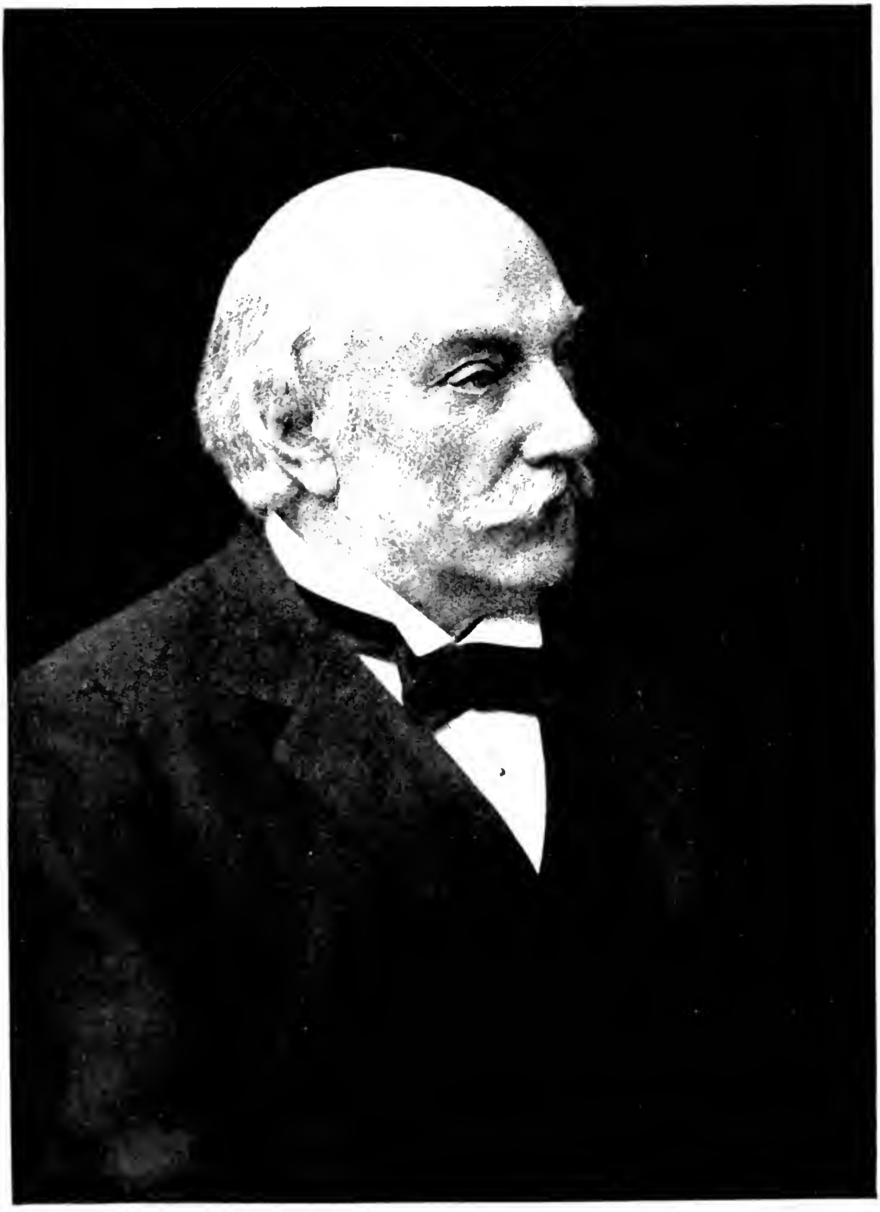
Preparations have now been completed for issuing a similar index, to cover the twenty-five volumes concluding with the December number of the present year, 1919. Professor Barrett also will undertake the preparation of this index.

The price of the volume, in paper covers, has been set, for subscriptions in advance, at $2.50, postage prepaid. The edition will be limited and a much smaller number will be printed than in case of the prior index. It will be of much assistance to the publishers, in determining the size of the edition, to receive orders in advance. Subscriptions should be addressed to the University of Chicago Press, Chicago, Illinois.

It is hoped that the volume will be ready for publication early in 1920.

There are still on hand a considerable number of copies of the first *General Index*, which is sold separately at $1.50. Individual subscribers and libraries which have not heretofore acquired the first index will probably wish to purchase both. For those ordering both indexes in advance of the publication of the new index the price will be $3.25, prepaid. Communications should be addressed to the University of Chicago Press, 58th Street and Ellis Avenue, Chicago, Illinois.

3.2



LORD RAYLEIGH

1842-1919

THE
ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME L — DECEMBER 1919 — NUMBER 5

LORD RAYLEIGH, 1842–1919

By SIR JOSEPH LARMOR and H. F. NEWALL

By the death of Lord Rayleigh on June 30, 1919, the last representative of the great age in theoretical physical science connected with the names of Kelvin, Helmholtz, Stokes, Kirchhoff, Maxwell, has been removed. The novel outlooks opened up by these men into the unfathomable yet intelligible constitution of the material universe have been made secure, though the fruits of their development by the combined labors of the scientific world are far from being yet exhausted. In the last quarter of a century new horizons have opened out, revealing facts and possibilities, in electrical science and in astronomical observation, even more marvelous than the ones which they reduced to intellectual order and system, and made amenable to practical utilization. Their complete comprehension will go yet deeper into the scheme of nature and is still largely a problem for the scientific thought of the future; however novel may be the paths into which it may lead, they will be approached only along the explored highways of the physical science that we have inherited.

John William Strutt, third Baron Rayleigh, was born at Langford Grove, Essex, on November 12, 1842, and succeeded his father in the title in 1873. His early education was frequently

interrupted by delicate health. He was for a very short time at Eton and then for a still shorter time at Harrow; but it was found necessary to send him to the milder climate of Torquay, and he was there put under the care of Mr. G. T. Warner, who had been a classical master at Harrow. In 1861 he went to Trinity College, Cambridge, and apparently notwithstanding this classical bent of his early education he graduated in January 1865 in mathematical science as senior wrangler and first Smith's prizeman. The economist Alfred Marshall was second wrangler in the list.

In 1866 he was elected to a fellowship of his college, which he vacated in 1871 on his marriage with Evelyn, daughter of James Maitland Balfour, of Whittinghame, the sister of Arthur James Balfour, formerly Prime Minister and now Foreign Secretary, and of Francis Maitland Balfour, who in a short life established a great reputation in the science of embryology and the evolution of the forms of life. Of his four sons two survive him: Robert, now professor of physics in the Imperial College of Science and Technology, who succeeds to the title, and Arthur, who was navigating officer on the flagship of the first battle squadron and fought in the Battle of Jutland.

After taking his degree at Cambridge he seems to have retired into a life of study and reflection—mainly in the country, first at Langford Grove, Essex, and, after his succession to the barony in 1873, at the family seat, Terling Place, Witham, Essex—which must have laid secure the foundations for his future eminence in the development of physical ideas. He seems to have made a voyage to Canada in 1867, before the days of swift passage across the Atlantic; for he recalls, in his presidential address to the British Association at Montreal, "the impression made upon me, seventeen years ago, by the wild rapids of the St. Lawrence and the gloomy grandeur of the Saguenay." Later in life, as if this experience had established his confidence in the efficacy of such a voyage as a relaxation and a restorative of vital energy, two other considerable voyages were undertaken by him: one to India in 1897–1898, after his term of office as secretary of the Royal Society, and another around Africa in 1908, after his resignation of the presidency of the Royal Society.

It was in the years following 1869 that he began the publication of important contributions relating to physical science, especially optics and acoustics, in a series of papers which were continued without intermission to the end of his life. They have been gathered together by himself, partly in the treatise on the *Theory of Sound*, published first in 1877–1878 and revised and greatly enlarged in 1894–1895, and mainly in the five volumes of *Scientific Papers*. These volumes, issued in 1899, 1900, 1901, 1902, and 1912 contain the papers, 349 in number, down to 1910; and a sixth volume is now in the press.

His earliest public scientific associations were with the London Mathematical Society, at that time a center of mathematical physics even more than of abstract analysis, as the numerous papers from Maxwell, Rayleigh, Kelvin, W. D. Niven, J. J. Thomson, and others testify. In those early days, at a time when the financial resources of the Society had become inadequate for the support of its publications, he effected insurance against the difficulty by a handsome benefaction, the proceeds of which have ever since been a substantial aid to its activities. His contributions to its *Proceedings* turned largely on hydrodynamics, especially. the theory of waves and of eddies and jets, and the causes of frictional resistance generally. A series of papers on general principles in statics and kinetics, which were afterward condensed into some early chapters of the *Theory of Sound*, started from universal theorems like the maximum and minimum theorems of energy connected with the names of Kelvin and Bertrand, and the reciprocal relations developed in the treatment of problems in sound by Helmholtz; and he widely expanded them in fruitful directions. They stimulated in other minds the utilization (cf. *Proc. Lond. Math. Soc.*, 1884) of the principle of least action as the necessary unique source of all general relations, in a manner that was systematically developed, with wider dependance on Hamilton's dynamical scheme, some years later by Helmholtz in his Memoir in *Crelle's Journal* on that principle.

Official connection with the university was resumed in 1876, when Rayleigh served as additional examiner in the Mathematical Tripos, then held in vacation time in January. In those days, now

remote, the examination papers of that Tripos circulated far and wide in many forms; they were immediately reproduced as a full-page supplement to the *Cambridge Chronicle*, the old-established local newspaper of county and university; later they were reprinted on smaller pages as part of the contents of the *University Calendar*, then an unofficial record, and in the *Cambridge University Almanac and Register*. It was through this newspaper sheet that in youthful days in Ireland one of the writers first came into touch with Rayleigh's name and work. Some years previously the complaints of Airy in regard to artificiality and want of utility of the questions, and of Kelvin, Tait, and others with regard to their unduly abstract character, had led to the institution of the additional examiner, who was a senior Cambridge graduate of distinction, invited usually from the outer world and probably more in touch with it, in order to assist the traditional two moderators and two examiners in conducting the examination, then of so great prestige. In this way the services of Maxwell, Kelvin, Tait, Rayleigh, Watson, C. Niven, Hopkinson, W. D. Niven, Greenhill, Darwin, were enlisted in successive years, with very marked influence on mathematical study. The new types of questions that were introduced directed attention to new branches of knowledge, and to unfamiliar and often novel aspects of other branches, and of necessity stimulated new ramifications in the teaching; and it may fairly be said that this result was as much due to the mere presence of the outside element in loosening the chains of tradition that had previously bound their resident colleagues as it was to their own direct efforts. The influence of men such as Kelvin, Maxwell, Rayleigh, provided the degree of guidance that was salutary; giving stimulus without dogmatic revolution, it justified on the whole the position of the Tripos papers as the controlling influence in mathematical study, notwithstanding minor abuses. Nowadays it has become fashionable in some circles to depreciate the office and the services of the public examiner as a dealer in artificially concise problems and elegancies that contrast with the sweeping march of a classical treatise. His function is often thought to be merely to verify in as wooden a manner as possible that the syllabus has been learned. Rayleigh has expressed or

implied his dissent from this view not infrequently; and he included his Tripos questions in the *Scientific Papers* (I, 280–286). So long as a meticulous counting up of marks, in order to evolve a minutely graded order of merit, is not insisted upon, and only broad views are taken, the canvassing of a new problem or a new turn of inquiry beyond what the student has been taught to master and reproduce affords even when unsuccessful a powerful stimulus to the better minds, and where it is combined with guidance beyond the handbooks of the day to the original and authoritative sources of knowledge it has been in fact the most potent incentive to progress and to transcend the grooves of the popular teaching of the time. The difficulty is that the field tends to expand so vastly; but restriction of each student to a narrow section of knowledge is not the true remedy.

On the urgent insistence of his friends, especially Kelvin and Stokes, Rayleigh consented to return to Cambridge in 1879 as Cavendish Professor of experimental physics, to take charge of the laboratory which had been left without a head by the premature and lamented death of its earliest chief, Clerk Maxwell; and he retained the chair for five years until considerations of health intervened. When he had thus succeeded to the control of a formal laboratory, he was not long in throwing himself into those problems of interconnected absolute measurement for which the methodical arrangements appropriate to such an institution are most essential. Our present age of electrical engineering was just discernible on the horizon; and it is not the least of the merits of the British Association for the Advancement of Science that it provided the leverage by which Kelvin and Maxwell and their coadjutors laid firm and exact foundations for the practical development that was to be so rapid. They secured the acceptance of a scientific nomenclature, based on the metric system, throughout the world, itself no slight achievement in the light of other experiences in similar directions; and they carried through the initial stages of experiment to place this unitary system on an absolute foundation by determining the numerical relations interconnecting the units. It was this latter problem that was taken over at the Cavendish Laboratory, and for the five years of Rayleigh's direction

it was, in the hands of himself and his sister-in-law, Mrs. Henry Sidgwick, along with Schuster, J. J. Thomson, Glazebrook, Dodds, Searle, and other colleagues, the recognized center of standard electrical determinations. In time the other nations took their full share of the work; later, on the foundation of the National Physical Laboratory with Glazebrook as director, the operations were naturally transferred there.

In the early years he developed the principle of Stokes, previously adumbrated by Sir W. R. Hamilton, that in a dispersive medium periodic disturbances travel as groups of waves with speed different from that of a simple wave-train. In a discussion (1881) in which Willard Gibbs intervened effectively, he deduced from the phenomena of remote variable stars and of occultations that radiations of all wave-lengths must travel through the celestial spaces at precisely the same speed. It seems to follow that the ether that is postulated as the frame for optical and electrical science must be a structureless medium; moreover, if that is so, and not otherwise, the light from distant stars will advance to the observer without being in time extinguished by scattering, other than may result from molecules or particles of matter obstructing its path. The visibility of stars through a nebula, presumably of vast depth, here invites consideration.

Forty years ago the refined use of the spectroscope was only beginning, and there was much misconception as to construction. It was not unusual in the arrangement of prisms to aim at mere dispersion of the lines in the spectrum, forgetting that, if the images of the lines are good, separation of them can be effected just as well by a magnifying eyepiece. This question was settled in the hands of Lord Rayleigh by a solution strikingly concise and so illuminating as to be now common form. He showed that the dispersion of colors depends solely on the aggregate effective thickness of the glass traversed by the beam of light, more precisely, is determined by the difference of the thicknesses, multiplied each by index of refraction, summed along the two terminal rays of the beam and divided by the breadth at emergence. This principle assumes its simplest form when the incident and emergent rays are in parallel beams. If the emergent beam is narrower than the incident, part

of the chromatic separation is really due to magnification, which is in the inverse ratio of their breadths, and not to dispersion, but every part is included in the Rayleigh rule. If the emergent beam is converging so that the spectrum is already focused, the breadth of the beam is not definite, but is becoming narrower as it proceeds; yet the universality of the rule is maintained, the angular dispersion of the wave-fronts getting larger when estimated at the narrower sections, as in fact must be so in order that the foci for the two colors that are compared should be the same whatever front is considered. The essential thing to aim at is sharp focusing of the detail of each line in turn, which is usually secured by the criterion of minimum deviation, very different from that of maximum dispersion. If that is gained there will be ample margin for as much subsequent magnification as may seem to be necessary.

The reasoning, though now familiar, will bear repetition as an illustration of the thesis that physical theory advances, not by elaborate calculations of algebra, nor by isolated guesses, but by the insight that can concentrate all the essentials in one view, and disentangle at sight the controlling principles. The result rests on the principle that the time of transit, proportional to $\Sigma\mu s$, must be the same for the two terminal rays AP and BQ, passing between the wave-fronts AB and PQ of the beam. When the color of the light is changed the paths of the rays from A to P and from B to Q are slightly changed also, and $\Sigma\mu s$ is no longer the same for both rays between these terminal points, for each sum is altered by $\Sigma\mu\delta s+\Sigma s\delta\mu$. But the definition of a ray as the path of *most rapid progress* (not always of shortest aggregate time) requires that for any small variation of path $\Sigma\mu\delta s$ must vanish. Thus the difference of the times of transit of the two rays for the new color is represented very simply by that of $\Sigma s\delta\mu$ taken along the rays. The wave-front for the new color must be PQ' when Q' is farther on along the ray BQ, so that QQ' multiplied by the final index of refraction is equal to $\Sigma s\delta\mu$ for the ray-path AP minus $\Sigma s\delta\mu$ for the ray-path BQ; and this means that the new front is inclined at the angle stated in words by the Rayleigh rule. Nowadays the other side of the matter has become practically essential: the scale of magnitude of the spectrum must be increased so much before the

rays reach the photographic plate that the size of grain of the emulsion shall not interfere with details of the impression; and to this degree it is not the same whether the spectrum, supposed to be sharp enough, is magnified by the instrument or by an eye-piece, as it would be for observation by direct vision in which the breadth of pupil is not a limiting factor.

The great advances in physical optics in the middle of last century, mainly in French hands but also very notably in those of Airy and Stokes, proceeded chiefly by utilization of interferences and changes of phase. Thus they implied a single source of light or different sources so nearly in unison that they may be regarded as emitting a single train of waves. Rayleigh's earliest note to the London Mathematical Society enforced the necessity of the consideration of independent molecular sources; and this subject, that of natural radiation as distinct from simple optical trains of waves, thus initiated from small beginnings, remained a predominant interest throughout his life. Why is it that in the explanation of solar coronas and halos the amplitudes of the radiation scattered by the various particles, being erratic in direction, do not simply cancel out and leave no result? In the current expositions he found no adequate reason why the particles scatter independently so that the energies are additive, and the mean resultant amplitude therefore not nothing, but definite through only $\sqrt{n}$ times a single amplitude, where n is the (very great) number of particles. This led him into lifelong refined discussions of the probabilities or expectations of different resultant values, when the component vibrations or other events are fortuitous, on which he based himself on the laws of probabilities elucidated by Pascal and James Bernoulli and systematically developed in Laplace's great *Treatise*.

Afterward the earlier resuscitation, and development on similar lines of precise statistical analysis, of the kinetic molecular theory of gases by Maxwell had brought the doctrine of equipartition of energy between the various degrees of freedom of the molecules into the foreground. In the hands of Maxwell and of Boltzmann the proof of equipartition did not stop short of application to any vibrating system whatever, obeying the Hamiltonian dynamical

relations; all its modes of freedom seemed to be of equal value. Yet these freedoms for systems involving continuous vibrating media are infinite in number, so that the thermal energy in such a system should fritter away indefinitely into modes of higher and higher frequency. Maxwell did not live to express his views on the paradox of the universal application of his principle of equipartition when Kelvin initiated and pressed home on many occasions an attack on its general theoretical validity. It was indeed difficult, as in most questions of refined probability, to specify exactly and universally what the theorem was; but of the various special systems propounded by Kelvin as instances of its failure there were none that really fell within the accurate enunciation of the principle. It was owing largely to Rayleigh's luminous and critical expositions, still the best account of the subject, that this result was established; though he repeatedly asserted his conviction that some mode of constraint must be at work, overlooked in the statistics, which really inhibits the energy from passing over into vibrations of the most rapid types. In particular he was suspicious of the introduction of mathematical constraints in the argument; for a geometrical constraint is physically merely an elastic connection whose elasticity is so stiff that its period of free vibration is extremely high, and in consequence it never gets into vibration at all. So also a rigid body is one in which waves of deformation equalize themselves with infinitely great velocity of propagation, and therefore deformations never get started into being. Later on this question of equipartition entered on a new phase from the anomalous values found for specific heats at extremely low temperatures, and also from the form of the experimental law of distribution of intensity among the components of natural radiation. In a brief but most remarkable note Rayleigh indicated what the law of distribution for natural radiation ought to be on an equipartition basis; and, when the coefficient in his expression had been numerically corrected for an oversight by Jeans, it was in precise agreement with experimental fact for very long optical waves, though not of course for visible light. Later the illustrative vibrating system used in his argument, the acoustical one of a rectangular box filled with gas, was replaced by Jeans by the more

relevant electric one, a rectangular region of ether isolated within perfectly conducting walls. Either illustration seems to be valid to the same degree to sustain the result; for it is assumed from considerations of general physical experience, as developed on the Stewart-Kirchhoff line of argument, that there is one unique distribution of energy among the wave-lengths which is universal. His mastery of this subject appeared at various times, as when he briefly refuted a subtle argument that rotatory polarization could introduce into such questions an element at variance with the laws of thermodynamics. But he seems to have maintained complete suspense of judgment in regard to the tentative hypotheses, usually of the type of unitary quantification of energy, by which efforts were made to correlate wide groups of new phenomena, particularly of low-temperature specific heat and of cathode emanations and the various connected types of radiation.

It was perhaps natural for him to proceed, in the early days, from the consideration of halos and coronas to the arresting and brilliant, though familiar, natural phenomenon in which scattering of sunlight by a fortuitous distribution of particles is particularly exhibited, the light of the sky and its azure color. The cause of its polarization, whose limitations were closely investigated by Tyndall in experiments on artificial fogs, had been already sagaciously indicated in a different connection by Stokes as early as 1852. In his first paper of 1871 Rayleigh proceeded, as was inevitable at the time, on the elastic-solid theory of the ether, and considered the scattering of a beam of radiation by obstructing elastic spheres small compared with the wave-length. The effect comes out inversely as the fourth power of the wave-length, as he readily verified in accordance with his custom by dimensional considerations, and this at once explains the predominant blue.

It was many years before he seems to have felt himself free (in 1899) to take the plunge of asserting that the molecules of the atmosphere, notwithstanding the vast number of them in the wave-length, are competent by themselves to scatter the blue light. It may have been on account of a difficulty in conceiving them as compact spherical obstacles. But on the electric theory of propagation of waves of radiation in a gas their influence is very simply

expressed; each molecule is polarized by the electric field and so interacts with the electric flux in the radiation. Expressions for the scattering by the molecules are readily obtained in which, when they are taken to be isotropic, the only quantity that occurs is the index of refraction of the medium constituted by their aggregate. In the analysis of Rayleigh the electric theory is not utilized, probably in order to preserve continuity with his early work; the formula for the index of refraction is obtained in a fundamental and illuminating way by determining the scattering along the way, in which direction the phases are all in unison, so that the scattered vibrations, instead of being independent light, combine with the main beam, introducing only a change of phase which is a measure of the delay in propagation. The very remarkable result emerges that the scattering depends only on the number of the molecules (supposed isotropic and all alike) and the index of refraction of the medium which they constitute. This formula of Rayleigh's ought to be applicable to estimate the number of molecules in a gas, i.e., the constant of Avogadro; and from Italian observations by Majorana of intensity of scattering at high altitudes made under the inspiration of Kelvin, but especially from the remarkable later work of the Smithsonian observers, the results have been found so concordant that this method of sky observations, apparently so remote, has even ranked as one of the ways of estimating the value of the principal absolute constant of chemical science.

There was, however, something lacking. The calculations of index of refraction and of the necessary scattering of the radiation had hitherto been made on the hypothesis that the molecules are isotropic and polarize as if they were spherules of a compact dielectric body. But the electrochemical notion of a molecule separable into two ions would rather be that it is electrically bipolar, after the analogy of a magnet; if so, it could not be isotropic. Yet the nearly complete polarization at high altitudes of the light of the sky seen in directions transverse to the solar rays seemed to afford strong presumption of actual isotropy, which the close verification of the Rayleigh formula for the amount of scattered light made almost compelling. Accordingly, when his son, Professor R. J. Strutt (now Lord Rayleigh), succeeded in

detecting and measuring the light scattered transversely, when a strong beam traversed various gases in his laboratory, feelings were in fact divided between surprise that most gases deviated somewhat from perfect polarization of the scattered light and satisfaction that helium deviated so much as to suggest a linear bipolar structure for its molecule. One of Lord Rayleigh's latest papers took up this side of the subject, that of gases with aeolotropic molecules, and provided the results on which further discussion must turn.

His interests extended to all aspects of meteorology and scenery and the sights of open-air life. There is an important paper, in part expository, on stellar scintillation, a phenomenon from which much can be learned concerning the minute irregular structure of the atmosphere; and on his return from a voyage to South Africa he delivered a delightful discourse at the Royal Institution on the colors of sea and sky and their interplay.

His practical knowledge of chemistry, especially in the direction of photography, as was also the case with Sir John Herschel, was extensive. In a footnote to a paper on another subject he threw out a hint as to the origin of certain sporadic color effects, which was realized independently some years later in Lippmann's definite process for color photography. The same topic developed into a mathematical theory in relation to Stokes's problem of the reflection from planes of multiple twinning in crystals of chlorate of potash; and in the case of sound he experimented with a structure consisting of parallel equidistant sheets of muslin which happens to be the exact analogue of the reflection of X and γ rays by the molecular strata of a crystal. He spent some time in developing the manufacture of copies of ruled diffraction gratings by photography and by taking impressions in soft material, a method which has been more recently so successful in the hands of Ives; but the most valuable results were the considerations on the quality and performance of gratings that the attempts suggested to his mind. More than once he expounded with the simplicity of complete mastery the laws of chemical thermodynamics, at a time when this beautiful subject had not shed the abstruse considerations out of which it emerged; a lecture on the "Dissipation of Energy" and

a paper on "The Energy That Is Dissipated in the Mixing of Gases" (1875) may be cited. He had much share (with Maxwell) in securing due recognition in Europe of the rather difficult work of Willard Gibbs, both on the thermodynamic and on the optical side.

He was greatly interested in the history of physical discovery, and perhaps better read in its original classical sources than any of his contemporaries. He ran many a tilt at current scientific authority in order to claim for discoverers, such as Young, Fraunhofer, Balfour Stewart, and Stokes, what he regarded as the recognition due to them; nothing could be more illuminating for the understanding of the historical development of scientific ideas. During his ten years of administration as secretary of the Royal Society, and in other ways, one of his constant preoccupations was to make sure that work of unknown investigators should not be overlooked, owing to imperfections or obscurities in its presentation; and to the end of his life he was always willing to devote valuable time to the assistance of other workers.

A main feature of Rayleigh's work was his sureness of touch. Even in his degree examination at the university it was remarked by Todhunter, no lenient critic, that his answers were fit to go straight to press for publication. No scientific writer of our time has been fertile on so great a scale, whose work could bear the ordeal of republication to any comparable degree. Hardly a line or an expression has had to be altered, except by way of expansion. In 1899 the Cambridge University Press conveyed to him their willingness to publish a collected edition of his scientific papers, a project which had been urged upon them by men of science; the result was three large but not unwieldy volumes, which appeared in 1901. Two volumes were added subsequently, as material accumulated, and a sixth volume will now complete the collection. As already mentioned, in the early time (1877) the *Theory of Sound* included a condensed summary of such papers, only in part reprinted, as had doubtless been evolved in making preparations for that work. A second edition, still in two volumes but of more than double the size, which appeared in 1894, became even to a greater extent a treatise on the fundamental theory of vibrations in

general as well as of sound in particular. The experimentally more tractable subject of electrical vibrations in circuits, telephonic transmission and the like became prominent, as well as the more complex domain of optical waves, of which the simpler and more definite phenomena of sound afforded in many ways apt guidance and illustration. Though the general dynamical principles that are requisite are all covered, regret must be felt that the same sure hand is not available to expound and forecast the modes and potentialities of that wonder of practical development of the present day, recalling the sudden invention of the telephone, where the emission of free electrons at enormous speed is put under simple and refined control for the delicate and almost timeless operations of electric telegraphy across vast distances of free space. In one respect he did intervene in the early stage, with his usual sagacity; when it was announced that wireless signals had been sent across the Atlantic he briefly inquired how the waves could manage to bend round the curved surface of the earth through so great an angle. Being merely optical waves of great length, they ought to be in analogy with waves of light traveling round the surface of a globe with radius reduced from that of the earth in the same proportion as their lengths. Now that the facts are established completely, even to the extent of reception of wireless messages at the antipodes, there is no difficulty in insisting on the mode of explanation that was at hand from the first, that the upper regions of the atmosphere are so much ionized by solar and other influences as to form a conducting screen that effectually deflects the waves before they can pass far into it and so suffer any great absorption. Incidentally the theory of diffraction of radiation around an obstacle, instead of at a sharp edge, has been greatly developed by himself and others, along with much valuable auxiliary mathematical analysis.

He never had occasion, nor perhaps did he possess the mechanical facilities, to acquire the technique of refined spectroscopy on any large scale, though he laid down the principles on which it has developed. But characteristically, at a time when the minute determination of the wave-lengths of standard lines was demanded, he effected some exact measures with simple homely appliances of

the Fabry type, fixed together as usual with sealing wax, a main object being to show how the most laborious part of the work could be safely circumvented by a sheer guess of the integral part of the numerical result, for which sufficient data were available.

He paid much attention to the corrections of systems of lenses though from the philosophical more than from the workshop side. In particular a systematic study of the von Seidel classification of aberrations may be mentioned, in which he rediscovered the apt and natural transformation of the Hamiltonian optical function from point co-ordinates to direction co-ordinates, which had been utilized by Bruns and by Schwarzschild under the name of the eikonal.

Much of the work on radiation was summarized in articles on optics and on wave theory contributed to the *Encyclopaedia Britannica* in the early eighties, which still remain the most concise and authoritative expositions of the subjects of which they treat.

The systematic determination of the electrical constants at Cambridge has been mentioned. In his presidential address to the Mathematical and Physical Section of the British Association at Bristol in 1882 he expressed a conviction that the next great advance might arise from a closer study of some of the simpler phenomena of chemistry, and announced an intention of undertaking a systematic revision of the densities of the principal gases, including, of course, their departures from the ideal standard laws. Within ten years the discrepancies found between nitrogen drawn from the air and the same gas drawn from combined chemical sources had pointed to a new atmospheric substance, and had gradually revealed argon notwithstanding the wholly unexpected absence of all chemical activity. Not the least part of his pleasure in the result arose probably from finding out that Cavendish, whom he admired almost as intensely as he did Young, had recorded his handling of the substance and given its precise amount, which he could not get rid of in the classical determination of the constitution of air. The way being thus pointed out, further scrutiny of air by the process of freezing out at very low temperatures, and of gaseous occlusions in crystals, gave to his energetic coadjutor Ramsay the discovery of helium and the other inert gases, thus

filling up at a single step many gaps in the system of the chemical elements.

The topics selected above in illustration of his work were chosen from memory. A glance through the volumes of the *Scientific Papers* reveals how inadequately they represent his incessant and many-sided activity. Thus there is the great memoir of 1870 on the *Theory of Resonance*, which extended and simplified Helmholtz' classical results of ten years earlier, and perhaps gave the stimulus to writing the *Theory of Sound*. It also introduced, incidentally, new and beautiful principles of approximate physical calculation, which Maxwell developed further in the second edition of *Electricity and Magnetism*. Rather earlier papers, in elucidation of the abrupt pulses and surges in inductance coils, utilized the magnetic indicator of Joseph Henry, which was afterward to become prominent again as a receiver for ether-waves. His early critical studies on double refraction, inspired by Stokes and followed up by Kelvin, formed the authoritative introduction to the subject for the younger generation. The incompleteness of the polarization by reflection from diamond, and from liquid surfaces, led to theory and experiment on the disturbing influence of thin sheets, such as films of oil on water, which in time rendered a complete account of these perplexing disturbances and of resulting uncertainties as to theory. The capillary phenomena of liquid surfaces and of films occupied his attention all through life and were elucidated in all directions with brilliant sidelights into molecular grouping, both by theory and by the introduction of experimental methods, such as observation of ripples, that evaded the many pitfalls. Inspired, as he afterward seemed to remember, by a Tripos problem proposed by Maxwell years before, he was in the field early (1872) as an expositor of the explanation of anomalous dispersion on the idea then broached afresh by Sellmeier, but glimpsed long previously by Young, that the change in the velocity of light which produces refraction is due to resonance and not to mere inertia of the molecules. He followed Langley's gliding experiments preparatory to a solution of the problem of flight with close interest; and at a time when the brothers Wright had just managed to stay up in the air, he announced to the Royal

Society in a presidential address that the year would be memorable as the one in which the problem of artificial flight had been solved. Later, as chairman of the National Physical Laboratory and especially of its Aeronautics Committee, he pointed the way to extended applications of the theory of physical dimensions to the precise interpretation and utilization of experiments on models, which thereby became the main source of progress in design in that branch of technical science.

In his own country Lord Rayleigh's scientific position was always secure, and the honors to which it led were limited only by his aversion to display and reluctance to divert time from what he must have felt to be his proper work. Consequently he was not very well known abroad except in the later years, though through his friend Lord Kelvin he early mixed in the Helmholtz circle; the *Theory of Sound* had been very cordially welcomed by Helmholtz on its appearance in 1877, in the pages of *Nature*. His main piece of administrative activity outside Cambridge was as secretary of the Royal Society for about ten years, where he left many traces visible to his successors of improvements effected on his initiative. He was lord lieutenant of the county of Essex for about ten years. When the Order of Merit was instituted he was selected as one of the twelve original members. For the last eleven years of his life he was conspicuously in place, by the unanimous suffrage of its members, in the great historical office of Chancellor of the University of Cambridge, thus serving as an ideal head for a seat of learning which has a tradition to maintain beyond the bounds of its own nation.

Cambridge, England
September 1919

THE RIGIDITY OF THE EARTH

By A. A. Michelson and Henry G. Gale

In 1914, in the March number of this *Journal* (**39**, 105, 1914) an account was published of a preliminary experiment to determine the rigidity of the earth. At that time it was announced that the experiment would be repeated, using an interference method. The new arrangements were completed and a new series of observations begun on November 20, 1916, and continued until November 20, 1917. The reduction of the observations was interrupted by the war in the summer of 1917 and could not be resumed until April 1919.

The same pits and pipes on the grounds of the Yerkes Observatory at Williams Bay, Wisconsin, were used as in the preliminary experiment. In that experiment pipes 502 feet long and 6 inches in diameter were placed 6 feet underground. One pipe was laid accurately N–S and the other E–W. The pipes ended in pits 10 feet deep and 8 feet square, walled with concrete. The pipes were carefully leveled, and half filled with water, so that an air space extended from end to end of each pipe, above the water. The pipes ended in air-tight gauges provided with windows through which the changes in water-level could be determined by measuring with microscopes the distance between pointers just below the surface of the water and their totally reflected images.

In the present experiment interferometers replaced the microscopes and pointers. The arrangement of the interferometers, one at each end of each pipe, is shown in Figure 1. The compensating glass serves to seal the pipe. The lower mirror is movable vertically and has also the usual adjustments for regulating the width and orientation of the fringes. The film of water over this mirror is kept thin, usually about 0.5 mm, as the viscosity of the water helps to dampen ripples and minor disturbances. The changing thickness of the water film, due to the tides, caused the shift of fringes. The arrangement for recording the fringes was as follows:

Horizontal fringes were projected by the lens *L* on a narrow vertical slit about 0.2 mm in width. Clockwork drew a moving-picture film behind this slit at the rate of about 2 cm per hour. In order to prevent the condensation of moisture on the optical parts the end of the pipe, interferometer, and camera were all inclosed in a galvanized iron box, in which large trays of calcium chloride were kept. An incandescent light was continually burning near the interferometer to keep the temperature slightly raised.

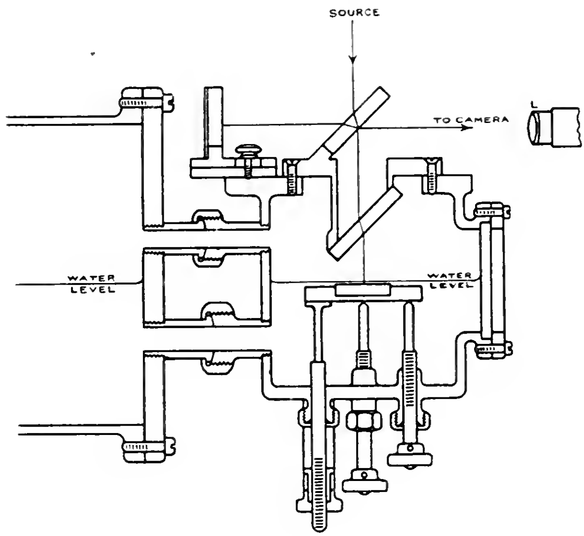


FIG. 1

Other interference arrangements are obvious which would give a displacement of a greater number of fringes, or permit the use of a shorter pipe; e.g., the fringes formed between the water surface and the lower mirror might be used, or the lower mirror might be dispensed with and use made of the fringes formed by the light reflected from the water surface and the vertical mirror. But the arrangement actually used was the most satisfactory, since the long pipes, 502 feet, were already installed.

The sources of light were commercial alternating-current Cooper-Hewitt mercury lamps. They proved very reliable and satisfactory. By using as filters thin films of a saturated solution of

esculin in water, all wave-lengths from the arc shorter than λ 4358 were absorbed, and the positive film used was not sensitive to the longer wave-lengths. The exposed portions of the films were removed and developed each week. The light was abundantly strong for satisfactory negatives, and it was possible to use 1.5 mm diaphragms on the projecting lenses. This gave sufficient sharpness to the fringes, even when there was a considerable change in their focus. It was necessary to readjust and refocus the fringes in only one pit during the entire year, although the width of the fringes was altered once or twice in two other pits. One of the mirrors required resilvering. One of the pits ran throughout the year without readjustment of the fringes or camera. The pits and cameras were in charge of Mr. George Monk and Mr. Frank Sullivan, of the Yerkes Observatory staff.

A relay which moved a shutter in front of the projecting lens was placed in each pit. The four relays were connected in series with a clock in the observatory, so that the time could be conveniently and accurately controlled. Once an hour the clock made contact, and a storage-battery circuit was closed through the relays and the light was cut off by the shutters for about 20 seconds. Very accurate time-marks were secured in this way. The control clock was kept six minutes faster than Central Standard Time in order to simplify the computations and bring the observations into agreement with them. (The longitude of Yerkes Observatory from Greenwich is $5^h\ 54^m\ 13^s$.)

The films were measured by sliding them on a lathe-bed beneath a low-powered microscope. The fringes, estimated to tenths, were counted as they moved up and down, and the numbers recorded for each hour. The difference in the motion at the two ends of each pipe gave the numbers for plotting the observed tides.

The calculated tides were drawn from the computed shift in fringes, the calculations being made for two-hour intervals. The calculations were made under the direction of Professor F. R. Moulton by Mr. Albert Barnett and Mr. Horace Olsen. The formulae are given in the accompanying article by Professor Moulton, "Theory of Tides in Pipes on a Rigid Earth." The value of μ for the water used was found to be 1.3408 for λ 4358, and this is

probably correct to within considerably less than 0.1 per cent for the range of temperatures used.

Calculated and observed curves for the period from March 24 to April 21, 1917, are reproduced in Figures 2 and 3. The dotted curve represents the observed and the full curve (displaced vertically to avoid overlapping) 0.7 of the calculated values of the tides. The ordinates are numbers of fringes, $N=\frac{2(\mu-1)d}{\lambda}$, and one fringe corresponds to 1/1564 mm.

The observed and calculated curves were plotted on long rolls of co-ordinate paper to the following scale: abscissae, 1 cm = 1 hour; ordinates, 1 cm = 2 fringes. In order to have the amplitudes approximately equal, 0.7 of the calculated values were plotted instead of the full amplitudes. Beginning with 10:00 A.M. November 20, 1916, the curves, both observed and calculated, were divided into periods of $12^{h}.42$ for the semi-diurnal and $25^{h}.82$ for the diurnal lunar tides. The principal solar tide, period twelve hours, was started at noon of the same day. In order to avoid a cumulative error in the case of the semi-diurnal lunar tide the period $12^{h}.4206013$ was put on a computing machine and added repeatedly to the initial time to get the exact beginning of each new period throughout the year. This process was repeated, using the period $25^{h}.8193409$ for the diurnal lunar tide.

The observations were reduced in groups of about a lunar month each, by dividing each period into ten equal parts (twelve in the case of the solar tide), and taking the mean of the first, second, third, etc., ordinates. The resulting values were plotted and any error in computation was usually indicated by the failure of a point to fall on a smooth curve. It is important to treat the observed and calculated tides both in the same way, as any distortion in the resulting sine curves due to lack of complete elimination of other periods affects the two alike. This is, of course, most noticeable in the case of the diurnal tide on account of its smaller amplitude and the smaller number of periods. Mr. Fred Pearson gave valuable assistance in measuring the films, in plotting the curves, and in deducing the various tides from the curves.

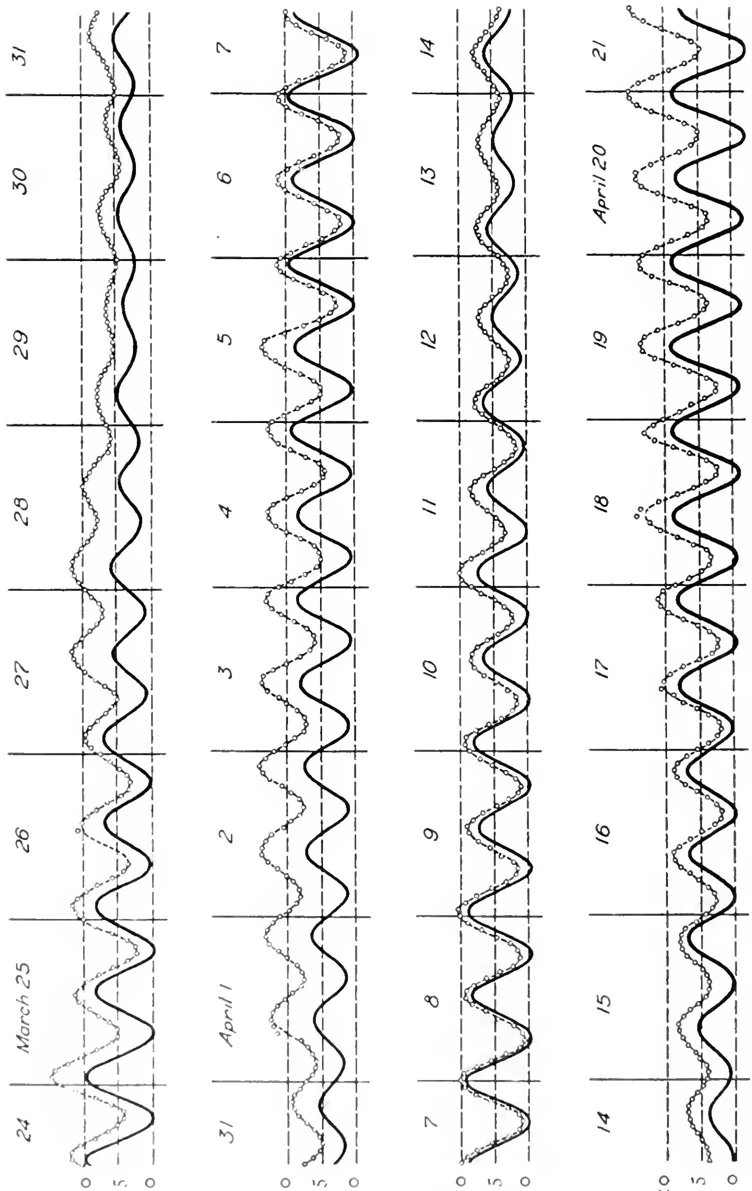


FIG. 2.—N-S tides, March 24 to April 21, 1917. Dotted curves, observed values. Full curves, 0.7 of calculated values

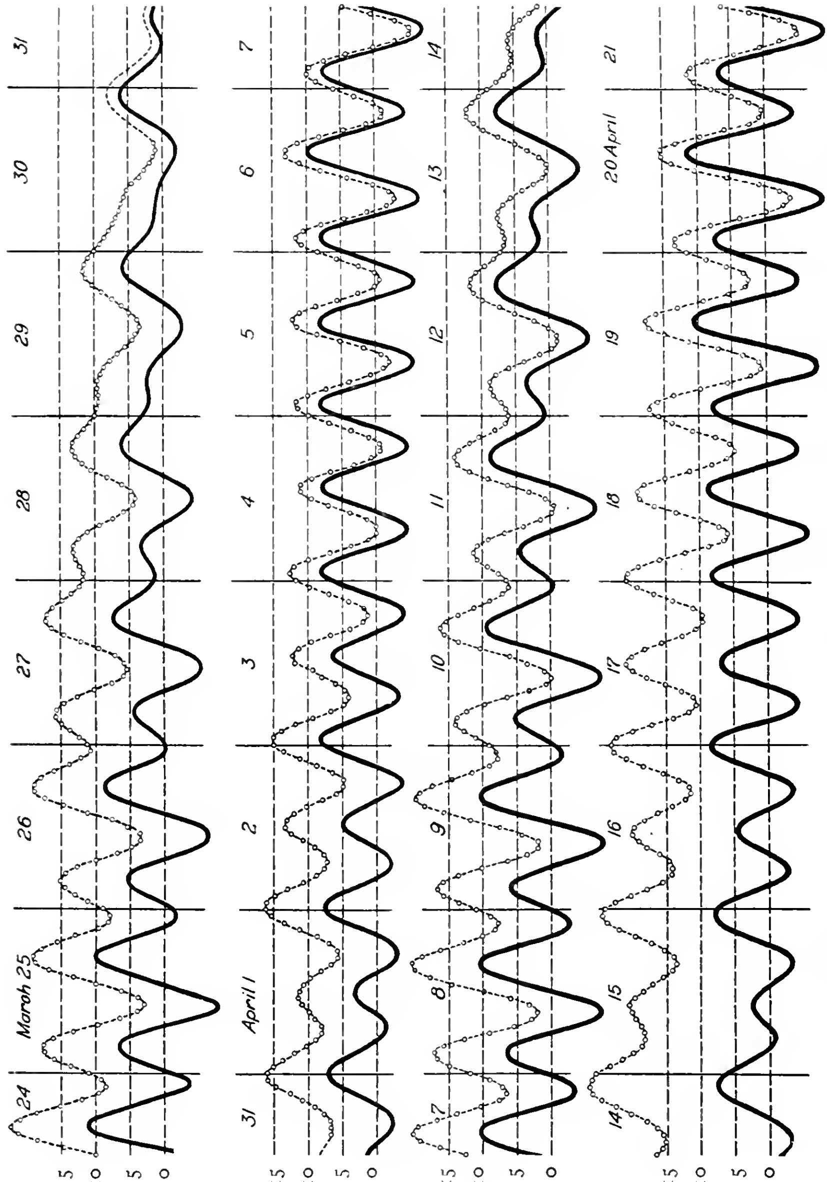


FIG. 3.—E–W tides, March 24 to April 21, 1917. Dotted curves, observed values. Full curves, 0.7 of calculated values

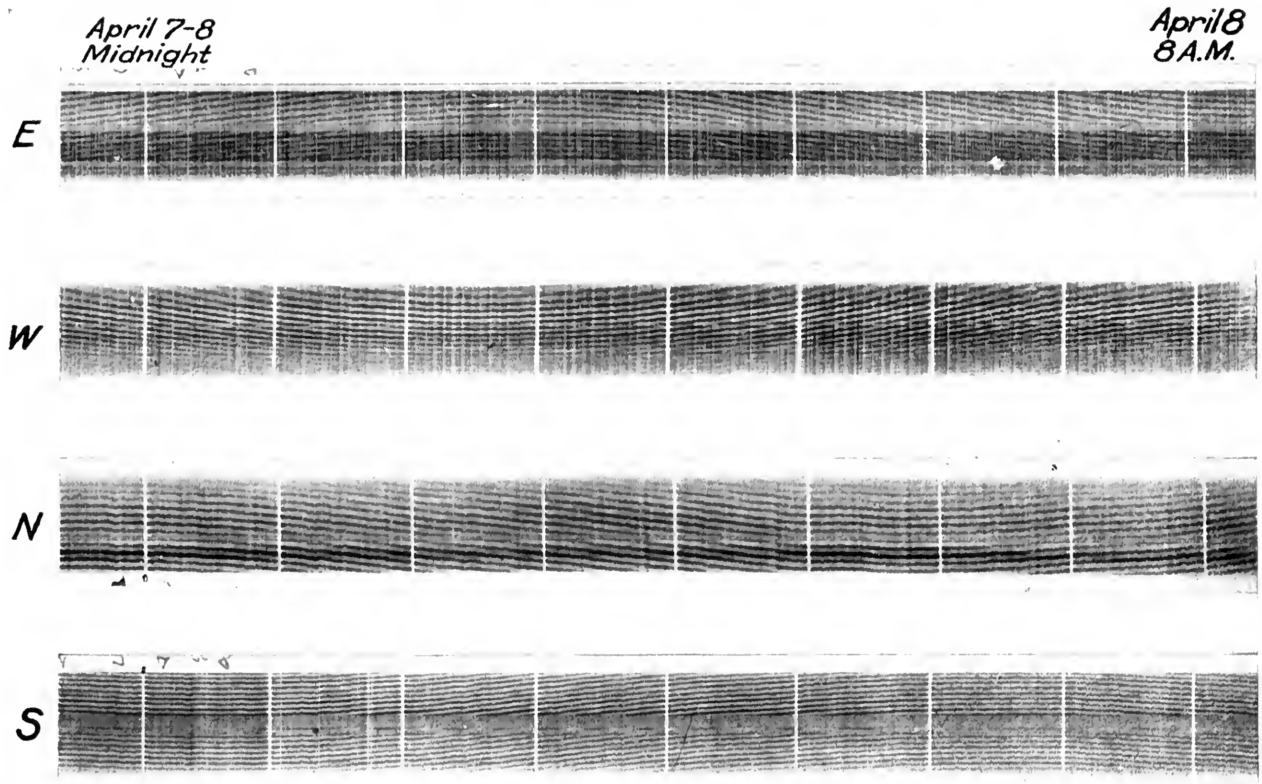
Very little trouble was caused by sudden erratic changes in the fringes. Occasionally, however, earthquakes would cause the fringes to disappear for from ten minutes to half an hour. Once the effects of an earthquake were evident for about six hours. During three hours of this time the fringes were completely obliterated.

The most serious disturbance was a gradual change in the slope of the observed curves. This would often be fairly uniform and gradual for a month or two. At some times the curves would rise and at others fall. Sometimes the N–S and E–W slopes had the same sign, and sometimes opposite signs. We have been able to discover nothing systematic about this drifting. It may have been caused by unequal settling at the ends of the pipes, by temperature changes in the pits, or by tilting in the earth's strata. There were always large shifts of the fringes when the lights came on after having been interrupted by the power company for a half-hour or so. The change of slope was eliminated in reducing each monthly tide, as given in Tables I–VI, and the tide for the whole year, opposite *Y* in the tables, by distributing the change of level uniformly throughout the period. This change of slope is quite conspicuous in Figure 3, where the observed E–W tide showed a fairly uniform and distinct downward trend throughout nearly the whole month. The change in slope of the N–S curves for the same period is comparatively small, as shown in Figure 2.

Plate XII is from four photographs taken simultaneously in the four pits. The reproductions are positives on the same scale as the originals, and represent fairly well the average quality of the films.

A graphical solution is excellent for detecting erroneous points and serves well to give the ratio of the observed to the calculated amplitude, but for determining the phase-difference of the two curves it is not so satisfactory. A least-squares method was therefore used to secure the ratio of amplitudes, R, and the displacement in phase $\Delta\phi$ of the observed with respect to the calculated tide. The following example illustrates the method of reduction. It is for the semi-diurnal tide, first month, E–W. For convenience the solar periods were divided into twelve parts instead of ten, but in other respects the method of reduction is the same.

PLATE XII



Photographic Records Taken Simultaneously

FIRST MONTH, SEMI-DIURNAL, E-W OBSERVED

θ	y	Sin θ	y Sin θ	Cos θ	y Cos θ
0°	17.58	0.0000	0.0000	1.0000	17.5800
36	18.74	.5878	11.0153	0.8090	15.1606
72	15.73	.9511	14.9608	.3090	4.8605
108	10.46	.9511	9.9485	− .3090	−3.2321
144	4.83	.5878	2.8390	− .8090	−3.9074
180	0.60	.0000	0.0000	−1.0000	−0.6000
216	−0.20	−.5878	0.1234	−0.8090	0.1698
252	2.58	−.9511	−2.4538	− .3090	−0.7972
288	7.76	−.9511	−7.3805	.3090	2.3978
324	13.48	−.5878	−7.9235	.8090	10.9053
Total	....	....	+21.1292	....	+42.5373

$$\alpha=\frac{21.1292}{5}=+4.2258 \quad \beta=\frac{42.5373}{5}=+8.5075$$

$$\tan\phi_c=\frac{\beta}{\alpha}=2.0132$$

$$\phi_c=63°35'$$

$$c=\sqrt{\alpha^2+\beta^2}$$

$$c=\sqrt{90.2350}$$

$$c=9.499$$

$$y=9.499 \sin(\theta+63°35')$$

FIRST MONTH, SEMI-DIURNAL, E-W CALCULATED

θ	y	Sin θ	y Sin θ	Cos θ	y Cos θ
0°	17.38	0.0000	0.0000	1.0000	17.3800
36	18.89	.5878	11.1035	0.8090	15.2820
72	16.73	.9511	15.9119	.3090	5.1695
108	11.58	.9511	11.0137	− .3090	−3.5782
144	6.13	.5878	3.6032	− .8090	−4.9591
180	0.90	.0000	.0000	−1.0000	−0.9000
216	0.00	−.5878	.0000	−0.8090	.0000
252	2.23	−.9511	−2.1209	− .3090	− .6890
288	7.16	−.9511	−6.8098	.3090	2.2124
324	12.91	−.5878	−7.5885	.8090	10.4441
Total	....	....	+25.1131	....	+40.3617

$$\alpha=\frac{25.1131}{5}=+5.0226 \quad \beta=\frac{40.3617}{5}=+8.0723$$

$$\tan\phi_d=\frac{\beta}{\alpha}=+1.6072$$

$$\phi_d=58°7'$$

$$d=\sqrt{\alpha^2+\beta^2}$$

$$d=\sqrt{90.3885}$$

$$d=9.507$$

$$y=9.507 \sin(\theta+58°7')$$

$$R=\frac{c}{d}\ \frac{0.7\times 9.499}{9.507}=0.699 \qquad \Delta\phi=\phi_c-\phi_d=+5°28'$$

In Tables I–VI the numbers in the columns under M denote the different approximate lunar months; under N is given the

TABLE I

N–S, Semi-Diurnal, Lunar

M	N	a	b	ϕ_a	ϕ_b	R	$\Delta\phi$
1......	60	6.217	6.373	149°14′	147°48′	0.683	1°26′
2......	54	5.703	6.489	147 53	144 4	.615	3 49
3......	48	7.261	6.780	146 15	145 56	.750	0 19
4......	45	6.750	6.739	148 44	146 23	.701	2 7
5......	55	6.041	6.269	148 37	148 54	.675	−0 17
6......	54	6.426	6.551	148 35	147 57	.687	0 38
7......	54	6.332	6.344	148 40	145 56	.699	2 44
8......	51	6.052	6.344	147 47	146 58	.668	0 49
9......	55	6.036	6.319	146 36	147 16	.669	−0 40
10......	54	5.821	6.210	149 27	147 32	.656	1 55
11......	54	5.875	6.140	148 14	148 16	.670	−0 2
12......	55	6.027	6.284	149 18	147 46	.671	1 32
13......	45	6.330	6.515	152 39	150 36	0.680	2 3
Av....	....	6.203	....	....	....	0.678 ±.019	1°15′ ±1 2
V....	684	6.173	6.401	148°24′	147°32′	0.675	0 52

TABLE II

E–W, Semi-Diurnal, Lunar

M	N	c	d	ϕ_c	ϕ_d	R	$\Delta\phi$
1......	60	9.499	9.507	63°35′	58° 7′	0.699	5°28′
2......	54	8.908	9.096	64 40	58 5	.686	6 35
3......	48	9.899	10.075	61 28	55 47	.688	5 41
4......	43	10.220	10.369	69 46	65 2	.690	4 44
5......	56	9.131	9.380	63 43	57 0	.681	6 43
6......	53	9.660	9.667	65 4	57 54	.700	7 10
7......	54	9.496	9.632	64 26	57 6	.690	7 20
8......	51	9.445	9.331	60 48	55 4	.709	5 44
9......	55	9.206	9.316	63 24	57 30	.692	5 54
10......	54	8.773	9.095	64 52	56 32	.675	8 20
11......	54	8.982	9.108	61 31	57 18	.690	4 13
12......	55	9.138	9.337	63 12	58 12	.685	5 0
13......	45	9.809	9.747	67 20	61 44	0.704	5 36
Av....	....	9.373	....	....	....	0.691 ±.007	6° 4′ ± 55
V....	682	9.380	9.485	63°56′	58°10′	0.692	5 46

number of periods used. For the solar tides missing portions of the observed curves were sketched in, following the computed tides,

thus giving a total of 730 periods for the year, but for the semi-diurnal and diurnal lunar tides only such portions of the observed

TABLE III

N-S SOLAR

M	*N*	*a*	*b*	ϕ_a	ϕ_b	*R*	$\Delta\phi$
1......	56	2.554	2.628	104° 8′	99°39′	0.680	4°29′
2......	62	2.392	2.294	100 13	85 45	.730	14 28
3......	48	4.883	4.272	93 34	82 59	.800	10 35
4......	70	2.778	3.132	88 41	80 35	.621	8 6
5......	58	3.840	3.744	100 26	97 14	.718	3 12
6......	54	3.595	3.485	102 43	99 59	.722	2 43
7......	56	2.818	2.570	109 29	103 27	.768	6 2
8......	54	2.513	2.312	109 1	99 49	.761	9 12
9......	56	2.771	2.506	87 23	84 18	.774	3 5
10......	56	2.944	3.108	89 35	86 54	.663	2 41
11......	56	3.562	3.456	99 10	95 51	.721	3 19
12......	56	3.154	3.291	108 6	103 40	.671	4 26
13......	48	3.775	3.889	98 55	99 38	0.679	−0 43
Av....	....	3.161	....	....	....	{ 0.714 ±.041	5°38′ ±3 16
Y....	730	3.140	3.063	99°25′	93°20′	0.718	6 5

TABLE IV

E-W SOLAR

M	*N*	*c*	*d*	ϕ_c	ϕ_d	*R*	$\Delta\phi$
1......	56	3.641	3.838	13°54′	9°40′	0.664	4°14′
2......	62	3.218	3.400	3 31	355 40	.663	7 51
3......	48	6.172	6.488	358 28	352 59	.666	5 29
4......	70	4.444	4.665	354 5	350 14	.667	3 51
5......	58	5.534	5.480	11 19	6 33	.707	4 46
6......	54	5.104	5.173	16 52	10 59	.691	5 53
7......	56	3.687	3.731	21 32	13 53	.692	7 39
8......	54	3.487	3.407	15 3	8 22	.716	6 41
9......	56	3.450	3.689	359 50	353 28	.655	6 22
10......	56	4.371	4.595	3 14	356 36	.666	6 38
11......	56	5.045	4.993	7 57	4 32	.707	3 25
12......	56	4.689	4.858	16 28	14 16	.676	2 12
13......	48	5.621	5.626	10 0	6 11	0.699	3 49
Av....	....	4.459	....	....	....	{ 0.681 ±.018	5°19′ ±1 28
Y....	730	4.399	4.522	8° 6′	2°23′	0.681	5 43

curves were used as gave complete periods. Under *a* and *c* are given the amplitudes and under ϕ_a and ϕ_c the phase constants for

TABLE V

N-S, Diurnal, Lunar

M	N	a	b	ϕ_a	ϕ_b	R	$\Delta\phi$
1	26	0.962	0.551	185°35′	193°24′	1.222	− 7°29′
2	28	0.653	.470	189 59	188 19	0.973	1 40
3	24	1.306	.550	319 49	201 31	1.662	118 18
4	20	2.042	.701	305 43	218 48	2.039	86 55
5	25	0.117	.509	189 00	203 36	0.336	−14 36
6	26	.347	.465	218 59	197 7	.522	21 52
7	26	.335	.499	190 27	194 25	.475	− 3 58
8	25	.279	.484	193 22	193 25	.404	− 0 3
9	26	.297	.463	213 32	195 58	.451	17 34
10	26	.538	.473	154 44	193 31	.796	−38 37
11	26	.277	.455	164 41	194 4	.426	−29 23
12	26	.282	.365	214 36	188 58	.541	+25 38
13	22	0.359	0.480	181 1	191 58	0.523	−10 57
Av_{13}		0.574				0.777 ±0.411	11°57′ ±30 49
Av_{12}		0.478				0.694 ±0.313	7 3 ±26 6
Av_{11}		0.408				.612 ±0.216	− 2 25 ±16 12
V_{13}	326	0.281	0.503	229°23′	197°57′	0.411	31 26
V_{12}	306	0.298	0.492	203 24	196 07	.424	7 17
V_{11}	282	0.411	0.489	188 14	196 56	0.588	− 8 42

TABLE VI

E-W, Diurnal, Lunar

M	N	c	d	ϕ_c	ϕ_d	R	$\Delta\phi$
1	26	4.107	3.819	282° 9′	284°56′	0.753	−2°47′
2	28	3.845	3.725	275 48	278 19	.723	−2 31
3	24	4.164	4.193	288 57	288 58	.695	−0 1
4	20	3.525	3.603	317 6	293 57	.685	23 9
5	25	3.925	4.134	287 31	285 43	.665	1 48
6	26	3.517	3.850	291 19	288 14	.639	3 5
7	26	3.925	3.900	281 44	285 13	.704	−3 29
8	25	4.032	3.968	281 54	281 24	.711	0 30
9	26	3.710	3.518	286 41	283 20	.738	3 21
10	26	3.480	3.595	282 29	284 21	.678	−1 53
11	26	3.383	3.442	281 40	283 59	.688	−2 19
12	26	3.439	3.470	278 57	281 36	.694	−2 39
13	22	4.526	4.485	275 17	280 35	0.706	−5 18
Av_{13}	326	3.807				0.699 ±0.022	0°29′ ±4 11
Av_{12}	306	3.825				0.700 ±0.024	−1 0 ±2 8
V_{13}	326	3.759	3.799	284°55′	283°22′	0.693	1 33
V_{12}	306	3.807	3.815	283 52	282 43	0.699	1 9

the observed curves and under b and d and ϕ_b and ϕ_d the corresponding quantities for the calculated curves. In taking the means the value for each month is weighted in proportion to the number of periods in the month. Below the mean R and $\Delta\phi$ in each case is given the average difference from the mean. At the bottom of each table opposite Y are given the results obtained by computing the tides for the entire year as a single period instead of for a month at a time. The results agree closely with the means for the thirteen months, and are shown graphically in Figures 4 to 8.

Violent storms broke down the electric wires during February and March on several occasions, interrupting the electric current for a few hours. The pits cooled, and there were resulting large shifts of the fringes after the current was re-established. It seems highly probable that the large difference of phase which is shown for the fourth month E–W diurnal tide is due to such disturbances. Moreover, this month, on account of the interruptions, contained but 20 periods instead of the usual 26. The mean is therefore given for twelve lunar months, omitting the fourth, on the line Av_{12} as well as for the thirteen months. A calculation for 306 periods was also made, omitting this month from both the observed and calculated data. This value is given opposite Y_{12}.

In the case of the N–S diurnal tide it will be noted that the amplitude is very small, about 0.5 fringe, as it should be since this tide has the coefficient cos $2l$, where l is the latitude. For Yerkes Observatory cos $2l=0.0848$. This tide is too small to admit of much accuracy in the determination, but the results are included, as they are not without interest. Mean values are added for the whole year, and the means omitting the fourth month, and also omitting both the third and the fourth. The calculations are also added for the year as a whole, 326 periods; omitting the fourth month, 306 periods; and omitting both the third and fourth months, 282 periods. The omission of these months is perhaps justified, since both R and $\Delta\phi$ are decidedly abnormal. The mean of the six values gives $R=0.584$ and $\Delta\phi=7°46'$. Probably the only conclusion which is justified for the N–S diurnal tide is that R is about $0.6\pm .2$ and that the difference of phase is small.

An effort was made to deduce the fortnightly lunar tide of period 13.66 days. Here the E–W tide should be zero, and the residual sine curves were less than 0.05 fringe from both the

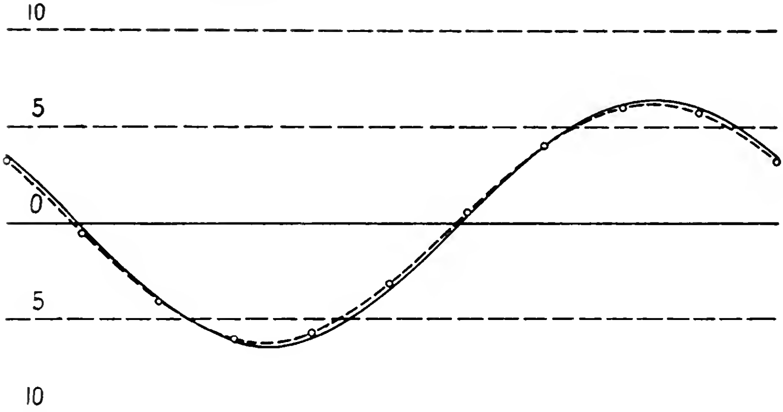


FIG. 4.—N–S semi-diurnal lunar tide for entire year. Dotted curve from observed values

$$y=6.173 \sin (\theta+148°24')$$

Full curve from 0.7 calculated values

$$y=6.401 \sin (\theta+147°32')$$
$$R=0.7 \quad a/b=0.675. \quad \Delta\phi=52'$$

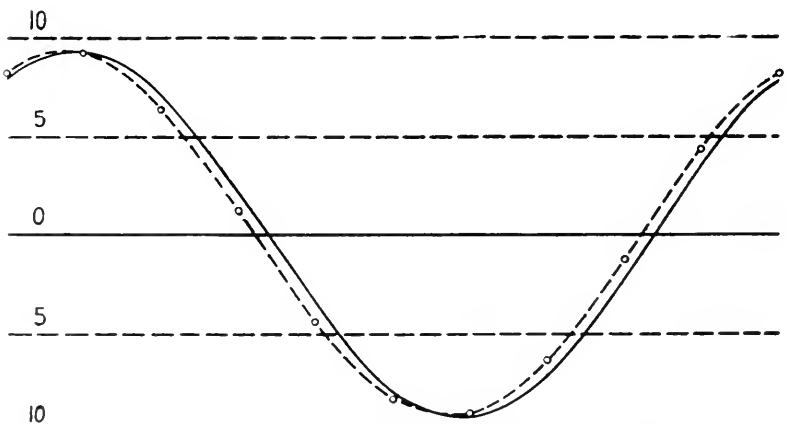


FIG. 5.—E–W semi-diurnal lunar tide for entire year. Dotted curve from observed values

$$y=9.380 \sin (\phi+63°56')$$

Full curve from 0.7 of calculated values

$$y=9.485 \sin (\theta+58°10')$$
$$R=0.7 \quad c/d=0.692. \quad \Delta\phi=5°46'$$

calculated and observed curves. The N–S tide, however, had an amplitude of about 1.8 fringes and gave $R=0.628$ and $\Delta\phi=-8°24'$, quantities which agree as well as could be expected with the shorter

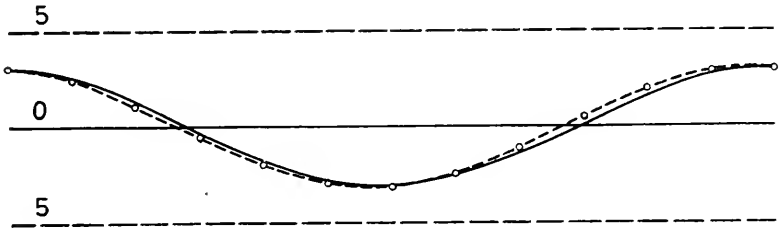


FIG. 6.—N–S solar tide for entire year. Dotted curve from observed values

$$y=3.140 \sin(\theta+99°25')$$

Full curve from 0.7 of calculated value

$$y=3.063 \sin(\theta+93°20')$$
$$R=0.7\ a/b=0.718.\quad \Delta\phi=6°5'$$

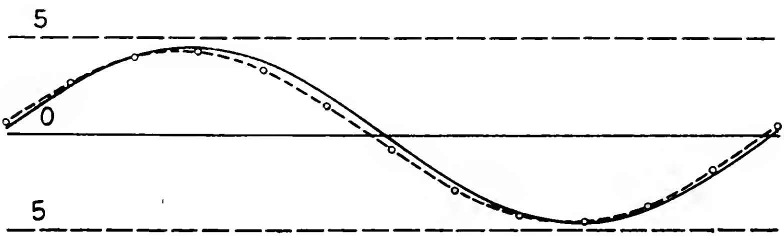


FIG. 7.—E–W solar tide for entire year. Dotted curve from observed values

$$y=4.399 \sin(\theta+8°6')$$

Full curve from 0.7 of calculated value

$$y=4.522 \sin(\theta+2°23')$$
$$R=0.7\ c/d=0.681.\quad \Delta\phi=5°43'$$

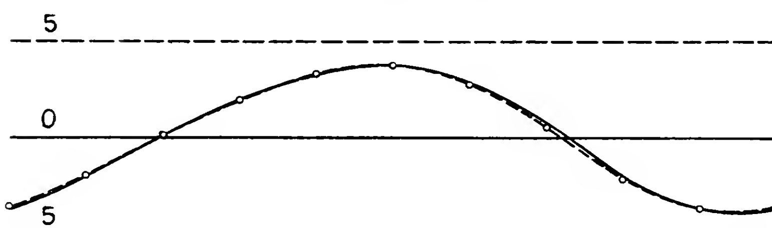


FIG. 8.—E–W diurnal lunar tide for entire year. Dotted curve from observed values

$$y=3.759 \sin(\theta+284°55')$$

Full curve from 0.7 of calculated values

$$y=3.799 \sin(\theta+282°53')$$
$$R=0.7\ c/d=0.693.\quad \Delta\phi=1°33'$$

periods. The negative sign of $\Delta\phi$ merely indicates that the uncertainty is considerable.

The results are collected in Table VII. The value given in each case is the average of the mean value for the thirteen lunar months and the value deduced by treating all the observations in a single set, except in the case of the N–S diurnal tide, where the values given are the mean of the six determinations mentioned above, and in the case of the E–W diurnal tide, which is the mean of four similar determinations.

TABLE VII

.681
.698

	N–S			E–W		
	Amp.	R	$\Delta\phi$	Amp.	R	$\Delta\phi$
Lunar semi-diurnal...	0.188	0.0765 ± .019	1°15′ ± 1°2′	9.376	0.6915 ± .007	5°54′ ± 56′
Solar...............	3.150	0.716 ± .041	5°53′ ± 3°12′	4.429	.681 ± .018	5°30′ ± 1°28′
Lunar diurnal	.408	0.584 ± 200	(7°46′ ± 15°)	3.799	.698 ± .023	38′ ± 3°28′
Weighted means...	......	0.6895	2°41′	......	0.6903	4°34′
Final values		**R = 0.690 ± .004**			$\Delta\phi$ = 4°	

The amplitudes are those for the observed curves averaged as indicated above. The errors indicated for the different tides are the average differences from the means of the thirteen months, except in the case of the N–S diurnal tide, where it is simply an estimate. In combining the different tides to get a mean in each direction, the ratios and phase differences were weighted in proportion to the amplitudes of the tides except that the N–S diurnal tide was omitted. The final means of R and $\Delta\phi$ are the mean values obtained by weighting the five determinations in this way. This seems to be the most logical procedure and is perhaps justified by the fact that the average difference from the mean for the different tides in each direction is roughly inversely proportional to the amplitude. The probable error, computed in the usual way, is given with the final value of R.

The final result indicates that the rigidity of the earth in the N–S and E–W directions is the same[1] and the ratio R is 0.690 with

[1] The preliminary experiment, through an error in computation, indicated a difference in the rigidities in the two directions. The ratio should have been 0.710 for both the N–S and E–W. See *Science*, October 3, 1910, p. 327.

a probable error of ± 0.004. That the viscous yielding of the earth is small is indicated by the small difference in phase between the observed and computed tides. It will be noted that the two solar tides appear to agree excellently in phase displacement with the E–W semi-diurnal tide and that for the N–S semi-diurnal, and probably also the E–W diurnal lunar tides the phase displacement is definitely smaller.

However, for lack of a better method of finding the means of the N–S and E–W phase displacement, each was averaged as the ratios were, that is, simply by weighting them in proportion to the amplitudes of the tides. This gives a displacement in phase of the water tides in the N–S direction of $+2^\circ 41'$ and in the E–W direction $+4^\circ 34'$. Although it seems certain that the difference in phase is slightly larger in the E–W than in the N–S direction, a mean displacement of $+4^\circ\!.0$ is probably correct to within 1°. If we take $R = 0.690$, the tides in the actual earth are 0.310 of what they would be if the earth were fluid, and the value of $\Delta\phi$ equal to $4^\circ\!.0$, for the displacement of the water tides means that the earth tides lag behind the impressed forces by this same amount.

It is desired to express appreciation of the interest taken in this work by Professor T. C. Chamberlin, Professor E. B. Frost, and Professor F. R. Moulton.

RYERSON LABORATORY
November 1919

THEORY OF TIDES IN PIPES ON A RIGID EARTH

By F. R. MOULTON

The moon and sun exert tidal forces upon the earth. The magnitudes of these forces depend upon the masses and distances of these bodies and upon the size of the earth, all of which are known. If the earth were perfectly rigid, the tides in surface fluids would have certain definite magnitudes. Since the earth as a whole yields somewhat to the tidal forces, the actual tides in surface fluids are less than they would be if the earth were perfectly rigid. The amount the earth is deformed and the character of its yielding can be determined from the differences in magnitudes and phases between the actual tides and what they would be on a rigid earth. The former are found from observations, and the latter can be determined only by computations. The theory of making these computations is the object of this paper.

Lord Kelvin inferred from the height of the oceanic tides that the earth as a whole has a considerable degree of rigidity. But oceanic tides are not suited to a precise numerical determination of the earth's rigidity because they are modified by many factors whose effects cannot be even approximately evaluated. Hence it is necessary to use relatively short channels of water, or their equivalent. This reduces the magnitudes of the quantities to be measured. Nevertheless, in the experiments of Michelson and Gale, in which the pipes are only about 500 feet long, the results are so precise that, in order that the theory shall be as accurate as the observations, it is necessary to develop the tidal theory to an order of precision not heretofore required.

The surface of the water in a partially filled pipe is an equipotential surface for all the forces which act upon it. The periods of any waves which might be produced in it would be so short that they could not be confused with tidal oscillations, and, moreover, they would soon be damped out. Hence it is necessary only to develop the expressions for the equipotential surfaces, and to find the way they vary with the time.

Take the origin O at the center of the earth and let

P_0 and P_1 be the positions of the ends of the pipe,
r be the distance from O to P_0,
ϕ be the latitude of P_0,
θ be the right ascension of P_0,
ω be the angular rate of rotation of the earth,
E be the mass of the earth,
a be the earth's equatorial semidiameter,
b be the earth's polar semidiameter,
e be the eccentricity of a meridian section of the earth,
M be the mass of the moon,
α be the right ascension of the moon,
δ be the declination of the moon,
ρ be the distance from O to the moon,
ρ_1 be the distance from P_0 to the moon,
ψ be the angle between r and ρ,
S be the mass of the sun,
A be the right ascension of the sun,
D be the declination of the sun,
R be the distance from O to the sun,
R_1 be the distance from P_0 to the sun,
Ψ be the angle between r and R,
k^2 be the gravitational constant,
V_0 be the potential function at P_0,
V be the potential function at the general point P.

The potential function is composed of four parts: V_1, that due to the attraction of the earth; V_2, that due to the rotation of the earth; V_3, that due to the attraction of the moon; and V_4, that due to the attraction of the sun. The first of these is numerically much more important than the remainder, and this fact will be used in certain expansions in series which follow. The earth is somewhat oblate, and this fact must be taken into account in order that the theory shall be at least as accurate as the observations of Michelson and Gale.

The first part of the potential function is[1]

$$V_1 = \frac{k^2 E}{r}\left\{1 + \tfrac{1}{10}\frac{b^2}{r^2}(\cos^2\phi - 2\sin^2\phi)e^2 \; . \; . \; . \; .\right\}. \qquad (1)$$

[1] *Introduction to Celestial Mechanics*, p. 122.

It follows from the formulae for centrifugal acceleration that the second part is

$$V_2=\tfrac{1}{2}\omega^2 r^2 \cos^2 \phi. \tag{2}$$

The third and fourth parts are[1] the similar functions

$$V_3=k^2 M\left\{\frac{1}{\rho_1}-\frac{r}{\rho^2}\cos\psi\right\}, \tag{3}$$

$$V_4=k^2 S\left\{\frac{1}{R_1}-\frac{r}{R^2}\cos\Psi\right\}, \tag{4}$$

and

$$V=V_1+V_2+V_3+V_4. \tag{5}$$

The condition that P_0 and P shall be on the same equipotential surface is

$$V-V_0=0. \tag{6}$$

The polar co-ordinates of V_0 are r, ϕ, θ; let the polar co-ordinates of V be represented by $r+\Delta r$, $\phi+\Delta\phi$, $\theta+\phi\theta$, where, in the case of pipes such as can be used in experiments, Δr, $\Delta\phi$, and $\Delta\theta$ are small quantities. Then the expansion of (6) as a power series in Δr, $\Delta\phi$, and $\Delta\theta$ is

$$\left.\begin{aligned}0=\frac{\delta V}{\delta r}\Delta r+\frac{\delta V}{\delta\phi}\Delta\phi+\frac{\delta V}{\delta\theta}\Delta\theta+\tfrac{1}{2}\frac{\delta^2 V}{\delta r^2}(\Delta r)^2+\tfrac{1}{2}\frac{\delta^2 V}{\delta\phi^2}(\Delta\phi)^2+\tfrac{1}{2}\frac{\delta^2 V}{\delta\theta^2}(\Delta\theta)^2\\+\frac{\delta^2 V}{\delta r\delta\phi}\Delta r\Delta\phi+\frac{\delta^2 V}{\delta r\delta\theta}\Delta r\Delta\theta+\frac{\delta^2 V}{\delta\phi\delta\theta}\Delta\phi\Delta\theta+\ \ldots\ .\end{aligned}\right\} \tag{7}$$

The problem is to find Δr from this equation when $\Delta\phi$ and $\Delta\theta$ are known. Equation (7) has the form

$$\left.\begin{aligned}0=c\Delta r+a_1\Delta\phi+a_2\Delta\theta+a_3(\Delta r)^2+a_4(\Delta\phi)^2+a_5(\Delta\theta)^2+a_6\Delta r\Delta\phi+a_7\Delta r\Delta\theta\\+a_8\Delta\phi\Delta\theta+\ \ldots\ .\end{aligned}\right\} \tag{8}$$

which can be solved for Δr as a power series in $\Delta\phi$ and $\Delta\theta$ provided c is not zero, a condition which is fulfilled in this problem, because $c=0$ would imply that gravity is zero at P_0. The solution of (8) has the form

$$\Delta r=c_1\Delta\phi+c_2\Delta\theta+c_3(\Delta\phi)^2+c_4(\Delta\theta)^2+c_5\Delta\phi\Delta\theta+\ \ldots\ . \tag{9}$$

[1] *Op. cit.*, p. 372.

On substituting (9) in (8) and equating coefficients of corresponding powers of $\Delta\phi$ and $\Delta\theta$, it is found that

$$\left.\begin{aligned}
c_1 &= -\frac{a_1}{c}, \\
c_2 &= -\frac{a_2}{c}, \\
c_3 &= -\frac{a_4}{c}+\frac{a_1 a_6}{c^2}-\frac{a_1^2 a_3}{c^3}, \\
c_4 &= -\frac{a_5}{c}+\frac{a_2 a_7}{c^2}-\frac{a_2^2 a_3}{c^3}, \\
c_5 &= -\frac{a_8}{c}+\frac{a_1 a_7}{c^2}+\frac{a_2 a_6}{c^2}-\frac{2a_1 a_2 a_3}{c^3}.
\end{aligned}\right\} \tag{10}$$

The higher terms are all negligible, and a number of those written will be shown to be numerically unimportant. In computing the second derivative of V with respect to r it will be sufficient to retain only the principal part of V_1.

It follows from the definitions of r, ρ, ψ, ρ_1, R, R_1, and Ψ that

$$\left.\begin{aligned}
\rho_1^2 &= r^2+\rho^2-2r\rho\cos\psi, \\
R_1^2 &= r^2+R^2-2rR\cos\Psi.
\end{aligned}\right\} \tag{11}$$

Then it follows from equations (1), ... (11) that

$$\left.\begin{aligned}
c &= \frac{\delta V}{\delta r} = -\frac{k^2 E}{r^2}\left\{1+\tfrac{3}{10}\frac{b^2}{r^2}(\cos^2\phi-2\sin^2\phi)e^2+\ \ldots\ \right\}+\omega^2 r\cos^2\phi \\
&\qquad -k^2 M\left\{\frac{1}{\rho_1^2}\frac{\delta\rho_1}{\delta r}+\frac{1}{\rho^2}\cos\psi\right\}-k^2 S\left\{\frac{1}{R_1^2}\frac{\delta R_1}{\delta r}+\frac{1}{R^2}\cos\Psi\right\}, \\
a_1 &= \frac{\delta V}{\delta\phi} = -\frac{k^2 E}{r^2}\frac{\delta r}{\delta\phi}-\tfrac{3}{10}k^2 E\frac{b^2 e^2}{r^3}\sin 2\phi-\tfrac{1}{2}\omega^2 r^2\sin 2\phi \\
&\quad -k^2 M\left\{\frac{1}{\rho_1^2}\frac{\delta\rho_1}{\delta\cos\psi}+\frac{r}{\rho^2}\right\}\frac{\delta\cos\psi}{\delta\phi}-k^2 S\left\{\frac{1}{R_1^2}\frac{\delta R_1}{\delta\cos\Psi}+\frac{r}{R^2}\right\}\frac{\delta\cos\Psi}{\delta\phi}, \\
a_2 &= \frac{\delta V}{\delta\theta} = -k^2 M\left\{\frac{1}{\rho_1^2}\frac{\delta\rho_1}{\delta\cos\psi}+\frac{r}{\rho^2}\right\}\frac{\delta\cos\psi}{\delta\theta} \\
&\qquad\qquad -k^2 S\left\{\frac{1}{R_1^2}\frac{\delta R_1}{\delta\cos\Psi}+\frac{r}{R^2}\right\}\frac{\delta\cos\Psi}{\delta\theta}, \\
a_3 &= \tfrac{1}{2}\frac{\delta^2 V}{\delta r^2}=\frac{k^2 E}{r^3}, \qquad a_4=\tfrac{1}{2}\frac{\delta^2 V}{\delta\phi^2}=\tfrac{1}{2}\frac{\delta a_1}{\delta\phi}, \qquad a_5=\tfrac{1}{2}\frac{\delta^2 V}{\delta\theta^2}=\tfrac{1}{2}\frac{\delta a_2}{\delta\theta}, \\
a_6 &= \frac{\delta^2 V}{\delta r\delta\phi}=\frac{\delta a_1}{\delta r}, \qquad a_7=\frac{\delta^2 V}{\delta r\delta\theta}=\frac{\delta a_2}{\delta r}, \qquad a_8=\frac{\delta^2 V}{\delta\phi\delta\theta}=\frac{\delta a_1}{\delta\theta}.
\end{aligned}\right\} \tag{12}$$

The east-and-west tide is obtained by putting $\Delta\phi=0$, and the north-and-south tide by putting $\Delta\phi=0$. Since in the tide experiment pipes are used only in these two directions, it is not necessary to compute c_5 and a_8. The parts of the other a_i which may be neglected can be determined only by deciding what accuracy must be attained and by passing to numbers. The degree of accuracy to be attained in the calculations depends upon that attained in the observations. In the Michelson-Gale experiments the individual observations are accurate to within about 1 per cent, and their averages, of course, are somewhat more exact. Consequently, it will be sufficient to determine the theoretical tides to within one-tenth of 1 per cent of their values. All terms will be included which affect the results by as much as this amount. In order to determine what terms must be included it is necessary to adopt numerical values for the various quantities involved.

The units of distance, time, and mass will be taken respectively as the mean distance from the earth to the sun, the mean solar day, and the mass of the sun. In these units

$$\left.\begin{aligned} E&=\frac{1}{330{,}000}, \\ \frac{M}{E}&=\frac{1}{81.8}, \\ \omega&=\frac{\text{sidereal day}}{\text{solar day}}\,2\pi, \\ \log k&=8.23558-10, \\ a&=\frac{3963.3}{92{,}900{,}000}, \\ b&=\frac{3950}{92{,}900{,}000}, \\ \text{mean } \rho&=\frac{238{,}860}{92{,}900{,}000}, \\ e^2&=0.00665, \\ \phi \text{ of } P&=42^\circ 34' 13''. \end{aligned}\right\} \tag{13}$$

Since

$$r=\frac{b}{[1-e^2\cos^2\phi]^{\frac{1}{2}}}, \tag{14}$$

it follows that

$$\left.\begin{aligned} r &= \frac{3950}{92{,}900{,}000}\left[1+\tfrac{1}{2}e^2\cos^2\phi \; . \; . \; . \; . \;\right] \\ &= \frac{3957}{92{,}900{,}000} \text{ at } \phi=42^\circ 34' 13''. \end{aligned}\right\} \tag{15}$$

Now consider the expression for c, equations (12). Let $-k^2E/r^2$ be taken out as a factor. Then the smaller quantities will be compared with unity. It follows from (14) that

$$\left.\begin{aligned} \tfrac{3}{10}\frac{b^2}{r^2}(\cos^2\phi-2\sin^2\phi)e^2 &= -\tfrac{3}{10}[2-(3+2e^2)\cos^2\phi+3e^2\cos^4\phi]e^2 \\ &= -0.00074 \text{ for } \phi=42^\circ 34' 13''. \end{aligned}\right\} \tag{16}$$

It is also found that

$$\left.\begin{aligned} \frac{-\omega^2 r^3\cos^2\phi}{k^2E} &= -0.00338(1+\tfrac{3}{2}e^2\cos^2\phi)\cos^2\phi = -0.00184 \text{ for} \\ &\phi=42^\circ 34' 13''. \end{aligned}\right\} \tag{17}$$

These quantities are constants for a given station.

Now consider the third term of c. It follows from (11) that

$$\left|\frac{\delta\rho_1}{\delta r}\right| = \left|\frac{-\rho\cos\psi+r}{\rho_1}\right| \leqq 1. \tag{18}$$

Hence, at the maximum,

$$\frac{k^2M}{k^2E}\left|\frac{r^2}{\rho_1^2}\frac{\delta\rho_1}{\delta r}-\frac{r^2}{\rho^2}\cos\psi\right| \leqq 0.000006.$$

This term is therefore quite insensible, and the same is true of the last one. Hence,

$$\left.\begin{aligned} \frac{\delta V}{\delta r} = c &= -\frac{k^2E}{r^2}[0.99601+0.00261\cos^2\phi] \\ &= -1.0027\frac{k^2E}{a^2}[1-0.00406\cos^2\phi] \\ &= -1.0005\frac{k^2E}{a^2} \text{ for } \phi=42^\circ 34' 13''. \end{aligned}\right\} \tag{19}$$

Now consider the expression for a_1. The first three terms are constant for a given station, and, since only the variations in the tides can be measured, they may be omitted.

Now consider the fourth expression in a_1. It follows from (18) that

$$\frac{\delta\rho_1}{\delta\cos\psi}=-\frac{\rho}{\rho_1}r. \tag{20}$$

It follows from the definition of the symbols ψ, ϕ, δ, α, θ, Ψ, D, and A that

$$\left.\begin{aligned}
\cos\psi&=\cos\delta\cos\phi\cos(\alpha-\theta)+\sin\phi\sin\delta,\\
\cos\Psi&=\cos D\cos\phi\cos(A-\theta)+\sin\phi\sin D,\\
\frac{\delta\cos\psi}{\delta\phi}&=-\cos\delta\sin\phi\cos(\alpha-\theta)+\cos\phi\sin\delta,\\
\frac{\delta\cos\psi}{\delta\theta}&=\cos\delta\cos\phi\sin(\alpha-\theta),\\
\frac{\delta\cos\Psi}{\delta\phi}&=-\cos D\sin\phi\cos(A-\theta)+\cos\phi\sin D,\\
\frac{\delta\cos\Psi}{\delta\theta}&=\cos D\cos\phi\sin(A-\theta).
\end{aligned}\right\} \tag{21}$$

Therefore the fourth term in a_1 becomes

$$-k^2M\left\{\frac{1}{\rho_1^2}\frac{\delta\rho_1}{\delta\cos\psi}+\frac{r}{\rho^2}\right\}\frac{\delta\cos\psi}{\delta\phi}=-k^2Mr\rho\left[\frac{1}{\rho^3}-\frac{1}{\rho_1^3}\right]\frac{\delta\cos\psi}{\delta\phi}. \tag{22}$$

Since ρ and ρ_1 are nearly equal it will be most convenient to expand the difference $\frac{1}{\rho^3}-\frac{1}{\rho_1^3}$ into a series. It follows from (11) that

$$\left.\begin{aligned}
\frac{1}{\rho^3}-\frac{1}{\rho_1^3}=\frac{1}{\rho^3}\left\{1-\left[1-\frac{2r}{\rho}\cos\psi+\left(\frac{r}{\rho}\right)^2\right]^{-\frac{3}{2}}\right\}=\frac{r}{\rho^4}\Big\{3\cos\psi\\
+\tfrac{3}{4}\frac{r}{\rho}(3+5\cos2\psi)+\tfrac{5}{8}\left(\frac{r}{\rho}\right)^2(9\cos\psi+7\cos3\psi)\ldots\Big\}.
\end{aligned}\right\} \tag{23}$$

The maximum numerical values of the terms included in the bracket are

$$3,\quad 6\frac{r}{\rho},\quad 10\left(\frac{r}{\rho}\right)^2,\quad 15\left(\frac{r}{\rho}\right)^3,\ \ldots.$$

Since r/ρ is about 1/60, it is sufficient, in order to secure an accuracy of one-tenth of one per cent, to retain the first three terms in the right member of (23). The last two terms of a_1 are therefore

$$\left.\begin{aligned}
&-k^2M\left\{\frac{1}{\rho_1^2}\frac{\delta\rho_1}{\delta\cos\psi}+\frac{r}{\rho^2}\right\}\frac{\delta\cos\psi}{\delta\phi}=-3k^2M\frac{r^2}{\rho^3}\left\{\cos\psi+\tfrac{1}{4}\frac{r}{\rho}(3+5\cos 2\psi)\right.\\
&\qquad\qquad\left.+\tfrac{5}{24}\left(\frac{r}{\rho}\right)^2(9\cos\psi+7\cos 3\psi)\;.\;.\;.\;.\right\}\frac{\delta\cos\psi}{\delta\phi},\\
&-k^2S\left\{\frac{1}{R_1^2}\frac{\delta R_1}{\delta\cos\Psi}+\frac{r}{R^2}\right\}\frac{\delta\cos\Psi}{\delta\phi}\\
&\qquad\qquad=-3k^2S\frac{r^2}{R^3}\left\{\cos\Psi+\;.\;.\;.\;.\right\}\frac{\delta\cos\Psi}{\delta\phi},
\end{aligned}\right\}\quad(24)$$

one term being sufficient in the right member of the second equation because r/R is about $1/23,000$.

The north-and-south tides of the first order in $\Delta\phi$ are therefore given by

$$(\Delta r)_{NS}=-\frac{a_1}{c}\Delta\phi=(Af+BF)r\Delta\phi,\qquad(25)$$

where

$$\left.\begin{aligned}
&A=-0.036454(1+0.0074\cos^2\phi)\left(\frac{a}{\rho}\right)^3\left\{\cos\psi+\tfrac{1}{4}\frac{r}{\rho}(3+5\cos 2\psi)\right.\\
&\qquad\qquad\left.+\tfrac{5}{24}\left(\frac{r}{\rho}\right)^2(9\cos\psi+7\cos 3\psi)+\;.\;.\;.\;.\right\}\\
&B=-984030(1+0.0074\cos^2\phi)\left(\frac{a}{R}\right)^3\left\{\cos\Psi+\;.\;.\;.\;.\right\}\\
&f=-\cos\delta\sin\phi\cos(\alpha-\theta)+\cos\phi\sin\delta,\\
&F=-\cos D\sin\phi\cos(A-\theta)+\cos\phi\sin D.
\end{aligned}\right\}\quad(26)$$

The east-and-west tides of the first order in $\Delta\theta$ are similarly given by

$$(\Delta r)_{EW}=-\frac{a_2}{c}\Delta\theta=(Ag+BG)\cos\phi\; r\Delta\phi,\qquad(27)$$

where

$$\left.\begin{aligned}
g&=\cos\delta\sin(\alpha-\theta),\\
G&=\cos D\sin(A-\theta).
\end{aligned}\right\}\quad(28)$$

It will now be shown that the terms of higher order in (9) are insensible for the present problem. Consider the third term, whose coefficient is given in (10), in comparison with the first term. The ratio is found from (10) to be

$$\frac{c_3}{c_1}\Delta\phi=\left(\frac{a_4}{a_1}-\frac{a_6}{c}+\frac{a_1a_3}{c^2}\right)\Delta\phi.\qquad(29)$$

It follows from (12) and (25) that the principal part of a_4/a_1 is

$$\frac{1}{2}\frac{\frac{\delta}{\delta\phi}(\cos\psi\cdot f)}{\cos\psi\cdot f},$$

which is of the order of unity. It follows from (11), (12), (22), and (24) that the principal part of $-\frac{a_6}{c}$ is

$$-6\frac{M}{E}\left(\frac{r}{\rho}\right)^3\cos\psi\,\frac{\delta\cos\psi}{\delta\phi},$$

which is very small compared to unity. Finally, the principal part of $\frac{a_1a_3}{c^2}$ is

$$-3\frac{M}{E}\left(\frac{r}{\rho}\right)^3\cos\psi,$$

which also is very small compared to unity. Since $\Delta\phi$ is less than 1/40,000 for a pipe 500 feet long, the term in question is wholly inappreciable. The same is true of the fourth term of (9), and the last vanishes for both north-and-south and east-and-west pipes.

The theoretical tides in the pipes on a rigid earth are given by equations (25), . . . , (28). The quantities $r\Delta\phi$ and $\cos\phi\, r\Delta\theta$ are the lengths of the north-and-south and east-and-west pipes respectively, and are constant in the computations for a given station. They may be expressed in any units which may be convenient in comparing the computed and observed results. Let

$$\left.\begin{aligned} k_1 &= -0.036454\, r\Delta\phi, \\ k_2 &= -0.036454\cos\phi\, r\Delta\theta, \\ K_1 &= -984030\, r\Delta\phi, \\ K_2 &= -984030\cos\phi\, r\Delta\theta \end{aligned}\right\} \qquad (30)$$

If the two pipes are equal in length, then $k_1=k_2$, $K_1=K_2$.

The latitude ϕ is constant for each station. The quantities α, δ, ρ, A, D, and R are taken from the *Ephemeris*. The sidereal time depends upon the times of observation. If they differ by equal intervals, say two hours of mean solar time, the values of θ differ by about $30°5'$, or, more exactly, $30°4'55''.7$. The quantities $\cos\psi$ and $\cos\Psi$ are computed from the first two equations of (21). Then equations (25), . . . , (28) give the desired results.

The terms

$$Q=\tfrac{1}{4}\,\frac{r}{\rho}(3+5\cos 2\psi)+\tfrac{5}{24}\left(\frac{r}{\rho}\right)^2(9\cos\psi+7\cos 3\psi) \tag{31}$$

of equations (25) and (27) are small corrections to cos ψ. Since they are relatively small, a table for Q with argument cos ψ was computed, which is given herewith. The use of this table made it unnecessary to compute Q for each value of cos ψ encountered in computing the tides.

TABLE I

Cos ψ	Q	Cos ψ	Q	Cos ψ	Q
1.00	0.0343	0.30	−0.0047	−0.40	−0.0015
0.95	.0300	.25	− .0059	− .45	− .0003
.90	.0260	.20	− .0068	− .50	− .0022
.85	.0222	.15	− .0075	− .55	− .0044
.80	.0186	.10	− .0080	− .60	− .0067
.75	.0153	.05	− .0083	− .65	− .0093
.70	.0122	00	− .0083	− .70	− .0120
.65	.0093	− .05	− .0082	− .75	− .0149
.60	.0066	− .10	− .0078	− .80	− .0181
.55	.0042	− .15	− .0073	− .85	− .0214
.50	.0019	− .20	− .0065	− .90	− .0249
.45	− .0001	− .25	− .0056	− .95	− .0285
.40	− .0018	− .30	− .0044	−1.00	− .0324
.35	− .0034	− .35	− .0031		

The variations in r and ρ produce no sensible variations in the last term of Q. The variations in ρ, however, in the first term of Q have small effects which must be taken into account in order to secure an accuracy of one part in a thousand. The ratio r/ρ is defined in the *Ephemeris* by the parallax of the moon. The following table gives the correction δQ to Q to be applied as a consequence of the variations in the moon's parallax.

TABLE II

Moon's Parallax	Cos ψ	Cos ψ	Cos ψ	Cos ψ	Cos ψ	Cos ψ	Cos ψ	Cos ψ	Cos ψ
	1.00	0.80	0.50	0.25	.00	−0.25	−0.50	−0.80	−1.00
60′	0.0017	0.0009	0.0001	−0.0003	−0.0004	−0.0003	0.0001	0.0009	0.0016
58′.5	.0008	.0005	.0000	− .0002	− .0002	− .0001	.0000	.0005	.0008
57′	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
55′.5	−.0008	−.0005	.0000	.0002	.0002	.0001	.0000	−.0005	−.0008
54′	−.0017	−.0009	−.0001	.0003	.0004	.0003	−.0001	−.0009	−.0016

University of Chicago
November 1, 1919

ON THE BRIGHTNESS OF THE SKY AT NIGHT AND THE TOTAL AMOUNT OF STARLIGHT[1]

By P. J. VAN RHIJN

1. INTRODUCTION

The following is an abstract of the results of an investigation which will appear in the "Papers from the Mount Wilson Observatory." It contains a discussion of observations of the brightness of the sky made at Mount Wilson during the summer and autumn of 1913. The main points investigated are the total amount of starlight per square degree for each galactic latitude and the distribution over the sphere of the illumination of the sky which is not due to the stars. The latter appears to vary with the latitude and longitude of the area observed; the evidence indicates that at least part of the light of the sky proceeds from a source probably identical with that of the zodiacal light.

2. THE OBSERVATIONS

The instrument used is that described by Yntema.[2] For details as to the method of measurement, reference may be made to the extended account of the investigation. Here it need only be remarked that the determination of the brightness of the sky has been made in two steps: (*a*) comparison of the light of various regions of the sky with that of an area at the North Pole; (*b*) comparison of the light at the pole with that of a star of known magnitude.

The first step has been made by varying the illumination of an annular screen in a measurable way until it disappeared against the background of the sky. The amount of light per square degree at the pole has been determined in terms of starlight by first comparing the surface brightness of the sky near the pole with that of an artificially illuminated disk of opal glass, and then estimating the stellar magnitude of the disk by observing it from a distance such that it had the appearance of a star.

[1] *Contributions from the Mount Wilson Observatory*, No. 173.

[2] *Groningen Publications*, No. 22, pp. 9–11, 1909.

The small electric lamps used in the photometers were fed by a two-cell storage battery. The constancy of the current was carefully controlled. The only photometric principle used in the measurements is the law of inverse squares.

The areas measured were identified by means of a map showing all the stars visible to the naked eye. The time of observation was noted in all cases.

The observations made in series are as follows: (1) measures of the brightness at the North Pole, which was determined several times each night; (2) measures of areas on a circle parallel to the horizon; (3) measures on a circle perpendicular to the horizon.

3. GENERAL PLAN OF THE INVESTIGATION

The observations have been used to investigate the following questions: (*a*) Yntema found that the brightness of the sky is not due exclusively to the stars,[1] a result confirmed by the present data. What is the distribution of this additional light over the sphere and what is its cause? (*b*) Is it possible to eliminate the additional light and thus discuss the distribution of starlight alone?

The first of these questions may be answered by investigating the skylight for galactic latitudes higher than 40°. Above this limit the light due to the stars is a relatively small percentage of the total brightness and can be computed with sufficient accuracy from the counts of stars given in *Groningen Publications*, No. 27, Table V. We thus find for all stars fainter than magnitude 5.5 the results shown in Table I. These may be considered as the quantities of starlight actually measured with our photometer.

TABLE I

LIGHT PER SQUARE DEGREE OF ALL STARS FAINTER THAN MAG. 5.5

(Unit = a star of mag. 1.00, Harvard Visual Scale)

Gal. Lat.	Amount of Light
40°	0.012
50	.011
60	.009
70	.009
80	.008
90	0.008

[1] L. Yntema, *op. cit.*, p. 35.

Yntema found that the so-called earthlight, i.e., the brightness not due to the stars,[1] changes with the zenith distance of the area observed, but that it is approximately independent of the azimuth. I first investigated, by means of measures on a circle parallel to the horizon, whether the earthlight is really the same for all points on such a circle. Only those areas were used whose galactic latitudes exceed 40°. The earthlight was found by subtracting the direct and scattered starlight from the total brightness. The differences vary clearly with the azimuth, being always larger in the eastern part of the sky. It will be shown that these variations are caused by a kind of zodiacal light extending over the whole sky. The observed brightness corrected for starlight depends distinctly on the latitude and longitude relative to the sun. The excess of this zodiacal brightness over its mean value will be found for each latitude and longitude,[2] and a correction equal to this excess with opposite sign has been applied to all measures. The values of the earthlight thus corrected are independent of the azimuth; in any series of measures parallel to the horizon we can thus find the starlight for areas in the Milky Way by taking the difference between the total observed brightness and the corrected[3] value of the earthlight, the amount of which has been found from areas in higher galactic latitudes. This gives the solution of the second problem stated above.

I next considered the dependence of the corrected earthlight on the zenith distance by means of the series of measures in high galactic latitudes along circles perpendicular to the horizon. The values increase with increasing zenith distance. This is partly due to earthlight scattered by the atmosphere, the scattering being greatest near the horizon. The amount of scattered earthlight was found by means of Abbot's determination of the ratio of direct and scattered sunlight.[4] After subtraction of the scattered earthlight, the residuals, corrected for the absorption of the atmosphere,

[1] The starlight scattered by the atmosphere is *not* contained in the earthlight. The determination of its amount will be described below.

[2] Longitude means always longitude relative to the sun.

[3] That is, corrected for the excess of zodiacal light over its mean value.

[4] *Astronomical Journal*, **28**, 130, 1914.

still increase toward the horizon. An empirical formula which represents these residuals was found and used to determine the earthlight for areas near the Galaxy. Subtraction of the values thus found from the observed brightness then gave the starlight in the lower galactic latitudes. This is another solution of the second question.

From the preceding lines it appears that I have based my conclusions as much as possible on a comparison of measures included in the *same* series of observations, which were always finished within one hour. This has been done in order to eliminate any variation of the earthlight with the time.[1]

4. CHANGE OF THE SKY-BRIGHTNESS NEAR THE NORTH POLE

The brightness at the North Pole has been measured repeatedly every night. Its value appears to be subject to large variations. Even during the same night the light near the pole may change as much as 20 per cent. These fluctuations are due to a variation of the earthlight. I have not, however, been able to find any systematic tendency in these variations, although the mean brightness is practically the same before and after midnight. The changes from one night to another are rather capricious.

5. DEPENDENCE OF THE EARTHLIGHT ON THE AZIMUTH. ZODIACAL LIGHT EXTENDING OVER THE WHOLE SKY

All series of observations parallel to the horizon have been treated in the way indicated in Table II, which serves only as an example.

The first six columns of Table II give the number, the galactic latitude (b), the latitude (β), the longitude relative to the sun (λ), the zenith distance (z), and the azimuth (A). The co-ordinates have been computed from the right ascension, the declination, and the time of observation. B denotes the observed amount of light per square degree expressed in terms of that of a star of magnitude 1.00 (Harvard Scale). The earthlight for areas whose galactic latitudes exceed 40° is given under the heading $B_{s', s}$. This has been formed by subtracting the direct starlight (s) and the scattered

[1] See section 4.

TABLE II

REDUCTION OF A SERIES OF OBSERVATIONS PARALLEL TO THE HORIZON

(Unit = a star of mag. 1.00, Harvard Visual Scale)

No. (1)	b (2)	β (3)	λ (4)	z (5)	A (6)	B (7)	$B_{s'}$ (8)	$B_{s',s}$ for $b>40°$ Uncorr. (9)	$B_{s',s}$ for $b>40°$ Corr.* (10)	$B_{s'}$ Corr.* $b<40°$ (11)	Corr. Zod. L. (12)	$B^z_{s',s}$ $b>40°$ (13)	$B^z_{s'}$ $b<40°$ (14)	Starlight $b<40°$ (15)
240	North Pole					0.118								
241	+56°	+47°	+82°	56°	108°	.108	0.101	0.092	0.103		+0.004	0.107		
242	+53	+65	+54	53	137	.104	.097	.088	.097		+.016	.113		
243	+49	+69	+42	53	144	.100	.093	.083	.091		+.019	.110		
244	−1	+47	−84	57	214	.156	.149			0.168	+.004		0.172	+0.061
245	+5	+54	−79	55	207	.132	.125			.140	+.010		.150	+.039
246	+11	+58	−72	54	199	.118	.111			.123	+.012		.135	+.024
247	−12	+40	−93	57	228	.140	.133			.150	+.002		.152	+.041
248	−55	+5	−130	57	291	.122	.115	.106	.120		−.010	.110		
249	−61	−2	−145	58	307	.128	.120	.112	.128		−.011	.117		
250	+25	+29	+122	55	72	.130	.123			.138	+.001		.130	+.028
251	North Pole					.119								
Mean												0.111		

* Corrected for atmospheric absorption, reduced to the zenith.

starlight (s') from the measured brightness B. As has been explained already, the direct starlight per square degree has been derived from counts of stars (see Table I). The scattered starlight has been computed by means of Abbot's observations already quoted. For the same zenith distance the earthlight, in nearly all cases, is larger for areas near azimuth 270° than for those near azimuth 110°. This is the case in the series of Table II. This difference is to be explained by a dependence of the earthlight on the position of the measured area relative to the ecliptic.

That this is really the case is shown in Table III, which contains the differences between the earthlight for the smallest and the largest latitudes occurring in each series. Excepting one case, for which the difference in β is only 12°, the lower latitude corresponds always to a higher illumination.

TABLE III

DIFFERENCES IN EARTHLIGHT, SMALLEST AND LARGEST LATITUDES IN EACH SERIES PARALLEL TO THE HORIZON

β_1	β_2	Diff.	β_1	β_2	Diff
22°	64°	+0.019	0°	70°	+0.032
21	69	+ .022	11	59	+ .041
27	69	+ .033	2	69	+ .037
58	70	− .005	9	67	+ .043
14	73	+ .025	2	23	+ .009
1	63	+0.037	7	19	+ .006
			10	63	+0.031

If now the brightness of the sky depends on the latitude, it is probably connected with the illumination usually called the zodiacal light, and must then also vary with the longitude relative to the sun. We have therefore to find the dependence of the earthlight on the latitude and longitude. It is impossible, however, to derive directly the absolute amount of the light which is related to the ecliptic and which for convenience will be called zodiacal light. For, as will be shown presently, it is very probable that the brightness of the sky is produced in part by other causes, whose contribution to the general illumination also varies with the zenith distance in a manner which is unknown. We therefore derive, first of all,

not the absolute amount of zodiacal light, but the excess of zodiacal light over its mean value for each latitude and longitude. This has been done on the supposition that the zodiacal light is the same north and south of the ecliptic; the sign of the longitude relative to the sun has also been disregarded.[1] The difference in the intensity of zodiacal light for various latitudes and longitudes can be found by subtracting any two values of the earthlight $B_{s',s}$ in the same series parallel to the horizon, for it can be shown that the variation of the earthlight with the azimuth is due to the zodiacal light alone. It follows, for instance, from the comparison of the mean $B_{s',s}$ for observations Nos. 241 to 243 with that for Nos. 248 and 249 that the zodiacal glow at $\beta=3°$, $\lambda=140°$, is $0.124-0.097=+0.027$ larger than at $\beta=60°$, $\lambda=59°$. By combining the differences thus found it is possible to compute for any latitude and longitude the excess of zodiacal light over a certain mean value.

The method used for the combination of the various differences has been described at length in the detailed investigation. It need only be stated here that the values of the excess of zodiacal light over its mean value were distributed among four groups having latitudes near $\beta=0°$, $20°$, $55°$, and $70°$, respectively. A small correction was applied in order to reduce the figures to the exact values of these latitudes. The corrected light intensities were then plotted as a function of the longitude. Finally, in order to find the absolute amount of zodiacal light, the values of the excess were increased by $+0.069$. The quantity $+0.069$, to be derived in a later section, is somewhat hypothetical and the absolute values are less trustworthy than their differences for any two values of the latitude and longitude.

The graphically interpolated values of the zodiacal light thus found are given in Table IV. It may be remarked that Fessenhoff's[2] observations have been used for the smaller longitudes near the ecliptic and that in my measures special attention has been given to the brightness of the counterglow at $\beta=0°$, $\lambda=180°$.

[1] The longitude relative to the sun has been counted from 0° to +180° and 0° to −180°; the longitude of the sun itself is, of course, zero.

[2] *La Lumière Zodiacale.* Thèse de doctorat, 1919.

We have now to derive an interpolation formula for the zodiacal light as a function of the latitude and the longitude. This formula will be used mainly to compute the excess of zodiacal light at any

TABLE IV

VALUES OF THE ZODIACAL LIGHT

(Unit=a star of mag. 1.00, Harvard Visual Scale)

Lat.	Long. from Sun	Zodiacal Light	Wt.	O−C	Lat.	Long. from Sun	Zodiacal Light	Wt.	O−C
0°.....	40°	0.329	15	+0.007	20°....	40°	0.183	15	−0.016
0	60	.184	15	− .001	20	70	.125	15	+ .012
0	70	.149	2	− .001	20	100	.081	7	.000
0	80	.127	5	.000	20	130	.069	9	− .002
0	100	.098	12	+ .001	20	160	.072	12	− .001
0	130	.084	11	+ .005	55	40	.069	5	+ .007
0	160	.083	13	+ .001	55	100	.058	10	+ .004
0	170	.083	15	− .005	70	40	.047	7	− .004
0	180	0.097	4	+0.001	70	100	0.047	3	−0.002

point (β, λ) over the mean value. After the application of this excess with the reversed sign to the observed brightness the quantity of zodiacal light is the same for all areas.

It is, of course, desirable to use a formula which has a physical basis. We shall, therefore, try the function resulting from Seeliger's theory of the zodiacal glow,[1] which supposes the zodiacal light to be due to a reflection of sunlight by small particles of meteoric matter. The cloud of particles is symmetrical relatively to a plane which contains the axis of the zodiacal light and extends equally far in all directions in this plane. Seeliger found that on this hypothesis the intensity of zodiacal light depends in the following way on the density D of the meteoric matter and the law of phase $f(\alpha)$:

$$\frac{H}{J}=\frac{C}{\sin\psi}\int_{\tau}^{\psi} D(x, y)\, f(\alpha)d\alpha \tag{1}$$

[1] "Ueber kosmische Staubmassen und das Zodiacallicht," *Sitzungsberichte der mathem. phys. Classe der kgl. bayer. Akademie der Wissenschaften*, **31**, Heft 3, 1901.

where

H = surface brightness of the zodiacal light

J = surface brightness of the sun

α = angle at the reflecting particle formed by the lines toward the sun and the earth. (See Fig. 1, where S represents the sun, O the observer, P a reflecting particle, and the plane SOP' the ecliptic. PP' is perpendicular to this plane)

ψ = Angle SOQ in Fig. 1 = supplement of the angle at the observer between the lines toward the sun and the particle.

τ = value of α at the exterior limit of the cloud

x and y = co-ordinates of the particle relative to the sun in a plane perpendicular to the ecliptic

$D(x, y)$ = density of the cloud as a function of the co-ordinates x and y

$f(\alpha)$ = function representing the dependence of the quantity of reflected light on the phase angle α

C = a constant

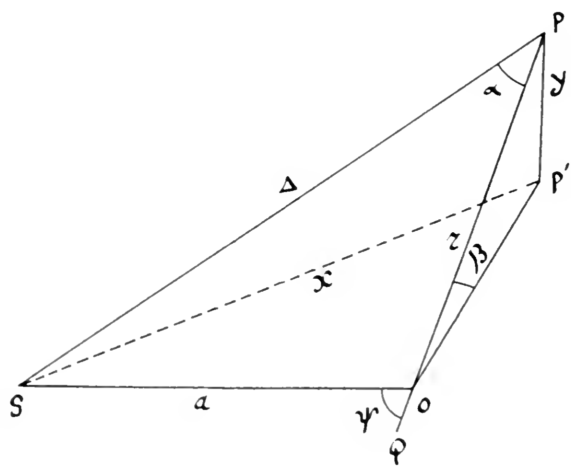


FIG. 1

If the function $f(\alpha)$ were known from terrestrial experiments, we might find the distribution of the density D by means of equations of the form (1), where the left-hand member is the observed quantity. This, however, is not the case; various laws have been

proposed,[1] but it is at present impossible to decide which of these is the right one.

Our observations, however, afford the material for a determination of the law of phases for small values of α, for in the neighborhood of the counterglow the variation of the zodiacal light with the position of the area depends almost exclusively on the law of phases, whereas the adopted density distribution is of little importance. This is clear a priori and has, moreover, been demonstrated by Searle.[2] The law of phases for small values of α can, therefore, be determined from the observed brightness of the zodiacal light near the counterglow. After some trials I found the fractions given in the third column of Table V. For large values of α the law of phases was made to agree approximately with the form of Lommel and Seeliger. Table V shows that my data yield a larger decrease of the brightness with the phase angle for small values of α than does Seeliger's formula. This is in agreement with the investigations made on the asteroids.[3] The adopted values of $f(\alpha)$ in the third column of Table V can be represented by the formula:

$$f(\alpha)=+0.655-0.711 \sin \alpha+0.345 \cos \alpha+0.406 \sin^2 \alpha \qquad (2)$$

TABLE V

THE LAW OF PHASES

α	Lommel and Seeliger	Adopted	α	Lommel and Seeliger	Adopted
0°	1.00	1.00	70°	0.54	0.46
10	.98	.88	90	.38	.35
20	.93	78	110	.24	.23
30	.86	.70	130	.12	.13
50	0.70	0.57	140*	0.08	0.10

* Phase angles larger than 140° do not occur in the present investigation.

[1] Müller, *Die Photometrie der Gestirne*, pp. 58–62, 1897. See also Otto Frh. v.u.z. Aufsess, "Experimentelle Untersuchungen über den Einfluss der Phase und der Rotation auf die Helligkeit vor Kugeln und beliebig gestaltene Körpern," *Astronomische Abhandlungen als Ergänzungshefte zu den Astronomischen Nachrichten*, No. 17, 1910.

[2] *Harvard Annals*, **19**, 233, 1893.

[3] Müller, *op. cit.*, p. 378.

Various formulae have been tried for the density, which is to be determined from the condition that the zodiacal brightness, computed by means of (1), should agree with the observations. I first considered only the measures in the ecliptic for various longitudes, but later the higher latitudes were also taken into account. It was found that the formula

$$\text{Density} = D_0(1 - 0.176\, x^2 - 1.406\, y^2) \tag{3}$$

satisfies the observations.

The brightness computed by means of formulae (1), (2), and (3) has been compared with the observed values in Table IV. The residuals O − C are all small. These considerations show that the present material at least does not contradict Seeliger's theory of the zodiacal light, although our lack of knowledge of the law of phases makes it impossible to put the theory to a severe test.

Formulae (1), (2), and (3), however, have been derived mainly for another purpose, viz., the computation of the zodiacal light at points of the sky for which no direct observations are available. The results of this computation have been given in Table VI, which represents the total amount of zodiacal light for different values of the latitude and longitude relative to the sun. It will be remembered that the values on which the table is based have been derived by increasing the excess of zodiacal light over a certain mean value by +0.069, and that this latter figure is somewhat uncertain on account of the hypothesis underlying its derivation. The same holds for the data of Table VI, the difference between any two values of the zodiacal light being more trustworthy than the values themselves. Fortunately it is the differences, in the main, which are needed for the reduction.

6. DETERMINATION OF THE STARLIGHT FOR GALACTIC LATITUDES LESS THAN 40° FROM THE OBSERVATIONS ALONG CIRCLES PARALLEL TO THE HORIZON

The method used to determine the starlight from series of measures parallel to the horizon is also illustrated by Table II. The first ten columns have already been explained. In the twelfth

column is given a correction for the excess of zodiacal light over a certain mean value. This has been derived by subtracting 0.069 from the figures of Table VI and reversing the sign of the difference

TABLE VI

COMPUTED VALUES OF THE ZODIACAL LIGHT

(Unit = a star of mag. 1.00, Harvard Visual Scale)

Long. from Sun	Latitude (β)									
	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
0°	......	......	......	......	0.112	0.077	0.060	0.050	0.046	0.045
10	......	......	......	......	.110	.076	.060	.050	.046	.045
20	......	......	......	......	.103	.076	.060	.050	.046	.045
30	......	......	......	......	.097	.073	.059	.049	.046	.045
40	0.325	0.258	0.191	0.132	.090	.072	.059	.049	.046	.045
50	.247	.206	.161	.113	.086	.071	.058	.049	.046	.045
60	.184	.168	.138	.105	.084	.069	.057	.049	.046	.045
70	.149	.139	.119	.097	.079	.066	.055	.049	.046	.045
80	.127	.119	.103	.087	.073	.063	.053	.048	.046	.045
90	.109	.102	.090	.079	.068	.061	.053	.047	.046	.045
100	.098	.090	.081	.075	.065	.060	.052	.047	.046	.045
110	.090	.084	.076	.071	.064	.059	.052	.047	.046	.045
120	.086	.080	.073	.068	.061	.057	.052	.047	.046	.045
130	.082	.077	.070	.066	.060	.057	.052	.047	.046	.045
140	.081	.078	.070	.066	.060	.056	.051	.047	.046	.045
150	.081	.078	.071	.066	.060	.056	.051	.046	.045	.045
160	.082	.079	.072	.066	.060	.056	.051	.046	.045	.045
170	.085	.082	.074	.067	.060	.056	.050	.046	.045	.045
180	0.097	0.090	0.075	0.067	0.060	0.055	0.050	0.046	0.045	0.045

thus found. These corrections a have been applied to the figures of the tenth and eleventh columns, thus giving the results in the thirteenth and fourteenth columns, respectively. The areas in small and large galactic latitudes have been treated separately, because for the former the amount of the starlight is unknown.

The thirteenth column thus contains the observed brightness corrected for direct starlight (s), scattered starlight (s'), and excess of zodiacal light (z), whereas the figures in the fourteenth column have been corrected for the last two quantities only. The values of $B^z_{s',s}$ and $B^z_{s'}$ include an amount of zodiacal light which is the same for all areas. The quantities $B^z_{s',s}$ appear to be independent of the azimuth. We therefore form their mean, for which the corresponding galactic latitude exceeds 40°, and subtract the result

from each value of $B^{z}_{s'}$ in the fourteenth column. The differences inserted in the fifteenth column represent the starlight per square degree, reduced for atmospheric absorption to the zenith.

Most of the series were treated in this way. In only a few cases was the number of observations in galactic latitudes greater than 40° too small to apply this method. In such cases I computed the mean value of $B^{z}_{s',s}$ from the measures whose galactic latitudes exceed 20°. The starlight between 20° and 40° required for the reduction of these cases was obtained from all other series parallel to the horizon which include a sufficiently large number of observations in the higher galactic latitudes. The results are averaged in Table VII, the probable errors being derived from the internal agreement of the observations.

TABLE VII

TOTAL LIGHT PER SQUARE DEGREE OF ALL STARS FAINTER THAN MAG. 5.5 DERIVED FROM SERIES PARALLEL TO THE HORIZON

(Unit = a star of mag. 1.00, Harvard Visual Scale)

Gal. Lat. Limits	Gal. Lat. Mean	Starlight	Prob. Error
0° to 9°	4°.2	0.068	±0.003
10 to 19	14.1	.042	.002
20 to 29	25.8	.032	.002
30 to 39	35.5	0.013	±0.002

7. DEPENDENCE OF THE EARTHLIGHT, CORRECTED FOR ZODIACAL GLOW, ON THE ZENITH DISTANCE. PROBABILITY OF A PERPETUAL AURORA

We pass to the consideration of the series of observations perpendicular to the horizon. The variation of the earthlight with the zenith distance will be investigated first. This can be done only for areas whose galactic latitudes exceed 40°, where the starlight is supposed to be known. An example of the method employed is given in Table VIII.

The first nine columns of Table VIII contain the same quantities as the corresponding columns in Table II. The tenth column

TABLE VIII

Reduction of a Series of Observations Perpendicular to the Horizon

No.	b	β	λ	z	A	B	$B_{s'}$	$B_{s',s}$ $b>40°$	Corr.* Zod. L.	$B^z_{s',s}$ $b>40°$	$B^z_{s'}$ $b<40°$	Corr. Sc.E.L.	$B^{z,e}_{s',s}$ $b>40°$	$B^{z,e}_{s'}$ $b<40°$	Starlight Uncorr.	Starlight Corr.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
194	+72°	+41°	+ 30°	73°	132°	0.158	0.147	0.141	−0.018	0.123	....	−0.041	0.082	....	....	....
195	+66	+46	+ 24	71	138	.139	.129	.122	− .012	.110	....	− .039	.071	....	....	....
196	+62	+56	+ 39	59	136	.133	.125	.117	+ .004	.121	....	− .028	.093	....	....	....
197	+55	+63	+ 49	51	137	.115	.108	.099	+ .013	.112	....	− .024	.088	....	....	....
198	+45	+70	+ 76	39	135	.115	.109	0.098	+ .020	0.118	....	− .020	0.098	....	....	....
199	+36	+71	+108	29	134	.115	.109	....	+ .022	....	0.131	− .018	....	0.113	0.020	0.020
200	+25	+70	+141	19	138	.124	.119	....	+ .022	....	.141	− .016	....	.125	.032	.032
201	+18	+68	+165	16	155	0.138	0.133	....	+0.022	....	0.155	−0.016	....	0.139	0.016	0.046

* This correction has been reduced to the zenith distance z for atmospheric absorption.

gives the correction for the excess of zodiacal light derived in the same way as in section 6. The corrected values in the eleventh column thus contain a constant quantity of zodiacal light, the scattered earthlight, and, in addition, any light of unknown origin. In the thirteenth column is given a correction for scattered earthlight, derived by means of Abbot's observations already referred to and the total amount of earthlight known from the present investigation. The fourteenth column, in which the scattered earthlight is subtracted from the data in the eleventh column, thus contains a quantity of zodiacal light constant over the whole sky and perhaps some light produced by unknown sources of illumination. In order to find whether there are such sources, I have investigated the variation of the values of $B^{z,e}_{s',s}$ with the zenith distance. The data are collected in Table IX; all measures similar to those of Table VIII, for which the galactic latitude exceeds 40°, have been used.

TABLE IX

MEAN BRIGHTNESS CORRECTED FOR STARLIGHT, EXCESS OF ZODIACAL LIGHT, AND SCATTERED EARTHLIGHT $\left(B^{z,e}_{s',s}\right)$ AS A FUNCTION OF THE ZENITH DISTANCE

Limits z	Mean z	$B^{z,e}_{s',s}$	Prob. Error	No.	Weight	$B^{z,e}_{s',s}$ in Zenith	Comp.
10° to 29°	24°	0.085	±0.004	8	1.6	0.087	0.087
30 to 39	36	.084	.003	11	2.2	.087	.089
40 to 49	45	.090	.003	14	1.4	.095	.092
50 to 59	54	.086	.003	20	2.0	.096	.096
60 to 69	64	.086	.003	21	2.1	.104	.103
70 to 77	73	0.077	±0.005	13	0.6	0.111	0.116

The seventh column of Table IX gives the values of $B^{z,e}_{s',s}$ corrected for atmospheric absorption. If these values included nothing but a constant quantity of zodiacal light for the whole sky, they would be independent of the zenith distance. Since they increase with increasing zenith distance, there must exist some other source of illumination whose intensity is greatest near the horizon. This conclusion is strengthened by the fact that Slipher has photographed the green auroral line in the spectrum of the background of the sky.[1] A study by Störmer of the auroras

[1] *Popular Astronomy*, **25**, 274, 1916.

occurring in 1913 showed that they originated in a comparatively shallow layer at a great height above the earth,[1] whence we conclude that the sky light due to this source should vary directly as the thickness of the auroral layer in the direction of the line of sight, i.e., as the secant of the zenith distance. This holds only for zenith distances smaller than say 70°.

We may therefore write

$$B_{s',s}^{z,e}=a+p \sec z,$$

where a represents the constant quantity of zodiacal light and $p \sec z$ the auroral brightness. A least-squares solution of the data of Table IX gives

$$a=+0.076 \qquad p=+0.0113.$$

The values computed with these constants are given in the last column of Table IX.

Other solutions were also made, which allow for the deviation of the thickness of the auroral layer from the secant law. Taking these into consideration the final value is $a=+0.072$. This differs little from the value $+0.069$, adopted in section 5 in accordance with a provisional solution. It is to be noted that the constant quantity of zodiacal light thus determined, which is the remainder after applying the correction indicated above by α, has been computed on the hypothesis that the only source of light, except the zodiacal glow, is the aurora borealis, and that the illumination contributed by the aurora varies as the secant of the zenith distance.

8. DETERMINATION OF THE STARLIGHT FOR GALACTIC LATITUDES LESS THAN 40° FROM OBSERVATIONS ALONG CIRCLES PERPENDICULAR TO THE HORIZON

From the third column of Table IX it appears that the observed brightness corrected for starlight, excess of zodiacal light, and scattered earthlight is independent of the zenith distance down to $z=70°$. This fact has been used as follows to determine the starlight near the galaxy.

[1] *Astrophysical Journal*, 43, 243, 1916.

The fifteenth column of Table VIII gives the observed brightness at low galactic latitudes, corrected for scattered starlight, excess of zodiacal light, and scattered earthlight,[1] and thus includes exactly the same sources of illumination as the values in the fourteenth column, plus the direct starlight. It has been stated already that the quantities in this column are independent of the zenith distance down to $z=70°$. In order to find the starlight at lower galactic latitudes, we therefore need only diminish the figures in the fifteenth column by the average value of the fourteenth column for $z<70°$. This has been done in the case of Table VIII. The values of the starlight inserted in the sixteenth column have been corrected for atmospheric absorption and appear in the seventeenth column.

All series of measures perpendicular to the horizon have been treated in this way, except that in a certain number of cases the mean value of $B^{z; e}_{s', s}$ has been computed from all areas whose galactic latitudes exceed 30°, or even 15°, because the series in question do not include a sufficient number of observations in galactic latitudes greater than 40° to afford a reliable result. The starlight for galactic latitudes 15° to 40° thus required for the derivation of the values of $B^{z; e}_{s', s}$ was found from the series perpendicular to the horizon which do contain a sufficient number of observations between 15° and 40° as well as above 40°. The results are given in Table X.

TABLE X

Total Light per Square Degree of All Stars Fainter than Mag. 5.5 Derived from the Series of Observations Perpendicular to the Horizon

(Unit = a star of mag. 1.00, Harvard Visual Scale)

Limits b	Mean b	Starlight	Prob. Error
0° to 9°	4°5	0.081	±0.003
10 to 19	14	056	.002
20 to 29	24	.030	.001
30 to 39	35	0.016	±0.001

[1] The seventh column gives the observed brightness; in the eighth column this brightness has been corrected for scattered starlight; in the twelfth column the excess of zodiacal light has been subtracted and in the fifteenth a correction for scattered earthlight has been applied.

9. FINAL RESULT FOR THE AMOUNT OF STARLIGHT AS A FUNCTION OF THE GALACTIC LATITUDE. COMPARISON WITH OTHER AUTHORITIES. TOTAL AMOUNT OF STARLIGHT OVER THE WHOLE SKY

Comparing Tables VII and X we see that the values of the starlight derived from the series parallel and perpendicular to the horizon agree pretty well. The difference between the two series of results for the region of the Milky Way is at least partially real, for Easton's observations[1] give even larger values for the starlight of those galactic areas occurring in the series perpendicular to the horizon than do my own measures.

The results of Tables VII and X have been combined with equal weight in Table XI. The light of the stars brighter than magnitude 5.5, which is not included in my measures, has also been

TABLE XI

TOTAL AMOUNT OF STARLIGHT PER SQUARE DEGREE

(Unit=a star of mag. 1.00, Harvard Visual Scale)

Gal. Lat.	This Paper	Yntema	*Gron.*, 27	*Gron.*, 18
0°	0.080	0.088	0.055	0.186
10	.065	.055	.043	.118
20	.044	.033	.027	.055
30	.026	.024	.020	.032
40	.015	.020	.015	.022
50	.014	.018	.014	.018
60	.012	.015	.012	.015
70	.011	.013	.011	.013
80	.010	.013	.010	.012
90	0.010	0.012	0.010	0.012

added. In addition Yntema's values[2] and those computed by means of the number of stars in *Groningen Publications*, No. 27, Table V, and *Groningen Publications*, No. 18, are also given. The agreement with Yntema's values is good; but the enormous difference between Yntema and myself, on the one hand, and *Groningen Publications*, No. 18, on the other, shows that the magnitude scale

[1] *Verhandelingen der Koninklijke Akademie van Wetenschappen te Amsterdam*, Eerste sectie, deel VIII, No. 3, 1903.

[2] *Groningen Publications*, No. 22, p. 31, 1909.

used for the latter is probably in error. The same conclusion was arrived at from other evidence in *Groningen Publications*, No. 27.

The total amount of light received from all the stars in both hemispheres as computed from the present material is equal to 1440 stars of magnitude 1.00, Harvard Visual Scale.

10. ACCIDENTAL AND SYSTEMATIC ERRORS

The mean of two measures of the brightness of the sky, made by moving the lamp in opposite directions, may be considered as a single observation. Its probable error, computed from the differences between the values found for the same area, is found to be 0.7 per cent of the amount. This does not contain the uncertainty due to errors in the magnitude of the artificial star. Including these, the probable error of a complete measure is about 2 per cent of the amount.

It is practically certain that the relative brightness of different areas, as measured in the present paper, is free from systematic error. In addition I have taken all possible precautions to obtain the correct value for the absolute amount of light received from one square degree at the North Pole. Nevertheless my value differs considerably from that found by Newcomb and Burns, which is as follows:

TABLE XII

VALUES OF THE EXTRA GALACTIC SKY BRIGHTNESS FOUND BY DIFFERENT OBSERVERS

Authority	Brightness per Square Degree
Newcomb, *Astrophysical Journal*, **14**, 297	0.029
Burns, *ibid.*, **16**, 166	0.050
Abbot, *Astronomical Journal*, **27**, 20	0.075
Yntema, *Groningen Publications*, No. 22	0.140
van Rhijn, present paper	0.130

Burns does not give all the details necessary to check his method of measurement and reduction. I do not feel sure that his results are free from objection. He compares the brightness of the sky with the extra-focal image of a star of known magnitude, the sky being observed with one eye and the extra-focal image, as seen through the telescope, with the other. I do not think that accurate

results can be expected if, as in this case, the sources of brightness compared are not in contact. The internal agreement of Burns's observations is, moreover, rather poor.[1] This method seems, therefore, inferior to that used in the present paper.

Abbot and Yntema used the same instrument and method as I myself have used. In view of the difficulties affecting all kinds of photometric measurements, further determinations of the absolute amount of the sky-brightness are very desirable. I have thought of various methods which may be used, and hope in the near future to give some of them a trial.

GRONINGEN ASTRONOMICAL LABORATORY
September 1919

[1] *Op. cit.*

PHOTOGRAPHIC EFFECTIVE WAVE-LENGTHS OF NEBULAE AND CLUSTERS

SECOND PAPER

By KNUT LUNDMARK and BERTIL LINDBLAD

Since our first results concerning the photographic effective wave-lengths of some bright spiral nebulae and globular clusters were published,[1] exposures of nebulae with the twin Zeiss-Heyde astrograph ($a = 15$ cm) of the Observatory of Upsala have been continued during the years 1917 and 1918 in order to derive color-equivalents for a greater number of objects. The general principles of measurement and reduction are given in our first paper mentioned. The wire grating in front of the objective I is the same in the new investigation, but here we have also used objective II, supplied with a grating of somewhat larger constant. Though for fixed stars the images given by objective II are inferior in quality to those given by objective I, the former objective has yielded useful results here, especially for some spiral nebulae. The great advantage of getting two images in the same exposure need not be pointed out, when the exposures in question are extended over several hours; in some cases, for instance for the spiral nebula M 51 and large globular clusters, it is very valuable to be able to put the grating spectra of the two objectives into different angles of position in order to discriminate more easily between the spectra and details of the object situated far from the center. Generally the two gratings are placed with the threads in right angles to one another, in the directions of right ascension and declination respectively. A rather large number of those different kinds which are included in the common name of nebulae has been photographed, but for a part of these objects we have not been able to obtain measurable images. In the estimation of the probability of securing a certain object the magnitudes of Holetschek[2] have been very

[1] *Astrophysical Journal*, **46**, 206, 1917; *Astronomische Nachrichten*, **205**, 161, 1917.

[2] *Annalen der k.k. Universitäts-Sternwarte in Wien*, Band **20**, 1907.

useful, but the chief condition for a successful result is that the increase of brightness toward the center is rather rapid, except for those objects the surface brightness of which is very great. This is the case especially for some planetary nebulae, which give very definite results. Their effective wave-lengths are, moreover, very characteristic. Generally, however, an object with a well-defined nucleus, i.e., most nearly resembling a fixed star, gives the best results. In this way the nebula N.G.C. 5473, of magnitude $10^{M}_{.}7$ in Holetschek's table, is well measurable, while many objects of magnitude $<9^{M}$ are impossible with the same time of exposure. Further, the atmospheric conditions sometimes have not permitted us to expose an object for all the time which was thought necessary in our estimations; sometimes again the images have been perceptibly deformed, especially near the margins of the plate.

It is a great pleasure to us to express here our gratitude to Mr. Sten Asklöf and Mr. Axel Corlin for the great interest they have had in our work, especially by taking a very considerable part in the long and tiresome exposures.

Table I contains a list of the plates taken for our purpose. The first and second columns contain the date of the plate and the sidereal time of beginning and ending the exposure, the third contains the times of exposure, the fourth the observers, the fifth the objects photographed.

The constant of the grating in front of objective II was measured by Lundmark. The positions of ninety-seven consecutive threads were measured on the millimeter scale of the Hartmann microphotometer, in which the grating was put into the frame of plates. Groups of five threads were also measured in the Repsold measuring machine. The result was: $c = 1.5258$ mm. The focal length of the same objective was found by Lundmark to be 1478.7 mm. Thus the effective wave-length is given by the formula

$$\lambda_{\text{eff}} = 515.93 \cdot s\ \mu\mu.$$

where s is the distance, expressed in mm, between the centers of density of the spectra. For the grating of objective I, Lundmark got the result $c_{\text{I}} = 1.3413$ mm in good agreement with that previously derived, $c_{\text{II}} = 1.3422$. Because the second value has been

TABLE I

Date		Sidereal Time	Exposure	Observers	Object	Remarks
1917	Aug. 27	$21^h20^m.0$— $0^h44^m.9$	3^h24^m	Lk., Ld.	N.G.C. 598	Cloudy
	Sept. 11	21 50.1— 0 50.1	3 0	Lk., Ld.	598	Hazy
	18		5 0		6946	Sky good
			15			
	19	21 26.0— 1 29.5	45	Lk.	205, 221, 224	Sky good, strong wind
			2 20			
	24	19 56.5—20 12.5	5	Lk., Ld.	7662	
	24	22 57.0—23 46.5	49.3	Lk., Ld.	7027	
	Oct. 2	21 20.5—22 13.5	10			
			16	Lk.	6826	Sky good
			24			
	5	20 39.0—21 48.0	5			
			15	Lk., Cn.	7009	Unsteady in the air
			45			
	6	23 40.7— 2 5.7	5			
			19	Lk., Cn.	6503, 6543	
			2 0			
	8	0 27.7— 1 13.2	45.5	Lk., Ld., Cn.	650–651	
	11	21 3.1—23 47.6	2 44.5	Lk., Ld., Cn.	6229	Hazy at the end of the exposure
	15	21 3.1— 0 15.6	3 10	Lk., Ld.	6720	
	28	1 25.5— 2 38.0	3			
			5	Lk., Cn.	1514	The air unsteady at the end of the exposure
			15			
			45			
	Nov. 3	21 34.8— 1 29.8	2 15	Lk., Cn.	7023	
	11	1 48.0— 2 22.0	34	Lk., Ld., Af.	205, 221, 224	Clouds interrupted the exposure
	15	21 27.1— 5 27.1	8 0	Lk., Ld., Af., Cn.	598	
	18	21 43.2— 1 13.2	3 30	Lk., Cn.	7814	Air unsteady
		2 6.2— 5 6.2	3 0	Lk., Cn.	1068	Air unsteady
	20	23 19.3— 1 5.7	1 36.4	Lk., Ld., Af., Cn.	205, 221, 224	
	Dec. 4	22 32.9— 2 35.9	4 3	Lk., Ld., Af.	650–651	Sky good
	14	23 45.2— 0 30.2	45	Lk.	650–651	Sky very fine

1918	Jan.	6..	0 56.0— 3 57.0	3 1	Lk.	N.G.C. 628	Temperature −21° Celsius
		13..	2 48.4— 6 3.4	3 15	Lk., Af.	2403	
	Feb.	1..	3 0.8— 0 32.8	3 0	Lk., Af.	2245, 2261	The plates were taken for stellar-statistical purposes
				3 0			
		15..	4 43.1— 0 15.1	4 32	Lk., Ld., Cn.	3031, 3034, 3077	
		17..	6 24.4— 0 40.4	3 22	Lk., Ld.	2415	The plate taken for obtaining λ_{eff} of Wolf's object
		28..	7 4.4—10 4.4	3 0	Lk., Ld., Af., Cn.	2841	Sky very good
	Mar.	2..	6 43.8—13 43.8	7 0	Lk., Af.	3377, 3379, 3384, 3412	
		3..	6 30.8—10 0.8	3 30	Lk., Af., Cn.	2903, 5	Sky very fine
		5..	10 16.8—12 16.8	2 0	Lk., Ld., Af.	4736	
		6..	8 25.1—15 48.8	6 3	Lk., Ld., Af.	$\alpha = 12^h25^m$; $\delta = +13°$	
		7..	7 36.4—11 5.8	3 30	Lk., Ld., Af.	3623, 3627	
		9..	7 6.8—15 38.0	3 30		3587	
		10..	7 30.0—13 0.8	5 30	Lk., Af.	$\alpha = 12^h30^m$; $\delta = +12°$	
		17..	8 34.2—15 40.0	7 0	Lk.	3587	Sky very good
		23..	10 46.3—11 48.3	30	Lk., Ld.	4736	
				30			
	April	4..	10 57.7—11 20.2	31 5	Ld.	$\alpha = 12^h28^m$; $\delta = +27°.5$	
		11..	11 21.2—15 52.7	5 31	Lk., Bd., Cn.	5422, 5457–8, 5473, 5485, etc.	
		16..	11 4.0—16 35.0	4 31	Lk., Ld., Af., Cn.	$\alpha = 12^h28^m$; $\delta = +27°.5$	Hazy at the end of the exposure
				30			
		25..	13 17.4—14 17.7	15	Ld., Cn.	6205	
				5			
		26..	12 7.8—14 7.8	2 0	Lk., Ld.	6205	
	May	2..	12 50.0—16 19.0	3 20	Lk., Ld., Cn.	5194, 5195	
		3..	14 13.0—15 13.0	1 0	Lk., Cn.	6341	
			15 37.7—16 37.7	1 0		6218, 6254	

* Lk. = Lundmark; Ld. = Lindblad; Af. = Asklöf; Cn. = Corlin; Bd. = Bergstrand.

adopted for some previous work it has also been used here. Thus we get the wave-lengths for objective I, the focal length being 1485.5 mm, by the formula

$$\lambda_{\text{eff}}=451.74 \cdot s \; \mu\mu.$$

The values of λ_{eff} derived in this way need some corrections. At first there is a systematic difference between the results from objective I and objective II. We have found on an average from our measures on the nebulae,

$$\lambda_{\text{II}}-\lambda_{\text{I}}=7 \; \mu\mu.$$

The values for objective II are reduced to the system of objective I by applying the correction $-7\ \mu\mu$. This value of the correction is confirmed by measures on spectra of twenty fixed stars effected by Mr. Ossian Vallin. By reason of the difficulties of international communications during the war we have not been able to use here the sensitive Imperial plates employed for our first paper. Instead we used at first Hauff, Ultrarapid, size 9×12 cm; then we found Wellington Press plate better for our purpose on account of a somewhat greater sensitiveness. For most objects in this investigation we have therefore used this sort of plate, of the size 9×12 cm. For stars of late type the correction from Wellington to Imperial has been found to be about $-3\ \mu\mu$. The correction is smaller for early types; further, this value of the correction is valid only for a certain rather strong intensity of stellar images. By comparing the results below for the effective wave-lengths of faint images of Saturn, derived from a Wellington plate with the corresponding values for an Imperial plate in our previous paper, we find the average correction $-1.3\ \mu\mu$ for intensities of images nearly corresponding to those of the nebulae. The difference is larger for the stronger images. We have applied the correction $-1.3\mu\mu$ to wave-lengths measured on Wellington plates and exceeding 420 $\mu\mu$. Other values of the effective wave-lengths we have left unchanged. For Hauff plates the correction to the Imperial system thus is assumed to be negligible. A possible error introduced by this assumption cannot be of any serious importance.

A matter of importance in this connection is the variation of the effective wave-length with the intensity of image. We have

tried to re-examine this effect by taking several exposures of Mars and Saturn on the same plate; the results are shown in Table II. The faintest images of Mars are somewhat stronger than those of exposure 3 sec. of Saturn. The shortest exposures of Saturn correspond to very faint images, even fainter than most of the nebular images measured. The change of λ_{eff} with the intensity seems to be in agreement with the results derived by Lindblad[1]

TABLE II

PLATE TAKEN MARCH 23, 1917

SIDEREAL TIME 11^h57^m—12^h26^m

Saturn					Mars				
Exposure	Number of Images	λ_c Lundmark	λ_c Lindblad	λ_c Mean	Exposure	Number of Images	λ_c Lundmark	λ_c Lindblad	λ_c Mean
s		$\mu\mu$	$\mu\mu$	$\mu\mu$	s		$\mu\mu$	$\mu\mu$	$\mu\mu$
$\frac{1}{3}$	2	433.8	432.3	433.0					
1	3	34.5	32.9	33.7					
3	2	34.7	36.1	35.4	$\frac{1}{3}$	3	434.2	435.6	434.9
10	2	33.4	35.0	34.2	1	3	34.7	32.8	33.8
25	1	434.3	434.9	434.6	3	2	33.5	32.5	33.0
					10	2	32.9	31.2	32.1
					30	1	35.5	35.1	35.3
					60	1	438.1	444.9	441.5

for stars of late type. A minimum of the effective wave-length for a certain intensity is rather conspicuous for the two planets; on the whole the change of λ_{eff} is very small. For the early-type stars, however, with small effective wave-lengths, the effect has been found stronger. We have no sure direct experience of how the effect may appear for a nebula of low λ_{eff}; for the planetary nebulae in Table III, however, the effect does not seem to be very serious. For spiral nebulae and clusters the effective wave-lengths are on an average rather large; the intensities of image may always be assumed to lie between those corresponding to the exposures, $\frac{1}{3}$ sec. and 10 sec. of Saturn. Thus the effect may be neglected for those objects, and consequently we may without great error apply the relations found for faint stellar images between the effective

[1] *Arkiv för Matematik, Astronomi och Fysik*, **13**, No. 26, Stockholm, 1918.

wave-lengths on Imperial plates and the spectral type, provided only that the relation between the appearance of characteristic spectral lines and the distribution of intensity in the continuous spectrum is really the same for the nebulae as for the fixed stars. That such an assumption is legitimate, at least for the photographic part of the spectrum, is shown by the coincidence of our computed spectral types with those determined from the spectral lines by Fath and by Wolf. The relation between the effective wave-lengths for faint stellar images and the spectral types of the usual Harvard sequence are given below according to the results of Lindblad.[1]

Spectral type:	B	A	F	G	K	M
λ_{eff} :	408	413	420	428	433	444 $\mu\mu$

For the three last types the values of λ_c are valid for the "giant" series. For "dwarfs" the values come out some units smaller. The discontinuous spectra of some objects do not now permit us to apply the usual sequence of spectral types, thus to assign, for instance, the spectral type B or A to a planetary nebula. Such objects we refer to a common class of gaseous nebulae, type P.[2] The planetary nebulae we know to be intimately connected with the Wolf-Rayet stars, type O, which have nearly the same color as the helium stars, type B; it is rather interesting that the distribution of intensity in the discontinuous spectra of the planetaries really agrees with the color of these two spectral types. Another interesting fact is that the novae seem to be attached to this group of objects. The effective wave-length of Nova Aquilae 3 was found by Lundmark[3] to be very nearly that of the planetaries during the time when the continuous spectrum weakened and was successively overwhelmed by the bright lines.

Objects with effective wave-lengths below that of type A0 we may probably always refer to the nebular class P; for somewhat greater wave-lengths there may be confusion between planetary nebulae on one hand and clusters or spiral nebulae of early types

[1] *Loc. cit.*

[2] *Harvard Annals*, **76**, 20, 1916.

[3] *Astronomische Nachrichten*, **205**, 73, 1918.

on the other. Here we may, strictly speaking, only assign correct spectral notations if we know in advance the character of the object. Spiral nebulae of early type, however, are certainly very exceptional, the clusters probably not departing widely from type F; on the other hand the greatest effective wave-length for a planetary nebula measured is 416 $\mu\mu$. The annular nebula in Lyra, another type of gaseous nebulae, has an exceptionally small effective wave-length. The separation of different nebular types by the effective wave-length thus is rather good, and it seems as if we may with a fair precision predict the character of an object by means of its effective wave-length.

The results of the measurements are shown in Table III. The first three columns contain the name of the object, its right ascension and declination, the fourth the magnitude according to Holetschek, the next three columns the time of exposure, the date of the plate, and the sort of plates used (I = Imperial, H = Hauff, W = Wellington). The next following columns contain the corrected effective wave-length λ_c, derived from cameras I and II respectively. Here we have also taken up the values of λ_c in our first paper. Generally the nebulae are measured by both of us, and the differences between our values, Lk. − Ld., are tabulated in the columns headed $\Delta\lambda$. The next column contains the adopted value of λ_c, derived as a weighted mean of the values in the preceding columns, where the weights are given according to various circumstances, the intensity and symmetry of the images, and the magnitude of the differences $\Delta\lambda$. Then we have tabulated the relative weights assigned to the final values; they are estimated by considering the number and weights of the separate images used. The next column contains the color-indices in the system of *Harvard Annals*, 80, where the color-indices for types A and K are put equal to $0^{M}.00$ and $1^{M}.00$, respectively. The color-indices were computed with the spectral type in the sequence B–M as argument, derived from the effective wave-lengths as if the objects were all fixed stars. This color-index is thus a direct translation of λ_c into another color-system, but it does not mean that the color-indices for the objects would necessarily be found to be those in the table, if determined in the usual way. This conclusion is obvious for the

TABLE III

Object N.G.C.	α_{1900}	δ_{1900}	H	Exposure	Date	Sort of Plates†	λ_c I	$\Delta\lambda$ (Lk.−Ld.)	λ_c II	$\Delta\lambda$ (Lk.−Ld.)	Adopted	Weight	Color-Index	Spectrum Inferred	Spectrum Observed	Class of Objects
					da. mo. yr.		μμ	μμ	μμ	μμ	μμ					
221*	0^h37^m3	+40°19′	8^M7	140^m	19. 9. 17	I	432.4	+0.1	......	......	......	......	......	......	......	Spiral
				45	19. 9. 17	I	32.4	−0.5								
				15	19. 9. 17	I	31.5	+2.9								
				96.4	20. 11. 17	H	30.6	+0.8	......	......	432	5	$+0^M91$	G8		
224*	0 37.3	+40 44	5.0	140	19. 9. 17	I	30.4	+5.5								Spiral
				45	19. 9. 17	I	31.0									
				96.4	20. 11. 17	H	28.2	+0.3	......	......	430	2	+0.74	G4	G–K	
1068*	2 37.6	− 0 26	8.7	180	18. 11. 17	H	20.2	......	415.3	+1.0	420	1	+0.28	Fo	G	Spiral
1514*	4 3.0	+30 31	......	45	28. 10. 17	H	11.8	−2.7								
				15	28. 10. 17	H	7.7				410	2	−0.14	P		
2245*	6 27.2	+10 14	9.5	180	1. 2. 18	H	0.5				422	1	+0.34	F2	G?	Cometic
2261*	6 33.7	+ 8 49	9.7	180	1. 2. 18	H	21.6				422	1	+0.34	F2	G?	Cometic
2841*	9 15.1	+51 24	9.1	180	28. 2. 18	W	33.4	+1.1			433	3	+1.00	Ko	......	Spiral
2903*	20.5	+21 50	9.0	210	3. 3. 18	W	23.6	−0.1			424	3	+0.42	F5	......	Spiral
3031	47.3	+69 32	8.0	120	16. 2. 17	I	35.8	+0.9	......	......	......	......	......	......	......	Spiral
				274	22. 2. 17	I	35.7	+0.1								
				272	15. 2. 18	W	......	......	427.3	+3.4	433	3	+1.00	Ko	K	
3034*	47.6	+70 10	8.8	274	22. 2. 17	I	25.0	−2.0			......	......	......	......	......	Spiral?
				272	15. 2. 18	W	......	......	18.6	+2.6	422	2	+0.34	F2		
3077*	9 55.3	+69 13	10.2	274	22. 2. 17	I	28.6	+2.3								
				272	15. 2. 18	W	......	......	15.6	......	424	1	+0.42	F5		
3377*	10 42.4	+14 31	9.8	420	2. 3. 18	W	32.8	−1.4	......	......	433	3	+1.00	Ko		
3379*	42.6	+13 6	9.1	420	2. 3. 18	W	32.1	+1.3	......	......	432	3	+0.91	G8		
3384*	43.0	+13 9	9.5	420	2. 3. 18	W	28.5	+3.3	......	......	428	3	+0.56	Go		
3412*	10 45.6	+13 56	9.5	420	2. 3. 18	W	26.8	−0.1	......	......	427	3	+0.53	F9		
3587*	11 9.0	+55 34	9.1	420	17. 3. 18	W	......	......	00.3	−2.8	......	2	......	......	......	Planetary
				210	9. 3. 18	W	10.1	+1.9	......	......	410	2	−0.14	O?		
Anonyma*	9.2	+55 31	13	420	17. 3. 18	W	14.3	+3.3	12.0	......	414	2	+0.05	O?	......	Star with nebulous rays
3613	12.8	+58 33	10.5	420	17. 3. 18	W	......	......	30.2	−3.6	439	2	+1.20	K6		
3619	13.6	+58 18	11.0	420	17. 3. 18	W	......	......	39.1	−4.9	439	1	+1.20	K6		
3623*	13.7	+13 38	8.9	210	7. 3. 18	W	34.0	−2.2	33.7	−0.5	434	4	+1.03	K1	......	Spiral
3627*	11 15.0	+13 32	8.6	210	7. 3. 18	W	20.7	−0.1	20.8	+1.6	421	4	+0.31	F1	......	Spiral
4251	12 13.1	+28 44	9.5	271	11. 4. 18	W	29.4	......	29.9	0.0	430	1	+0.74	G4		
4278	15.1	+29 50	9.6	271	11. 4. 18	W	25.0	−0.6	21.1	+0.5	424	2	+0.42	F5		
4494	26.4	+26 20	9.2	271	11. 4. 18	W	28.3	−1.5	428.2	−3.4	428	2	+0.56	Go		
4725	45.5	+26 3	8.7	100, 100	19. 4. 17	I	30.4	+3.3	......	......	430	1	+0.74	G4	G	Spiral

4736*	12 46.2	+41 40	7.7	105	15. 3. 17	I	26.7	+1.2	......	......	427	1	+0M53	F9	......	Spiral
				105	15. 3. 17	I	32.0	+0.2	......	......	433	1	+1.00	K0		
				120	5. 3. 18	W	31.5	+3.6								
				30	23. 3. 18	I	33.0									
				30	23. 3. 18	I	431.3	......	......	......	432	4	+0.91	G8	G	
4826	12 51.8	+22 13	8.6	100	19. 4. 19	I	433.8	−0.1	......	......	434	1	+1.03	K1	G	Spiral
				100												
5194	13 25.7	+47 43	8.4	180	27. 3. 19	I	30.7	+2.0								
						W	35.2	−3.6	410.0	−4.6	436	2	+1.11	K3	K	
5195	25.8	+47 47	8.6	180	27. 3. 19	I	26.3	0.0	......	......						Spiral
						W	32.5	−3.2	35.0	−1.8	431	3	+0.82	G6		
5272	37.6	+28 53	......	......	......	I	25.1	+0.3			425	1	+0.15	F6	A G	Globular cluster
5423	57.3	+55 39	10.2	300	11. 4. 18	I	......	......	26.7	−3.6	427	1	+0.53	F9		
5457	13 59.7	+54 50	9.2	300	11. 4. 18	I	30.0	+2.3	32.0	......	431	2	+0.82	G6	......	Spiral
5471	14 0.0	+54 53	......	300	11. 4. 18	I	30.5	−2.0	......	......	431	1	+0.82	G6		
5473	14 1.2	+55 23	10.7	300	11. 4. 18	I	28.4	−2.2	429.2	−4.6	429	3	+0.64	G2		
5485	14 16.6	+57 11	11.0	300	11. 4. 18	I	30.3	−3.7	......	......	430	2	+0.74	G4		
5904	15 13.5	+ 2 27	6.7	120	11. 5. 17	I	19.1	−0.0	......	......	419	1	+0.25	A9	A G	Globular cluster
6229*	16 44.2	+47 42	8.6	164.5	11. 10. 17	H	15.0	......	......	......	415	1	+0.08	A3	A G	Globular cluster
6341*	17 14.1	+43 15	6.2	60	3. 5. 18	W	15.7	+2.0	......	......	416	2	+0.11	A4	A G	Globular cluster
				60	3. 5. 18	W	16.5	−1.7	......	......	417	1	+0.17	A6		
6543	17 58.6	+66 38	7.6	120	6. 10. 17	H	12.0	−0.9	......	......	......	......	......	......	......	Planetary
				10	6. 10. 17	H	08.6	+1.8								
				5	6. 10. 17	H	412.2	−2.0	......	......	412	4	−0.05	P	P	
6720*	18 49.8	+32 54	8.9	90	25. 4. 17	I	394.0	0.0	......	......	......	......	......	......	......	Planetary
					15. 10. 18	H	397.6	−3.0	397.8	−7.3	397	1	......	P	P	Planetary
6826	19 42.1	+50 17	8.1	21	2. 10. 17	H	417.0	−2.4								
				16	2. 10. 17	H	14.6	+0.5	......	......	416	4	+0.11	P	P	Planetary
7009*	20 58.7	−11 46	7.2	45	5. 10. 17	H	12.6	+0.4	409.3	−0.3	411	4	−0.10	P	P	Planetary
7027	21 33	+41 50	8.5	49.3	24. 9. 17	H	11.5	−2.1	409.3	−0.5	411	1	−0.10	P	P	Planetary
7662	23 21.1	+41 59	7.6	51	24. 9. 17	H	18.5	−0.5								
				15	24. 9. 17	H	413.3	0.0	......	......	415	4	+0.08	P	P	Planetary

* See corresponding note at end of paper.

† I = Imperial; H = Hauff; W = Wellington.

objects with discontinuous spectra, but according to the results of Hertzsprung[1] as to the distribution of intensity in the spectrum of M 3 it seems that we must make this restriction also for the color-estimations of composite systems such as star-clusters and, perhaps, spiral nebulae. It seems that we must determine color and spectral type from the same spectral region in order to get the proper correspondence. However, an indication of a more extended validity for the computed color-indices of spirals and planetaries may be found in the very interesting results of F. H. Seares,[2] derived by comparison of ordinary photographs with those taken on isochromatic plates with yellow filter. The last three columns contain the spectrum calculated according to the principles set forth in the preceding, the spectrum observed by Fath and by Wolf, and the observed character of the object. The objects marked with an asterisk have a corresponding note at the end of the paper. There are also some notes for objects photographed which have not yielded satisfactory results.

The agreement between the observed spectra of spiral nebulae derived by Fath and by Wolf and those computed from the effective wave-lengths by applying the relations found for fixed stars is confirmed by the new results in this paper. According to an investigation of Lundmark, which will soon appear in print,[3] the probable parallax of the great nebula in Andromeda, the second object in Table III, is

$$\pi = 0\overset{''}{.}0000057 .$$

Because other spiral nebulae are probably at still greater distances, our results seem to confirm the opinion of Shapley that no sensible absorption exists in space. Even if we assume an improbable displacement of one interval in the spectral classification between the spectrum observed and computed, corresponding to $0\overset{M}{.}33$ in color-index, we should have the coefficient of absorption d, the change in color-index for one parsec:

$$d \leq 0\overset{M}{.}00000019 .$$

[1] *Astrophysical Journal*, **41**, 10, 1915.

[2] *Proceedings of the National Academy of Sciences*, **2**, 553, 1916.

[3] Will appear in *Astronomische Nachrichten*, **209**, 369, 1919.

The list of possible objects for our instrumental means is not exhausted by the objects of Table III, and therefore investigations on the subject will be continued at the Observatory of Upsala.

NOTES

N.G.C. 221. Spiral[1] with very sharp nucleus. The accuracy of the measurements for this nebula cannot be estimated less than for an average fixed star. The four values of the effective wave-length obtained here for different intensities of image confirm our conclusion that the photographic Purkinje-effect is of quite subordinate importance for relatively "red" objects.

N.G.C. 224. The great nebula in Andromeda. By reason of the sharp nucleus tolerably distinct grating spectra were obtained, and consequently the value of λ_c is valid for the center of the nebula.

N.G.C. 598. On the plate of 11. 9. 17, exposure 180^m, there are very weak spectra of the most luminous part of the nebula (*Yerkes Publ.*, **2**, Plate XXV, $X=+34$ mm, $Y=-28$ mm), but the measurements are very uncertain and have not been adopted in Table III. The plate of 15. 11. 17, exposure 480^m, has a strong veil; the spectra are exceedingly faint.

N.G.C. 628. Though this nebula has a sharp nucleus, the time of exposure, 181^m, did not suffice to give a measurable image.

N.G.C. 650–651. We have not obtained measurable images with an exposure of 243^m.

N.G.C. 1068. According to Fath the spectrum of this nebula shows bright emission lines. The relatively low value of λ_c obtained here may possibly be explained by the assumption that the center of intensity in the photographic spectrum is displaced toward the value 413 $\mu\mu$, the mean value of λ_c for the planetary nebulae measured here. In the same way the low wave-lengths obtained for N.G.C. 2903 and 3627 may perhaps be caused by an appearance of bright lines in their spectra.

N.G.C. 1514. This nebula is a very remarkable object. It was classified by Herschel as a planetary, but on Mr. Isaac Roberts' photographs[2] it has been found to have spiral structure. The value of λ_c shows that very probably we have to do with a planetary nebula.

N.G.C. 2245, 2261, 2245 resembles a star and shows nebulous rays. 2261 is Hubble's variable nebula.[3] The wave-lengths measured are in both cases valid for the nuclei of the objects.

N.G.C. 2403. This nebula has small surface intensity and resembles N.G.C. 598 as to its structure. The time of exposure, 195^m, is too small to give measurable images.

[1] A. R. Hinks, *Monthly Notices*, **71**, 588, 1911.

[2] *Monthly Notices*, **74**, 234, 1914. [3] *Astrophysical Journal*, **44**, 190, 1916.

N.G.C. 2841. The measures on the plate taken with objective I are very sure. Because there is an irregularity in the image given by objective II and, in contrast to what is usually the case, the wave-length falls out lower than on the plate of objective I, it has not been used in deriving the definitive value of λ_c.

N.G.C. 2903, 2905. The nebula 2905, which is a condensation in one of the spiral arms of 2903, is not measurable. A somewhat longer exposure, however, would certainly make it possible to determine its wave-length and thus to derive the difference in color between the two objects.

N.G.C. 2976. This nebula, which consists of a round nebulous patch with small surface intensity, is found on the plates containing 3031, 3034, and 3037, but only the central image is perceptible. The total intensity is estimated by Kritzinger to $10^{\mathrm{M}}.0$ and by Holetschek to $10^{\mathrm{M}}.7$.

N.G.C. 3034. On account of an error of reduction in our first paper the wave-length of the nebula had been given to be 415 $\mu\mu$ instead of 425 $\mu\mu$, which is the right value according to the measurements. However those measurements were rather uncertain, and the value here given must be considered better, though it is subject to some uncertainty due to the faintness and irregular form of the object.

N.G.C. 3077. The difference between our new and old determinations is considerable. None of the measurements are very sure, but a greater weight ought to be ascribed to the value from our first paper.

N.G.C. 3377, 3379, 3384, 3412. These nebulae are reproduced on the Wolf-Palisa chart No. 109. Though it is impossible to discern their structure plainly, they seem to be small spirals, which also follows from our values of their effective wave-lengths. See also *Mount Wilson Report for 1917*, p. 214.

N.G.C. 3587.[1] If we refer the central star to type O, which is a manner of interpreting the low effective wave-length, and assume its absolute magnitude to be $-1^{\mathrm{M}}.0$, according to the mean value for O-type stars in Kapteyn's, Hertzsprung's, Gyllenberg's, and van Maanen's investigations, we get the parallax $\pi_{\mathrm{M}97} = 0''.0001$. If we make the same assumption for the object: R.A. $11^{\mathrm{h}}9^{\mathrm{m}}.2$, Decl. $+55^{\circ}31'$, which is a star connected to nebulous masses, we get a hypothetical parallax of the same order of magnitude. The similarity of the appearance of the latter object[2] with the present appearance of Nova Persei[3] makes it possible that the nebulous star may be a late nova, and thus probably a star with the characteristics of type O.

N.G.C. 3623, 3627. Spirals with well-exposed spectra. The difference in effective wave-length obtained by us is confirmed by the circumstance that while according to Holetschek 3627 is $0^{\mathrm{M}}.3$ brighter visually than 3623, the difference in photographic magnitude is much greater. Thus we find from

[1] *Monthly Notices*, **67**, 543, 1907.

[2] See Ritchey, *Astrophysical Journal*, **32**, 26, 1910; *Mt. Wilson Contr.*, **2**, 283, 1910, Plate XVII.

[3] *Publications of the Astronomical Society of the Pacific*, **30**, 163, 1918.

estimations performed by means of the grating spectra the photographic difference

$$M_{3623}-M_{3627}=+0\overset{M}{.}7$$

and consequently a color-index about $\frac{1}{2}^{M}$, which is in tolerably good agreement with the result $0\overset{M}{.}7$ obtained from the color-indices in Table III.

N.G.C. 4736. The first and second values in the table are taken from our first paper and are valid for the nebula in its entirety and the central nucleus respectively. For the new images measured such a discrimination has not been possible.

N.G.C. 6205. The short exposures of this cluster give too faint and diffuse images of the condensations measurable in our previous paper. For the long exposure, 120^{m}, spectra and central image of the entire cluster begin to overlap.

N.G.C. 6229. This object has sometimes been classified as a planetary nebula, sometimes as a globular cluster. Shapley's investigations have shown that the object is a dense globular cluster containing about 1500 stars. The grating spectra on our plate are very weak and the measurements rather uncertain.

N.G.C. 6341. This cluster has given rather distinct and easily measurable spectra. The first value in Table III refers to the cluster in its entirety, the second to a conspicuous condensation situated in the northern edge of the cluster.

N.G.C. 6503. Irregular nebula. Not measurable on our plate, exposed 120^{m}.

N.G.C. 6720. Though our measurements on different plates give rather concordant results, they must be considered very uncertain. It is evident, however, that the nebula gives a lower wave-length than any other measured by us. The stars *c* and *d* mentioned by Burns,[1] with the spectral types A and F respectively, are measurable and gave the accordant wave-lengths 417.5 $\mu\mu$ and 421.4 $\mu\mu$.

N.G.C. 6946. On account of an erroneous statement as to the magnitude of Nova Ritchey, 1917, this nebula was exposed 300 minutes. The nebula has very small surface intensity and seems to be of the same type of spirals as N.G.C. 598 and 2403. Not the faintest grating spectra are to be found on our plate.

N.G.C. 7009. The great value of the zenith-correction makes the value of λ_c somewhat uncertain.

N.G.C. 7023. This irregular nebula envelops the star[2] B.D. $67\overset{\circ}{.}1283$. On our plate there is a faint trace of the nebula but not sufficiently strong for a determination of λ_c. The star is overexposed, and therefore we have still no value of its effective wave-length.

N.G.C. 7814. With an exposure of 210 minutes the spectra are too weak to be measured with any accuracy.

[1] *Lick Observatory Bulletin*, No. 193, 1911.

[2] *Astronomische Nachrichten*, **204**, 41, 1917.

SUMMARY

Photographic effective wave-lengths of nebulae and clusters, in all 44 objects, have been obtained, whereby it has become evident that different kinds of objects correspond to certain characteristic intervals of wave-length. Thus the planetary nebulae have wave-lengths below 416 $\mu\mu$, and the mean value for six objects, excluding the Owl nebula and the Ring nebula in Lyra, is 413 $\mu\mu$. The four globular clusters investigated have wave-lengths ranging between 415 $\mu\mu$ and 425 $\mu\mu$ and the mean value is 419 $\mu\mu$, corresponding to the spectral type A8. Fourteen nebulae known as spirals give the mean value 429 $\mu\mu$, spectral type G2; the values of λ_c range between 420 $\mu\mu$ and 436 $\mu\mu$. Several objects in Table III may with great probability be referred to the spiral class by considering the values of their effective wave-lengths.

From the effective wave-lengths we have computed spectral types and color-indices, using the relations found for fixed stars. For clusters and spiral nebulae the convertion of λ_c into a spectral type of the usual sequence B–M seems to be legitimate; on the other hand we have seen that a low value of λ_c for a nebulous object indicates with a fair probability the character of a planetary nebula, a fact which may be expressed by assigning the nebular spectral class P to such objects.

From the facts mentioned we may draw the conclusion that at least a rigorous discrimination between planetary and spiral nebulae may easily be effected by measuring photographic effective wave-lengths, a circumstance which may be of importance for the future classification of faint nebulae.

ASTRONOMICAL OBSERVATORY
UPSALA
October 10, 1919

THE ORBIT OF THE SPECTROSCOPIC BINARY BOSS 2285[1]

By ALFRED H. JOY AND GIORGIO ABETTI

The variable velocity of the star Boss 2285 = Pi 8^h108 ($\alpha = 8^h30^m.5$, $\delta = +6°58'$; 1900.0) was discovered at this observatory from plates taken in 1918.[2] With Boss 2286 = Lal. 16895 it forms the physical system β G.C. 4677 = Σ 1245. The measures of position angle and distance show no change since the time of the observations by Dembowski. The lack of relative motion is confirmed by van Maanen from measures of the two components on his parallax plates which give: 1916.23, P.A. 25°.0, dist. 10″.49. The Harvard apparent magnitudes, the proper motions by Boss, the Mount Wilson spectral types, and the absolute parallaxes with corresponding absolute magnitudes by van Maanen are:

	Harvard M	μ	Spec.	π	M
Boss 2285 . . .	6.04	0″.195	F7	+0″.022	+2.8
Boss 2286 . . .	7.15	0.201	G3	0.026	4.2

The absolute magnitudes determined by Adams and Joy from the spectra are 4.5 and 5.5, which would give a mean parallax of 0″.048. Measures of the four plates of Boss 2286 give radial velocities ranging from +23.4 km to +31.0 km with a mean of +27.9 km which may be compared with the velocity of +25.2 km found in this investigation for the center of mass of the system Boss 2285. The spectrum of the secondary star is not visible on any of the plates.

Table I gives a list of the photographs. The Greenwich mean time refers to the middle of the exposure; the phase is derived from the period 14.296 days and refers to the epoch J.D. 2421598.0. The plates have been taken with the 60-inch reflector and the 18-inch single-prism spectrograph. The various plates have been

[1] *Contributions from the Mount Wilson Observatory*, No. 172.

[2] *Publ. Astr. Soc. of the Pacific*, **31**, 41, 1919.

measured by Adams, Joy, Abetti, Strömberg, and Miss Burwell, and the number in the table indicates the number of measures of each plate. No systematic difference between the measures was found, so that the mean result has been used in the table.

TABLE I

Plate No.	Date	G.M.T.	Phase	Velocity	No. of Meas.	O−C	Remarks
				km		km	
7853.....	1919, Feb. 8	18^h40^m	0^d490	− 0.4	3	+0.1	
5770.....	1917, May 6	15 42	0.686	− 2.0	4	+1.8	
6818.....	1918, Mar. 31	17 41	0.961	− 3.3	4	−2.1	
6870.....	1918, Apr. 29	16 51	1.334	+ 3.3	3	+1.6	
7936.....	1919, Mar. 10	18 26	1.888	+ 6.5	3	−2.6	
6886.....	1918, Apr. 30	17 22	2.356	+16.5	3	−0.1	
7943.....	1919, Mar. 11	17 25	2.846	+25.9	3	+1.6	
7944.....	1919, Mar. 11	17 52	2.864	+25.0	3	+0.5	
6575.....	1918, Jan. 21	21 22	3.594	+35.4	3	+2.0	
6580.....	1918, Jan. 22	20 41	4.565	+38.5	3	−1.7	
7864.....	1919, Feb. 12	20 55	4.584	+40.7	3	+0.4	
7772.....	1919, Jan. 15	21 20	5.199	+47.9	4	....	Weak
6597.....	1918, Jan. 23	21 16	5.590	+41.7	5	−1.3	
7786.....	1919, Jan. 17	22 28	7.240	+44.5	4	+2.6	
6504.....	1917, Dec. 29	21 40	9.199	+32.8	4	−2.5	
6615.....	1918, Jan. 27	20 17	9.549	+35.4	3	+1.8	
7975.....	1919, Mar. 18	16 03	9.789	+29.9	3	−2.5	
6515.....	1917, Dec. 30	23 47	10.287	+32.2	4	+2.5	
6925.....	1918, May 23	15 59	11.002	+16.9	3	....	7-in. camera
7914.....	1919, Feb. 19	20 17	11.557	+20.3	4	−1.0	
6659.....	1918, Jan. 30	19 10	12.503	+15.2	3	+1.5	
6799.....	1918, Mar. 29	17 29	13.248	+ 6.7	4	−0.6	
6744.....	1918, Mar. 15	19 30	13.628	+ 2.9	4	−1.1	

The velocity-curve was first drawn through all the observations with the aid of a preliminary period determined from a comparison of the times of recurrence of approximately equal velocities. Since the eccentricity was shown to be considerable, the method of Lehmann-Filhés was used for determining the approximate elements of the orbit, which were then slightly adjusted so as to make the residuals as small as possible. Plate 7772, which was very weak, and 6925, which was taken with the 7-inch camera, showed rather large differences and were omitted in the final solution and are not shown in the figure. A least-squares solution, with twenty-one equations of condition of the form given by Lehmann-Filhés and Schlesinger, was used for improving the approximate elements. The following corrections were found:

$$\begin{aligned} \delta P &= +0.005 \text{ days} \\ \delta K &= -0.26 \text{ km} \\ \delta\omega &= -13^\circ.00 \\ \delta e &= +0.006 \\ \delta T &= -0.526 \text{ days} \\ \delta\gamma &= +0.03 \text{ km} \end{aligned}$$

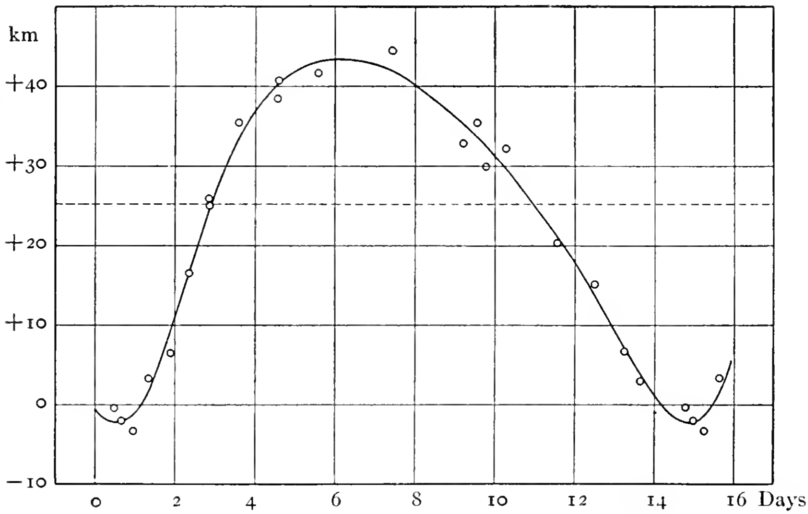


FIG. 1.—Velocity-curve of Boss 2285

After applying these corrections, we have for the final elements:

$$\begin{aligned} P &= 14.296 \pm 0.003 \text{ days} \\ K &= 22.74 \pm 1.20 \text{ km} \\ \omega &= 220^\circ.80 \pm 1^\circ.98 \\ e &= 0.276 \pm 0.021 \\ T &= \text{J.D. } 2421599.474 \pm 0.157 = 1918, \text{ Jan. } 5, 11^h 22^m \text{ G.M.T.} \\ \gamma &= +24.23 \pm 0.62 \text{ km} \\ a \sin i &= 4{,}300{,}000 \text{ km} \\ \frac{m_1^3 \sin^3 i}{(m+m_1)^2} &= 0.015 \odot \end{aligned}$$

The residuals O–C given in Table I were computed with these elements. The computed curve and the observed velocities are shown in Fig. 1.

MOUNT WILSON OBSERVATORY
April 1919

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